Section Notes 7

Covering material from Lecture on February 2^{nd}

<u>Class Outline</u>

1. Constrained Optimization
2. The Cobb-Douglas Utility Function

1 Constrained Optimization

With the assumption of diminishing MRS, we have shown how to find the bundle of goods that optimizes a consumer's utility, but there's a more formal way of getting it done. Assume a person's preferences can be represented by the following utility function: $U(X, Y)$, where they have an income of I, and the prices of the two goods are given. Here, the **constraint** is essentially given by income. Formally, we can set up the problem as:

$$\max_{X,Y} \quad U(X,Y), \qquad \text{s.t. } P_X X + P_Y Y = I \tag{1}$$

We solve this problem by building the constraint into the problem and creating a single function, which is referred to as the **Lagrangian**. We can then maximize this single function over the three variables in the standard way, or:

$$\Phi(X, Y, \lambda) = U(X, Y) - \lambda(P_X X + P_Y Y - I) \tag{2}$$

Notice, this gives us a system of three equations and three unknowns, so that we can solve. Also, notice, that we can get the condition that we saw two sections ago with relative prices and the MRS.

Problem: (P&R Chapter 4, Appendix, Exercise 5)
Maurice has the following utility function:

$$U(X,Y) = 20X + 80Y - X^2 - 2Y^2$$

where X is his consumption of CDs with a price of \$1 and Y is his consumption of movie videos, with a rental price of \$2. He plans to spend \$41 on both forms of entertainment. Determine the number of CDs and video rentals that will maximize Maurice's utility.

Question: Is there only a single utility function for any set of preferrences?

Problem: (P&R Chapter 4, Appendix, Exercise 2)
Show that the two utility functions given below generate identical demand functions for goods X and Y:

a. $U(X,Y) = \log(X) + \log(Y)$
b. $U(X,Y) = (XY)^{.5}$

What's the value of recognizing this when maximizing utility?

2 The Cobb-Douglas Utility Function

The Cobb-Douglas Utility function is a particularly useful one because of it's nice properties. We will simply introduce it now, but will come back to it again when we move from consumers onto producers.

Cobb-Douglas Utility Function: Utility function of the form $U(X, Y) = X^{\alpha}Y^{(1-\alpha)}$, where $0 < \alpha < 1$.

Problem: (P&R Chapter 4, Appendix, Exercise 4)
Sharon has the following utility function:

$$U(X, Y) = \sqrt{X} + \sqrt{Y}$$

where X is her consumption of candy bars, with price $P_X = \$1$, and Y is her consumption of espressos, with $P_Y = \$3$.

a. Derive Sharon's demand for candy bars and espresso.

b. Assume that her income $I = \$100$. How many candy bars and how many espressos will Sharon consume?

c. What is the marginal utility of income?