Lecture 20

Interest Rates, Investments, and Capital Markets

Key issues

1. comparing money today to money in the future: interest rates
2. choices over time: invest in a project if return from investment > return on best alternative

Capital and durable goods

- *durable goods*: products that are usable for years
- if durable good or capital is rented, rent up to the point where the marginal benefit = *MC*
- if bought or built rather than rented, firm compares current cost of capital to future higher profits it will make from using capital

Interest rates

- assume no inflation: consuming $1 worth of candy today is better than consuming $1 worth in 10 years
- how much more you must pay in future to repay a loan today is specified by an *interest rate*:

    percentage more that must be repaid to borrow money for a fixed period of time

"Look at it this way—this is the first day of the rest of your money."

General compounding formula

Frequency of compounding

- for a given *i*, more frequent compounding, greater payment at end of a year
- annual interest rate is $i = 4\%$
- if bank pays interest 2 times a year,
  - half a year's interest, $i/2 = 2\%$, after six month:

    $$\$(1 + i/2) = \$1.02$$

  - at end of year, bank owes:

    $$\$(1 + i/2) \times (1 + i/2) = \$(1 + i/2)^2 = \$(1.02)^2 = \$1.0404$$

Interest rates connect present and future

- future value (*FV*) depends on the present value (*PV*), the interest rate, and the number of years
- put *PV* dollars in bank today and allow interest to compound for *t* years:

$$FV = PV \times (1 + i)^t$$

Present value

- 2 equivalent questions:
  - how much is \$1 in the future worth today?
  - how much money, *PV*, must we put in bank today at *i* to get a specific *FV* at some future time?
- answer:

$$PV = FV/(1 + i)^t$$

Example

When is future money nearly worthless?

- at high interest rates, money in future is virtually worthless today
- $1 paid to you in 25 years is worth only 1¢ today at a 20% interest rate

Stream of payments forever

Stream of payments for *t* years

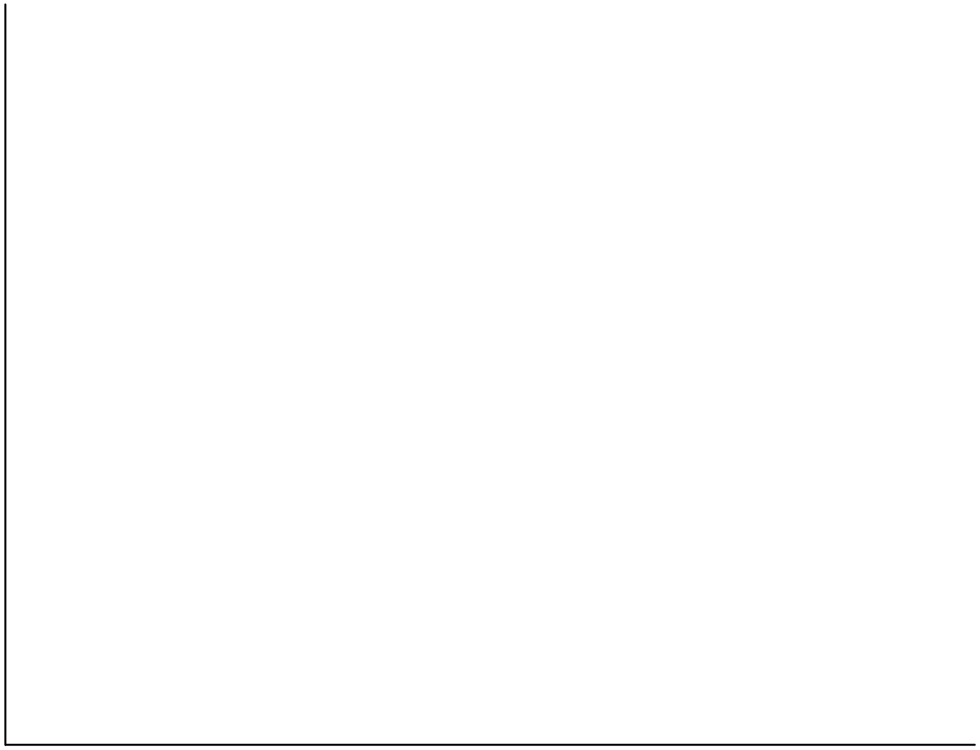
- What's *PV of* payments per period of f made every year?
- you agree to pay \$10 at end of each year for 3 years to repay a debt (i = 10%)

$$PV = \$10/1.1^1 + \$10/1.1^2 + \$10/1.1^3 \approx \$24.87$$

- generally:

$$PV = f\left[\frac{1}{(1+i)^1} + \frac{1}{(1+i)^2} + ... + \frac{1}{(1+i)^t}\right]$$

Present Value of a Dollar in the Future



Future value of payments over time

- What's FV after t years if you save f each year?
- year 1: put f in account
- year 2: add a second f, so you have first year's payment + accumulated interest of $f(1 + i)^1$ or $f[1 + (1 + i)^1]$ in total
- year 3: total is $f[1 + (1 + i) + (1 + i)^2]$
- after t years:

$$FV = f[1 + (1+i)^1 + (1+i)^2 + \ldots + (1+i)^{t-1}]$$

Inflation and discounting

- we've been assuming inflation rate = 0%
- suppose general inflation occurs: nominal prices rise at a constant rate γ over time
- by adjusting for rate of inflation, we convert nominal prices to real prices

Adjusting for inflation

Nominal and real rates of interest

- to calculate *PV* of this future real payment, we discount using real interest rate
- without inflation, $1 today is worth $1 + i$ next year
- with inflation rate of γ, $1 today is worth

  $(1 + i)(1 + \gamma)$ nominal dollars tomorrow
- if $i = 5\%$ and $\gamma = 10\%$, $1 today is worth 1.05x1.1 = 1.155 nominal dollars next year

Nominal vs. real interest rates

Real interest rate

- depends on inflation and nominal rate

$$i = \frac{\tilde{i} - \gamma}{1 + \gamma}$$

- if inflation rate is small $\gamma \approx 0$, then we can closely approximate the real rate by $i = \tilde{i} - \gamma$
- if nominal rate is 15.5%, γ = 10%, real rate is (15.5%-10%)/1.1=5% and approximation is 5.5%

Real present value

Investment decision

1. net present value approach
2. internal rate of return approach

Net present value approach

depends on PV of revenues, R, and cost, C

$$NPV = R - C$$

$$= \left[R_0 + \frac{R_1}{(1+i)^1} + \frac{R_2}{(1+i)^2} + \ldots + \frac{R_T}{(1+i)^T} \right]$$

$$- \left[C_0 + \frac{C_1}{(1+i)^1} + \frac{C_2}{(1+i)^2} + \ldots + \frac{C_T}{(1+i)^T} \right]$$

$$NPV = \left[R_0 - C_0 + \frac{R_1 - C_1}{(1+i)^1} + \frac{R_2 - C_2}{(1+i)^2} + \ldots + \frac{R_T - C_T}{(1+i)^T} \right]$$

$$= \left[\pi_0 + \frac{\pi_1}{(1+i)^1} + \frac{\pi_2}{(1+i)^2} + \ldots + \frac{\pi_T}{(1+i)^T} \right]$$

NPV rules

- invest if *NPV* > 0
- it isn't necessary for cash flow in each year, π_t (loosely, annual profit), to be positive

Internal rate of return approach

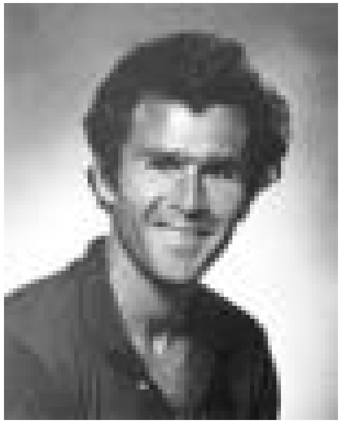
- at what discount rate (rate of return) is firm indifferent between making investment and not?
- *internal rate of return* (*irr*) is discount rate where *NPV* = 0
- replacing *i* with *irr* and setting *NPV* = 0, find *irr* by solving for *iir*

$$NPV = \left[\pi_0 + \frac{\pi_1}{(1+irr)^1} + \frac{\pi_2}{(1+irr)^2} + \ldots + \frac{\pi_T}{(1+irr)^T} \right] = 0$$

IRR for flow

Human capital

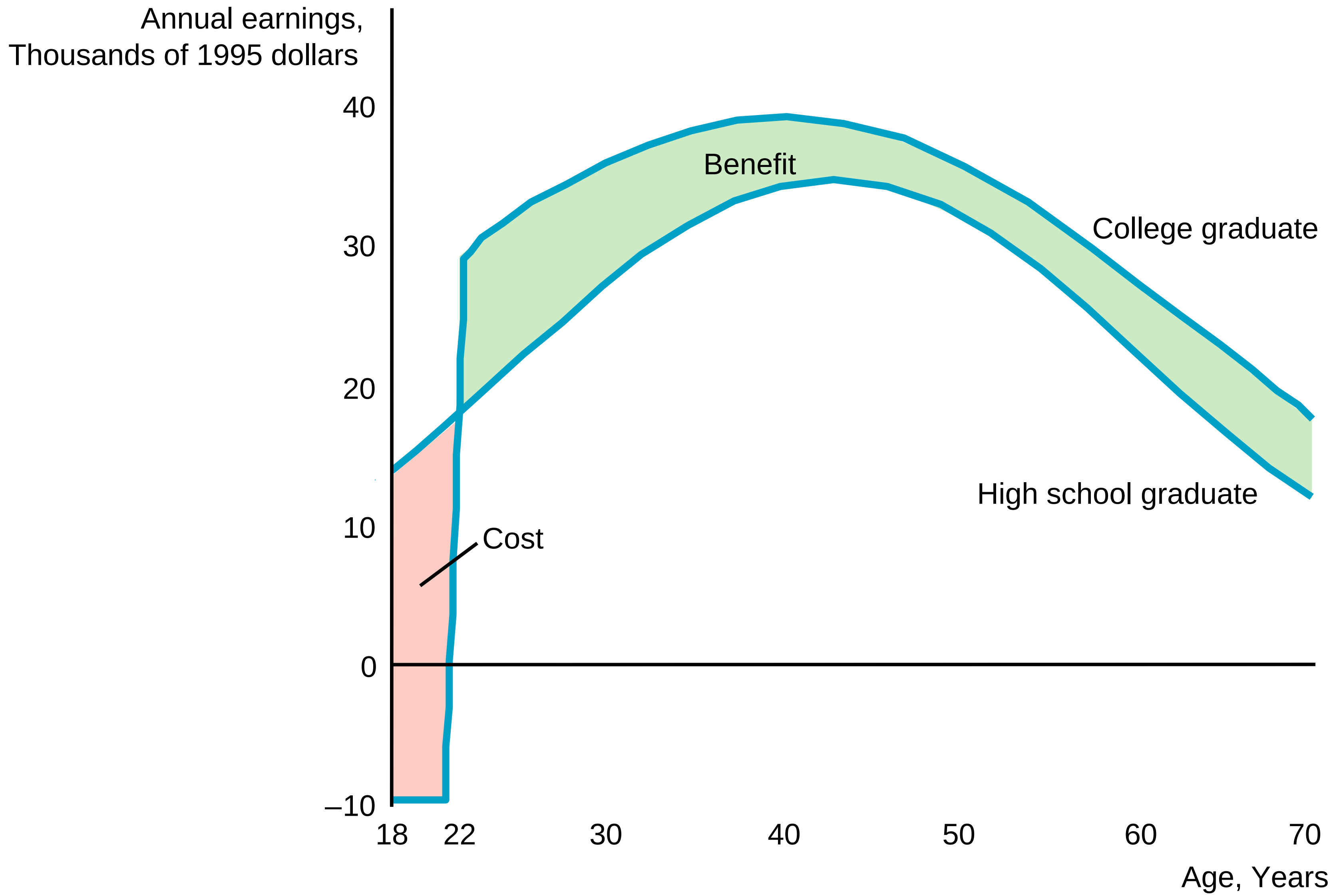
- individuals decide whether to invest in their own human capital
- does going to college increase your lifetime earnings?
- graduate high school at 18 years old and either go to work or go to college



Retire at 70

- suppose you
  - graduate from college in 4 years
  - do not work when in college
  - pay $10,000 a year for school expenses: tuition, books, fees
- opportunity cost of college: tuition payments plus 4 years of foregone earnings (at HS grad wage)
- at age 22
  - typical college grad earns $29K ($1995)
  - HS grad earns $18K

Annual Earnings of High School and College Graduates



Compare earning streams

- earnings peak
  - for college grad at 40 years of age at $39K
  - for HS grad at 43 years at $34K
- decide whether invest in college by comparing *PV* at age 18 of the two earnings streams

Present Value of Earnings

	Present Value, Thousands of 1995 dollars	
Discount Rate, %	High School	College
0	1,333	1,531
1	1,033	1,170
2	817	911
3	657	720
4	538	578
5	448	471
6	378	388
6.9	329	329
7	324	323
8	281	272
9	246	231
10	218	197

Exhaustible resources

- discounting determines how fast we consume exhaustible resources
- *exhaustible resources*:
  - nonrenewable natural assets that cannot be increased, only depleted
  - examples: oil, gold, copper, uranium

Intuition

- storing coal in the ground is like keeping money in the bank
- if you sell a pound today, net $p_1 - m$, invest the money in a back, you have $(p_1 - m)(1+i)$ next year
- if that's less than $(p_2 - m)$, sell coal now

Many periods

Interpretation

- if coal is sold in both years, price next year must exceed price this year by
  - $i(p_t - m)$
  - interest payment if you sold a pound this year and put profit in the bank at *i*
- so price grows at

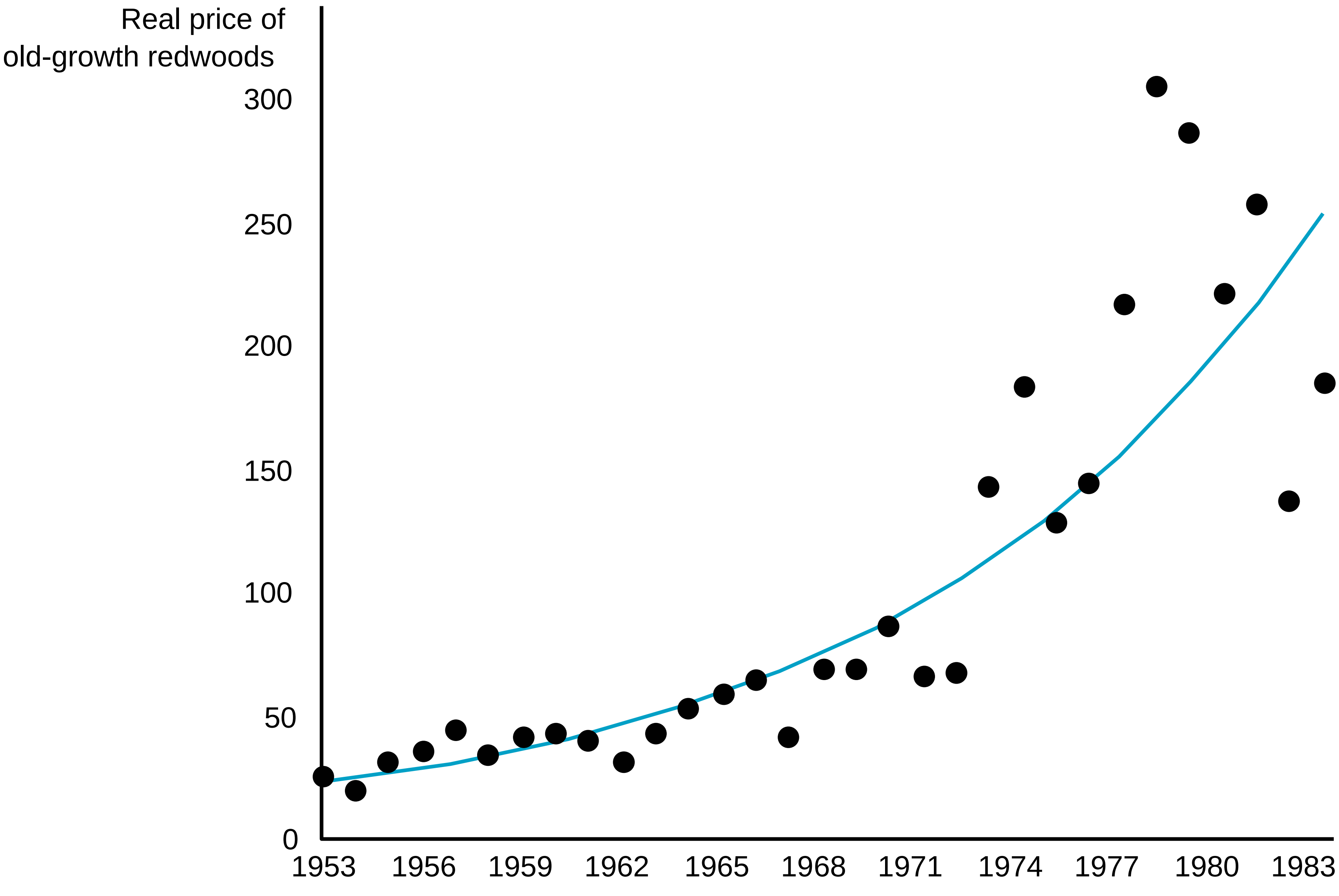
$$\Delta p = p_{t+1} - p_t = i(p_t - m)$$

Gap over time

- gap between this year's price and next year's price increases as cash flow, p_t-m, increases
- thus price gap grows exponentially over time

Price of an Exhaustible Resource

Redwood Trees



1. Comparing money today to money in future

- people value money in the future < money today
- interest rate reflects how much more people value $1 today to $1 in the future
- compare a payment made in future to one today by expressing future payment in terms of current dollars using interest rate
- similarly, a flow of payments over time is related to present or future value of payments by interest rate

2. Choices over time

pick investment option (cash flow over time) by

- picking one with highest discounted present value if net present value > 0
- if internal rate of return > interest rate