PREFERENCES AND UTILITY

Fundamental Problem of Micro-Economics:
satisfying unlimited wants with scarce resources

What a consumer wants (preferences)

What a consumer can afford (budget)

=> What a consumer actually consumes (choice)

The Approach: perfectly-rational consumers pursuing their self interest

RATIONAL BEHAVIOR

THREE KEY ASSUMPTIONS

Consumer can make a decision ("completeness")

Either *a* preferred to *b*, or *b* preferred to *a*, or indifferent between the two:

$$a \succsim b \text{ or } b \succsim a \text{ or } a \sim b$$

Consumer is consistent ("transitivity")

If *a* preferred to *b*, and *b* preferred to *c*, then *a* preferred to *c:*

$$\text{if } a \succsim b \text{ and } b \succsim c, \text{ then } a \succsim c$$

Consumer prefers more to less ("monotonicity")

If *a* has more of all goods than *b*, then *a* is preferred to *b*:

$$\text{if } X_a > X_b \text{ and } Y_a > Y_b, \text{ then } a \succsim b$$

Rational Behavior?

- Limitation of *homo economicus*
  - Transitivity experiments
  - Biases caused by “anchoring,” status quo, regret, “halo” effects
- Interdependencies
  - One consumer’s preferences depend on how another ranks bundles, or on their consumption (bandwagon, snob effects)
- Habit formation
  - A consumer’s preferences depend on how much of the good they consumed in the past

The Indifference Curve

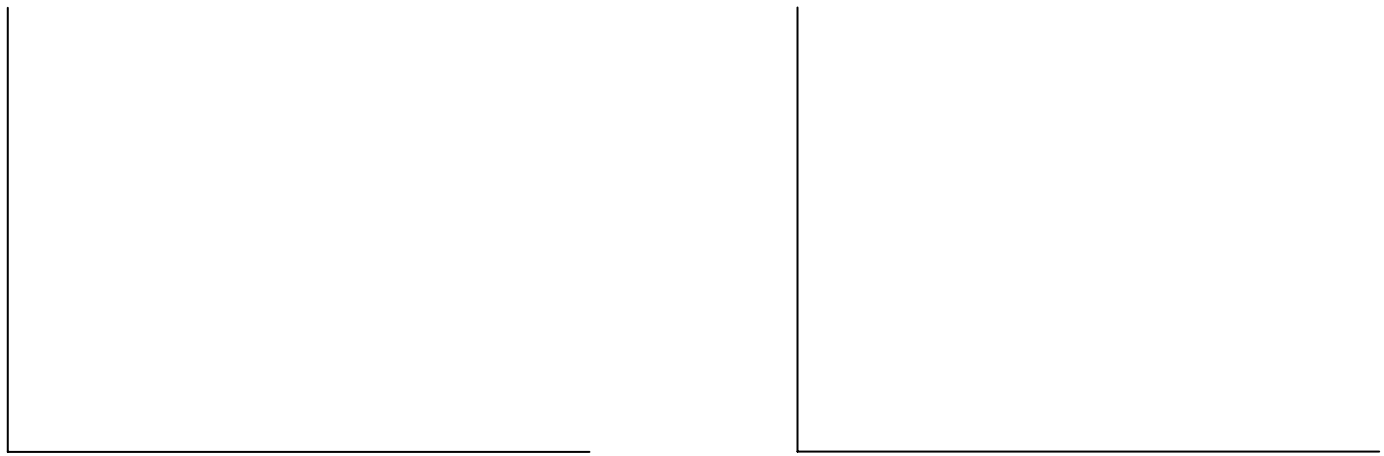
- All bundles among which a consumer is indifferent
  - "Indifference map" is all of a consumer's indifference curves
  - All bundles (X_a, Y_a) such that:

$$(X_a, Y_a) \sim (X_0, Y_0)$$

Properties of Indifference Curves

1. Every bundle is on some indifference curve
2. Two indifference curves never cross
3. An indifference curve is not “thick”
4. Indifference curve slopes downward
4. a bundle that has more of all goods is on a higher indifference curve (“no satiation”)

Properties of Indifference Curves



Marginal Rate of Substitution

- Question
  - How much more of a good (e.g., Y) would a consumer require to compensate them for loss of a unit of another good (e.g., X)
- Measurement
  - MRS measures willingness to make this substitution:

$$MRS_{XY} = -dY/dX$$

Marginal Rate of Substitution

Utility

- The holy grail for 19th century economists
  - Measure a person's happiness in "utils"
  - Make comparison of different bundles, and between consumers
- Modern notion of utility
  - Indicates relative (ordinal rankings, not strength of preferences
  - Each indifference curve assigned a different number, with higher indifference curves getting higher numbers

Utility

Utility (Cont'd)

- Utility function
  - Assigns a number to each bundle that represents a consumer's preferences:

$$a \succsim b \text{ if and only if } u(a) > u(b)$$

  - Utility number can take any value, even negative
  - Along an indifference curve:

$$u(X,Y)=u_0 \text{ (a constant)}$$

Utility Functions

- Examples of utility functions
  - Perfect substitutes: $u(X,Y) = 2X+3Y$
  - Perfect compliments: $u(X,Y) = \text{minimum } \{X,Y\}$
  - Smooth, symmetric: $u(X,Y) = XY$

Properties of Utility Functions

- Invariant to an increasing transformation:

$$v(X,Y) = f(u(X,Y))$$

Where f is an increasing function: $df/dx > 0$

Ex: a positive, linear transformation:

$$A + B*u(X,Y) \text{ where } B > 0$$

- Increasing in any good (holding all others fixed):

Marginal utility: $MU_x = du/dX = du(X,Y)/dX > 0$

MRS, AGAIN

Expressing MRS mathematically

- recall definition of an indifference curve:

$$u(X, Y) = u_0 \text{ (a constant)}$$

- totally differentiating, we get:

$$dU_X / dX * dX + dU_Y / dY * dY = du_0 = 0$$

- rearranging:

$$MRS_{XY} = - dY / dX$$

$$= (dU_X / dX) / (dU_Y / dY)$$

$$= MU_X / MU_Y$$

MRS, AGAIN

Diminishing MRS

- a consumer needs less of a second good (e.g., Y) to compensate for giving up a unit of some good (e.g., X), the more of that good she has to begin with
- mathematically, MRS_{XY} decreases with increases in X (along a given indifference curve)
- indifference curve is “convex to origin” = prefer “mixtures”

Examples

- $u(X,Y) = 2X + 3Y \Rightarrow MRS = MU_X/MU_Y = 2/3$
- $u(X,Y) = XY \Rightarrow MRS = MU_X/MU_Y = Y/X$

Summary

1. Preferences of a “rational consumer” assumed to satisfy 3 conditions: completeness, transitivity, monotonicity.
2. Indifference curves summarize a consumer’s preferences, and indifference maps have certain properties if they satisfy the 3 conditions
3. Utility functions represent a consumer’s preferences by assigning numbers to bundles to indicate rank, but they are unique only up to an increasing transformation.
4. Marginal rate of substitution measures a consumer’s willingness to trade off between two goods, expressed as the ratio of the marginal utilities of the two goods.

The Budget & Consumer Choice

Major Issues

- Characterize the consumer's "opportunity set" and her "budget line"
- Examine how budget line changes when income and prices change
- Write down and solve the consumer's choice problem as utility maximization to budget constraint

Consumer's Budget

- Money income
  - **I** is money income available to buy goods
  - Could include loans, credit cards, money value of assets (even "knowledge")
- Nominal prices
    - p_X = price of X = music albums (CDs, downloads)
    - p_Y = price of Y = movies (tickets, rental, PPV)
    - Note that 1/price is the number of units that can be bought with $1
- Expenditures
    - $p_X X + p_Y Y$ = expenditure on entertainment
    - Money can also be saved. All **I** does not have to be expended

U.S. HOUSEHOLD EXPENDITURES, 2001

Yearly after tax income: $42,362
Yearly total expenditures: $40,900

Source: U.S. Census Bureau

Food	$5,904
Housing	$12,248
Transportation	$8,672
Health care	$2,239
Entertainment	$1,958

OPPORTUNITY SET & BUDGET LINE

- The “opportunity set”
  - all bundles that are affordable (given income and prices) $p_X X + p_Y Y \leq \mathbf{I}$
  - Both income and prices both assumed known
- The “budget line”
  - bundles on the “frontier” of the opportunity set:

    $P_X X + p_Y Y = \mathbf{I}$
  - intercepts equal maximum of a good that can be purchased
  - e.g., if all money spent on music (none on movies), then can buy $\mathbf{I} / p_X$ albums

BUDGET LINE

CONSUMER'S BUDGET

- Slope of the budget line gives "terms of trade" between two goods
- Slope equal to (negative of) the price ratio:

$$dY/dX = - p_X / p_Y$$

- Example: give up 1 album:

  => frees up $\$p_X$ of money income

  => can then buy $p_X * (1/p_Y)$ movies

AN EXAMPLE

- Suppose $p_X = \$12$, $p_Y = \$6$, $\mathbf{I} = \$180$
- Budget line: $12*X + 6*Y = 180$

Bundle	Albums	Music expenditures	Movies	Movie expenditures	Total expenditures
a	4	$48	1	$6	$57
b	3	$36	5	$30	$61
c	2	$24	6	$36	$60
d	0	$0	12	$72	$72

An Example (Cont'd)

- *BL slope* = dY /dX = $- p_X / p_Y = -$ \$12 / \$6 = - 2

An Example (Cont’d)

CHANGES IN PRICES AND INCOME

Changes in prices (holding income fixed)

– increase in one price:
  - swing budget line toward origin
  - new price ratio
– increase in all prices:
  - shift budget line toward origin
  - new price ratio, UNLESS all prices change same percentage amount

Changes in income (holding prices fixed)

increase/decrease ==> shift out / in budget line

no change in price ratio

same as proportional change in all prices

Change in both prices and income: combination of above

INCOME CHANGE

- Recall: $p_X = \$12$, $p_Y = \$6$, $\mathbf{I} = \$180$
- Compare: $p_X = \$12$, $p_Y = \$6$, $\mathbf{I} = \$240$

INCOME CHANGE

PRICE CHANGE

- Recall: $p_X = \$12$, $p_Y = \$6$, $\mathbf{I} = \$180$
- Compare: $p_X = \$12$, $p_Y = \$9$, $\mathbf{I} = \$180$

PRICE CHANGE

MORE COMPLICATED BUDGETS

1. Volume Discounts: price falls as more purchased
   - X sold at a constant price
   - Y sold at two-block price:
     - first Y_0 units sell at p_{Y0}
     - units beyond Y_0 sell at $p_{Y1} < p_{Y0}$

MORE COMPLICATED BUDGETS

MORE COMPLICATED BUDGETS

2. Membership Fees
   - example: Costco, Sam's Club
   - fee ($ F) paid for the right to purchase at (discounted) unit price (p_X)
   - membership fee deducted from money income: **I** - F
   - member's budget line: $p_X X + p_Y Y = \mathbf{I} - F$

MORE COMPLICATED BUDGETS