

HW 4: Monopoly and Price Discrimination  
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1. The inverse demand curve facing a monopoly is  $p = 20Q^{-1/2}$ . Its cost curve is  $C(Q) = 10Q$ . What is the firm's profit maximization price and quantity?

*The profit maximizing price  $p^* = 20$ , and the profit maximizing quantity  $Q^* = 1$ . To profit maximize the firm sets  $MR = MC$ . We need MR and MC. To find MR, first find  $TR = pQ = (20Q^{-1/2})Q = 20Q^{1/2}$ . Now,  $MR = dTR/dQ = 10Q^{-1/2}$  (by the power rule). Similarly,  $MC = dTC/dQ = 10$ . Setting  $MR = MC$  and solving for  $Q^*$ ,  $10Q^{*-1/2} = 10$ , so  $Q^* = 1$ . Filling  $Q^* = 1$  into the inverse demand curve produces  $p^* = 20(1)^{-1/2} = 20$ .*

2. What is the monopoly's demand for labor if it uses a fixed-proportions production function in which each unit of output takes one unit of labor (which costs  $w$ ) and one unit of capital (which costs  $r$ )?

*The monopoly's demand for labor ( $L^*$ ) is  $L^* = [a - (w + r)]/2b$  where  $p = a - bQ$  is assumed to be the demand for the monopoly's product. To profit maximize the monopoly sets  $MR = MC$ . Since the firm uses a fixed-proportions production function in which each unit of output takes one unit of labor and one unit of capital, the firm's  $MC = w + r$ , which is what it pays for one unit of labor and one unit of capital. Assuming a linear demand for the monopoly's product  $p = a - bQ$ , the firm faces a marginal revenue curve that is twice as steep with the same price intercept. Thus,  $MR = a - 2bQ$ . Setting  $MR = MC$  produces a relationship of  $Q^*$  (the number of units a firm produces) and  $w$  (the current wage).  $a - 2bQ^* = w + r$ , so  $Q^* = [a - (w + r)]/2b$ . Since the firm will produce  $Q$  in the most efficient way, it will hire exactly as many units of labor as the number of units of the good that it wants to produce [remember here that it takes one unit of labor and one unit of capital to produce a unit of the good. In other words,  $L^* = Q^* = [a - (w + r)]/2b$ .*

3. An upstream monopoly sells its product at a price  $p_u$  to a downstream monopoly, which in turn sells it to final customers at price  $p_d$ . The upstream monopoly's marginal cost is constant at \$2 per unit. The inverse demand of final customers is  $p_d = 10 - Q_d$ .

A. Suppose that the two firms merge ("integrate"). What price will the new monopoly charge customers and how many units will it sell?

*The new monopoly will charge customers  $p^* = 6$  and sell  $Q^* = 4$  units. Again, to profit maximize the firm sets  $MR = MC$ .  $MR = 10 - 2Q$  and  $MC = 2$ . Thus,  $Q^* = 4$ , and filling into the inverse demand of consumers finds  $p^* = 6$ .*

$$\begin{aligned} MR &= MC \\ 10 - 2Q^* &= 2 \\ Q^* &= 4 \\ P^* &= 10 - Q^* = 10 - 4 = 6 \end{aligned}$$

- B. Now assume that the two firms act independently. Determine the derived demand curve facing the upstream firm. [*Hint: Think about the role of the downstream firm's marginal revenue curve.*]

*The derived demand curve facing the upstream firm is  $p_u = 10 - 2Q_u$ . Like any firm, the downstream firm sets  $MR = MC$  to maximize its profit. The downstream firm sells to the final customers, so it faces a marginal revenue curve  $MR = 10 - 2Q_d$ . The downstream firm doesn't produce its good; it simply buys it from the upstream firm at a price  $p_u$ . Thus, the downstream firm's  $MC = p_u$ . Setting  $MR = MC$ , produces  $p_u = 10 - 2Q_d$ . Since the downstream firm just resells all of the quantity it purchases from the upstream firm without producing any extra,  $Q_u = Q_d$ . Thus, the derived demand curve facing the upstream firm is  $p_u = 10 - 2Q_u$ .*

$$\begin{aligned} MR_d &= MC_d \\ 10 - 2Q_d &= p_u \\ \text{since } Q_d &= Q_u \\ 10 - 2Q_u &= p_u \end{aligned}$$

- C. Next, determine the price that the upstream firm charges the downstream firm. What price does the downstream firm charge the final customers? How many units are sold?

*The upstream firm charges the downstream firm a price  $p_u^* = 6$ . The downstream firm charges the final customers  $p_d^* = 8$ , and  $Q_u^* = Q_d^* = 2$  units are sold. To maximize profits the upstream firm sets  $MR_u = MC_u$ . Again using the trick that for a linear inverse demand curve the marginal revenue curve is twice as steep and has the same price intercept,  $MR_u = 10 - 4Q_u$ . The question states that  $MC_u = 2$ . Setting them equal to solve for  $Q_u^*$ ,  $10 - 4Q_u^* = 2$ , so  $Q_u^* = 2$  units are sold to the downstream firm. Filling  $Q_u^* = 2$  into the derived demand curve facing the upstream firm, produces  $p_u^* = 10 - 2(2) = 6$ . The downstream firm now has  $Q_d^* = 2$  units to sell to the final customers. It sells these units for the highest price it can get. Using the customers demand curve, the downstream firm can sell its two units for a price of  $p_d^* = 10 - 2 = 8$ .*

$$\begin{aligned} MR_u &= MC_u \\ 10 - 4Q_u^* &= 2 \\ Q_u^* &= 2 \\ p_u^* &= 10 - 2(2) = 6 \\ Q_d^* &= 2 \\ p_d^* &= 10 - 2 = 8 \end{aligned}$$

- D. Compare the results for the integrated firm to the two firms that impose successive markups.

*The integrated firm produces more and sells it to the final customers at a lower price than the two firms that impose successive markups.*

4. There are two groups of consumers that purchase a monopoly's good. The inverse demand for one group is  $p_1 = 10 - Q_1$ , and the inverse demand for the other group is  $p_2 = 12 - 2Q_2$ . If the monopoly's marginal cost is constant at \$2 and it can prevent resales, how many units does it sell to each group at what prices?

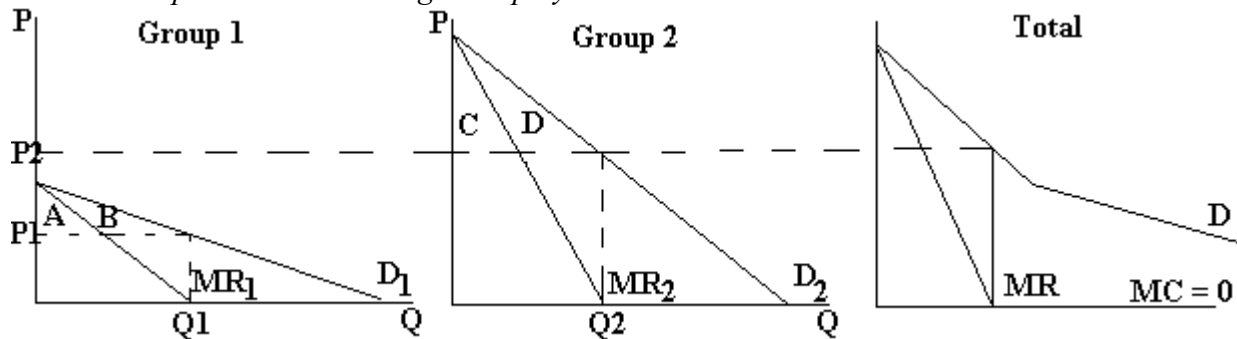
*The monopoly sells 4 to group 1 at a price of 6, and it sells 2.5 to group 2 at a price of 7.*

*Strategy (without using MR trick): Set  $MR_1 = MC$  to find  $p_1$  and  $Q_1$ , and set  $MR_2 = MC$  to find  $p_2$  and  $Q_2$ . [Notice that this implies  $MR_1 = MR_2$ .] To get  $MR_1$ , first need  $TR_1 = p_1 * Q_1 = (10 - Q_1) * Q_1 = 10Q_1 - Q_1^2$ .  $MR_1 = \partial TR_1 / \partial Q_1 = 10 - 2Q_1$ . Set  $MR_1 = MC$ :  $10 - 2Q_1 = 2$ , so  $Q_1 = 4$ . Now find  $p_1$  from group 1's inverse demand:  $p_1 = 10 - 4 = 6$ .*

*Similarly, to get  $MR_2$ , first need  $TR_2 = p_2 * Q_2 = (12 - 2Q_2) * Q_2 = 12Q_2 - 2Q_2^2$ .  $MR_2 = \partial TR_2 / \partial Q_2 = 12 - 4Q_2$ . Set  $MR_2 = MC$ :  $12 - 4Q_2 = 2$ , so  $Q_2 = 2.5$ . Now find  $p_2$  from group 2's inverse demand:  $p_2 = 12 - 2 * 2.5 = 7$ .*

5. Suppose that there are two types of customers. Show diagrammatically that welfare can be either higher or lower under simple (one-price) monopoly compared to multimarket price discriminating monopoly.

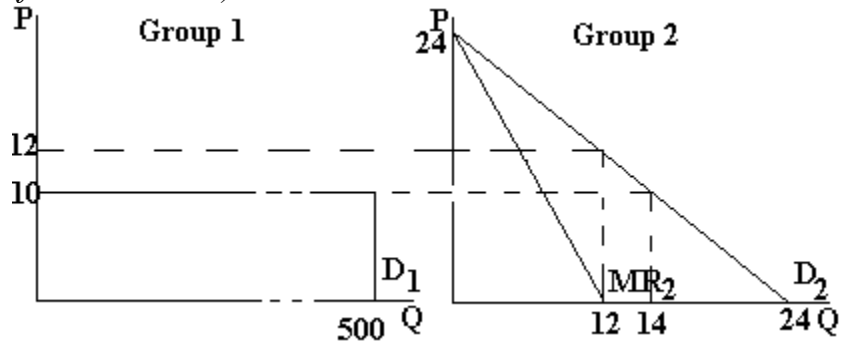
*An example (others possible) where welfare is lower under a one-price monopoly compared to a multimarket price discriminating monopoly:*



*In this example, there are two types of consumers with different demand curves, and  $MC = 0$  for simplicity. The single price monopoly sets quantity in the Total market where  $MR = MC = 0$ , and it chooses its price from the demand curve at that quantity. This example is constructed so that when the single price monopoly maximizes profits, it sets a price  $P_2$  that is too high for any consumer in group 1 to buy anything. A multimarket price discriminating monopoly can sell to group 1 at a lower price  $P_1$ , while maintaining its price and quantity for group 2. Thus, consumer welfare rises by  $A+B$  under the multimarket price discriminating monopoly. Producer surplus also rises by selling to group 1 at a lower price, so total welfare rises with the multimarket price discriminating monopoly.*

An example (others possible) where welfare is higher under a one-price monopoly compared to a multimarket price discriminating monopoly:

(from TA Kostis )



In this example, there are two type of consumers with different demand curves, and  $MC = 0$  for simplicity. The multimarket price discriminating monopoly sells 500 units to group 1 at a price of 10, and it sells 12 units (where  $MR=MC$ ) to group 2 at a price of 12. The single price monopoly sets quantity in the Total market (not pictured) to maximize profits. Since the first group of consumers won't pay any more than 10, but they will buy 500 units if the price is at most 10, the monopoly will make 5000 in profit from group 1 alone. The single price monopolist chooses a price of at most 10 in order to gain this big profit. The single price monopolist won't charge any less than 10 because doing so would lower revenues from both groups and raise cost. At the profit-maximizing price 10, the single price monopoly produces 514 units. Welfare from group 1 is no different in each case, while welfare from group 2 is higher when they receive the lower price and higher quantity produced by the single price monopoly (the lost producer surplus is transferred entirely to consumers). Thus, total welfare is higher with the single price monopoly in this case.

6. Would a price-discriminating monopoly ever sell less than a single-price monopoly?

Yes, it's possible that a price discriminating monopoly would sell less than a single-price monopoly. In the second example in question 5 this is true (other examples possible).

7. A local gym sells an annual membership for \$500. Any member can use the gym for \$5 a visit. Why does the gym charge both an annual fee and a per visit fee? Some people love exercise and use the gym frequently, while others are less enthusiastic and rarely use the gym. Can you think of a different pricing scheme that might increase the gym's profit? What would the gym need to know to make this pricing scheme feasible?

(Answers may vary) The gym's charging both an annual fee and a per-visit fee is a way to price discriminate, a strategy that can produce higher profits than charging a single per-visit price. This type of price discrimination is called a two-part tariff.

Perfectly price discriminating might increase the gym's profit. In order to perfectly price discriminate, the gym would have to know each customer's reservation price, that is the highest price each customer is willing to pay to use the gym.

*The gym might also try bundling its product, such as selling monthly memberships that include “free” fitness courses or personal training. The gym might also charge a sky-high one-time use fee, while having a much more reasonable unit price for a membership contract for an extremely long period (a la cell phone companies).*

8. Document Senator Kerry’s and President Bush’s views on importing drugs from Canada and elsewhere in the United States. What reasons do they give? What are the economic implications of a ban?

*Senator Kerry wants to allow importing drugs from Canada, which are currently selling at prices remarkably lower than in the US. He argues that it will lower the price for US consumers.*

*President Bush’s administration argues that the drugs from Canada aren’t certified by the US FDA, and therefore could be unsafe.*

*The ban allows drug companies to price discriminate, and thus earn higher profits.*

References: <http://www.johnkerry.com/issues/seniors/prescription.html> (Kerry)

<http://www.globalaging.org/health/us/2004/thompson.htm> (Bush)

*Notably, in my quick search I couldn’t find anything about this issue on Bush’s campaign site.*