PS#1 Solutions

1) Use the Internet to determine the views of President Bush and at least two of the leading Democratic candidates for President about whether they want to raise or lower the minimum wage and why (you'll probably only find a phrase or a sentence on their campaign Web sites). Give at least two reasons (someone benefits) for raising it and two reasons for leaving it alone or lowering it (someone is harmed).

a) President Bush advocates raising the minimum wage by $1.
http://www.whitehouse.gov/omb/legislative/sap/106-2/HR3846-h.html
He argues that the "real" minimum wage (i.e. the minimum wage after adjusting for inflation) has fallen over the past twenty years and should be adjusted as such.

b) Senator John Kerry advocates raising the minimum wage and index it to inflation.
http://www.johnkerry.com/communities/african_americans/position.html
He also claims that inflation has eroded the value of the minimum wage.

c) Senator John Edwards advocates raising the minimum wage.
http://www.johnedwards2004.com/page.asp?id=524
Senator Edwards wants to help "working families make ends meet."

d) Reverend Al Sharpton advocates raising the minimum wage to $7.15.
http://www.ontheissues.org/2004/Al_Sharpton_Jobs.htm
According to Sharpton, $5.15 per hour is not "realistic" in today's America (whatever that means).

e) It is safe to say that almost all the candidates for president advocating raising the minimum wage. The question is, how many of them are actually going to try and do something about it?

Two reasons for raising the minimum wage:

1) People who keep their jobs earn higher wages.
2) Under certain circumstances, the wage bill may increase (see problem # 2).

Two reasons against raising the minimum wage:

1) Raising the minimum wage *may* increase unemployment, particularly among minorities and teens.
2) Raising the minimum wage hurts businesses by increasing labor costs.

2) Show that an increase in the minimum wage may (but does not necessarily) cause the total wage bill, *wL* (where *w* is the wage and *L* is the hours worked by all employed workers) to rise. Under what conditions does it rise? You may show using graphs or math (calculus and/or elasticities).

Math version:

The wage bill is equal to w*L. Assume that w is determined exogenously by whomever determines the minimum wage, and that L is quantity of labor (in hours) demanded by employers for a given level of wage. In other words, L is a function of w, and we write this is as L(w). Then a change in the minimum wage leads to the following equation:

$$wage\ bill = wL(w)$$

$$\frac{\partial(wage\ bill)}{\partial w} = L(w) + w\frac{\partial L}{\partial w} = L(w)\left(1 + \frac{w}{L(w)}\frac{\partial L}{\partial w}\right)$$

Notice that L(w) – or the demand for labor – is always non-negative. Assume L(w) is positive (i.e. at least one person is employed). This implies that $L(w)\left(\frac{w}{L(w)}\frac{\partial L}{\partial w}\right)$ >0 whenever $\left(\frac{w}{L(w)}\frac{\partial L}{\partial w}\right)$ >0.

Also notice that the second term in the parentheses – $\left(\frac{w}{L(w)}\frac{\partial L}{\partial w}\right)$ – is just an elasticity. Instead of quantity of a good, we have quantity of labor hours. Instead of the price of a good, we have the price of labor per hour (i.e. wage). So we can rewrite this term as ε, which denotes the elasticity of labor demand. Then:

$$\frac{\partial(wage\ bill)}{\partial w} = L(w)\left(1+\frac{w}{L(w)}\frac{\partial L}{\partial w}\right) = L(w)(1+\varepsilon) > 0$$ which implies

$$(1+\varepsilon) > 0$$
$$\varepsilon > -1$$

In words, we have just shown that an increase in the minimum wage will lead to an increase in the wage bill whenever the elasticity of labor demand is greater than –1 (i.e. whenever demand is inelastic). Intuitively, this says that whenever the percent change in the quantity of labor is less than the percent change in the wage, the wage bill will increase.

Graphical version:
If you tried to show this graphically, then you needed to draw a very inelastic (i.e. steep) demand curve for labor and show that the change in the minimum wage did not decrease the quantity of labor employed by very much.

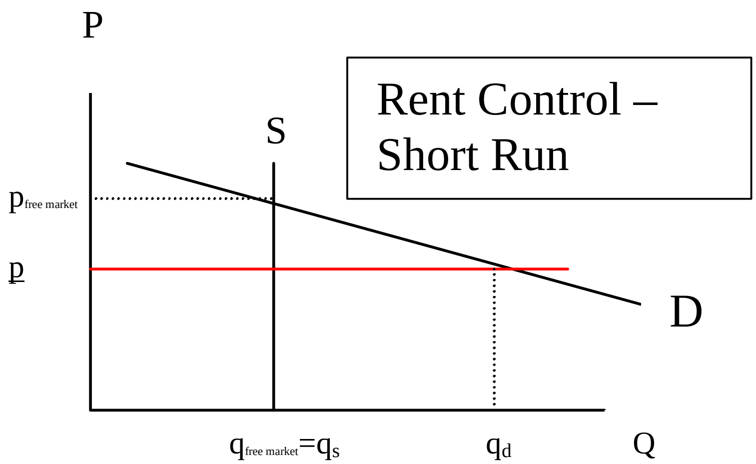
3) Briefly (less than a page) make the cases for and against rent control (a price ceiling on the Price that can be charged to rent an apartment). Discuss both efficiency and equity issues (consider short-run and long-run effects). *Hint*: Who benefits and who is harmed by the law? Note: The magnitude of the effects may depend on the shapes of the demand and supply curves—an empirical question.

In the short run, the supply of apartments is probably inelastic. This implies that rent control will have little effect on the quantity of apartments available, but it will affect the quantity demanded and the price. In the graph below, the two solid black lines denote supply and demand. Supply is drawn as a vertical line to show that it is inelastic. In the absence of any rent control, the market allocation is given by ($q_{free\ market}$, $p_{free\ market}$).

The red (horizontal) line denotes rent control. As a result of this policy, the rent falls to $\underline{p}$, and consumers *who have an apartment* are helped by the law because it lowers their cost of renting. However, at $p = \underline{p}$, the quantity demanded (q_d) exceeds the quantity supplied (q_s). The net result is excess demand – there exists consumers willing to pay for an apartment at $p = \underline{p}$, but there is not enough supply. The amount of excess demand is given by the difference between the quantity of consumers who demand housing at $\underline{p}$ and those that receive an apartment: q_d - q_s.

This is somewhat inefficient, but not dramatically so since the quantity of apartments does not change in the presence of rent control (i.e. $q_s = q_{free\ market}$).

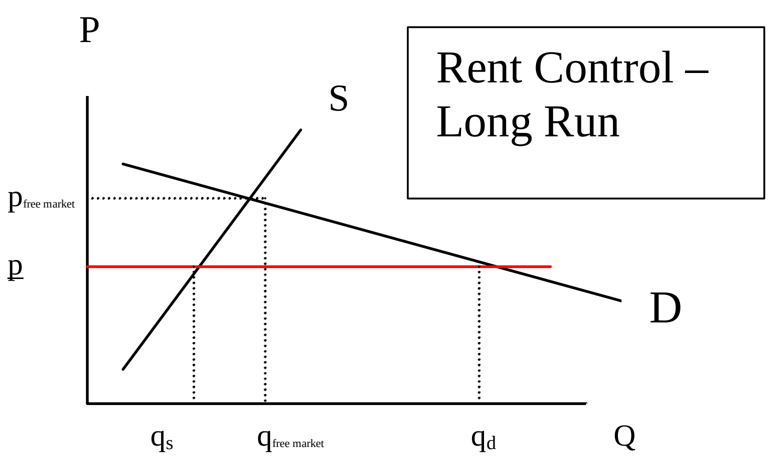
Rent control hurts those consumers who do not have an apartment, as these people are forced to search for alternative forms of housing. The lower rent also unambiguously hurts suppliers of apartments, because it lowers the amount of money they can earn on an apartment.



In the long run, the supply of apartments is more elastic and therefore slopes up (instead of being vertical). For example, property owners may convert apartments to condos, or they may build more apartments. This implies that rent control will impact both the quantity supplied and demanded (see graph below). As before, the free market allocation is denoted by ($q_{\text{free market}}$, $p_{\text{free market}}$).

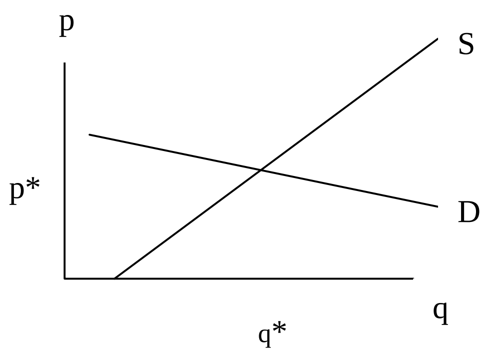
In the presence of rent control, the rent falls to $\underline{p}$. As before, those consumers with an apartment are better off, because they pay less for an apartment. Those that cannot find an apartment are worse off, since they cannot get an apartment even though they are willing to pay for one. Producers are also worse off, because they earn less return from renting the apartment.

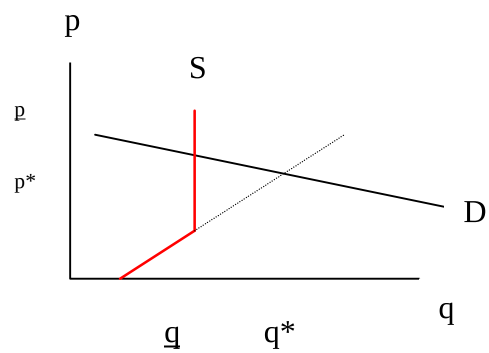
Since supply is more elastic, rent control reduces the number of available apartments – $q_s < q_{\text{free market}}$. So now there are even more consumers who cannot find apartments, even though they are willing to pay for one at a price $p = \underline{p}$. This large excess demand makes the market much less efficient.



4) Initially in an unregulated, competitive market, the competitive price is *p and the competitive quantity is *Q**. The entire supply is imported. The government imposes a binding quota at $\overline{Q} < Q^*$. Describe the effects of this quota on the market price and quantity. Who gains and who loses?**

The competitive market is show in the figure below on the left. The market price is denoted by p* and q*. When the government imposes a binding quota at $\underline{q}$, the supply curve changes. In particular, for all values of q less than $\underline{q}$, the supply is just as before. For all values of q greater than q, the supply is fixed at q. The market price rises. As a result, consumers are unambiguously hurt from the higher prices. The effect on producers depends on certain assumptions we make on producer behavior. If the only suppliers to the market are importers, than the increase in price may help those that continue to import by increasing their profits. However, it will hurt those who are no longer allowed to import, since they are excluded from the market. If, as a result of the quota, domestic producers start supplying the market, then domestic firms are better off, and the effect on importers is still ambiguous.





5) The demand function for roses is *Q = a – bp + fpc*, and the supply function is *Q = c + ep*, where a, *b*, *c*, *e*, and *f* are positive constants and p_c is the price of chocolate. Write a formula that shows how the equilibrium quantity and price of roses vary with the price of chocolate.

Setting demand equal to supply yields:

$$Q = a - bp + fp_c = c + ep$$

$$(e + b)p = a - c + fp_c$$

$$p = \frac{a - c + fp_c}{e + b}$$

This tells us how the price of roses varies with the price of chocolate. Plugging this into the supply function (or demand function – you'll get the same solution) tells us how the quantity of roses varies with the price of chocolate:

$$Q = c + ep = c + e\left(\frac{a - c + fp_c}{e + b}\right) = \frac{ae + bc + efp}{e + b}$$

6) The market demand and supply curves are (respectively):

$$Q_d = 90p^{-2} \qquad Q_s = 10p$$

A. What are the equilibrium price and quantity?

Equating supply and demand:

$90p^{-2}=10p$

$9=p^3$

$p=9^{1/3}$

$Q = 10*p = 10*9^{1/3}$

B. What is the formula for the elasticity of demand at various possible points along this demand curve? What is the elasticity of demand at the equilibrium?

The elasticity of demand is:

$$\varepsilon = \frac{\%\Delta Q_d}{\%\Delta p} = \frac{dQ_d}{dp} * \frac{p}{Q} = -180p^{-3} * \frac{p}{90p^{-2}} = \frac{-180}{90} = -2 = \varepsilon$$

This implies that the elasticity of demand is constant (see below for demand curves that satisfy the constant elasticity of demand property). So the elasticity of demand at the equilibrium, as well as everywhere else on the demand curve, is ε=-2.

Constant Elasticity Demand Curves:
Supposed demand is of the form $Q_x = Ap^{\varepsilon}$ where A is a constant and ε is a negative constant. Then:

$$\frac{dQ_d}{dp} = A\varepsilon p^{\varepsilon-1}$$

$$\frac{dQ_d}{dp} * \frac{p}{Q} = A\varepsilon p^{\varepsilon-1} * \frac{p}{Ap^e} = \frac{A\varepsilon p^{\varepsilon-1+1}}{Ap^e} = \varepsilon$$

So, in the above example where $Q_d = 90p^{-2}$, we know (without needing to take a derivative) that the elasticity of demand is ε =-2.

C. What is the formula for the elasticity of supply? What is the elasticity of supply at the equilibrium?
The elasticity of supply is:

$$\eta = \frac{\%\Delta Q_s}{\%\Delta p} = \frac{dQ_s}{dp} * \frac{p}{Q} = 10 * \frac{p}{Q}$$

At the equilibrium, the elasticity of supply is:

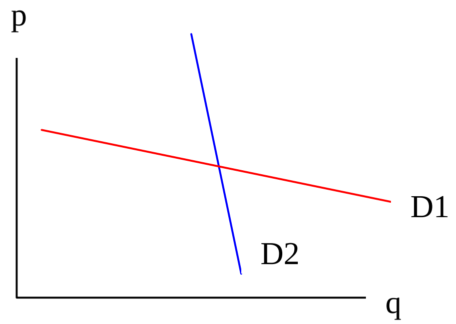
$$\eta = 10 * \frac{p}{Q} = 10 * \frac{9^{1/3}}{10 * 9^{1/3}} = 1$$

D. What is the incidence of a specific tax on consumers?

The incidence of tax is given by:

$$tax\ incidence = \frac{\eta}{\eta - \varepsilon} = \frac{1}{1-(-2)} = 1/3$$

7. Suppose two linear demand curves cross at the point (p^*, Q^*). Which demand curve (the steeper or the flatter one) is more elastic at that point, and why?



The flatter demand curve, D1, is more elastic than the steeper curve, D2. Recall that the elasticity of demand is given by:

$$\varepsilon = \frac{\%\Delta Q_d}{\%\Delta p} = \frac{dQ_d}{dp} * \frac{p}{Q}$$

At the optimum (p^*, Q^*), the more elastic demand curve is the one in with the larger $\frac{dQ_d}{dp}$ (in absolute value). Since we are graphing the demand curve in (q,p) space (i.e. q is on the x-axis), this implies that the flatter demand curve has a smaller absolute value of $\frac{dQ_d}{dp}$. Intuitively, ask yourself the following question – starting at p^*, if I increase (or decrease) p a little, along which demand curve will q fall (rise) the most? The answer, of course, is D1.