Chapter 19

INTERTEMPORAL CONSISTENCY ISSUES IN DEPLETABLE RESOURCES

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1. Introduction

The critical feature of an exhaustible resource is that it is in finite supply, so that if one unit is consumed today, there is one less unit available for consumption in the future. As a consequence it commands a scarcity value, and its opportunity cost is the sum of two elements. The first is the standard marginal cost of production (in this case extraction), and the second is the scarcity value or the rent. The theory of exhaustible resources, from Hotelling (1931) on, summarized elsewhere in this volume, is largely the theory of the determination of this rental element. The larger is the rental element relative to extraction costs, the more important is this theory in explaining the opportunity cost of the resource. Natural resources which are relatively abundant (as measured by the ratio of economically recoverable stocks to annual consumption) and/or are costly to extract will have a low rental element in their opportunity cost, and conversely those which are relatively scarce and yet cheap to extract will have a high rental element. Oil is a good example of the latter.

Scarcity can be artificially induced by the exercise of market power, and there are rents to the exercise of such power. Exhaustible resources generate rent in a competitive economy, as do durable factors in fixed supply, such as land, and renewable resources such as forests, or fishery stocks. The critical feature of rent determination which will concern us in this chapter is that the current level of rent depends in an essential way on future levels of rent, which in turn will depend on future demand and supply conditions.

An ordinary monopolistic supplier of a produced perishable good (air travel between two points on a given flight, for example) will normally be unaffected by

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future demand and supply conditions when setting the current price. As such the problem can be treated as a normal static optimization exercise. The owner of a piece of land who knows its current and future market price (or has expectations about the future price) must decide whether to sell the land now, or to use it for the current year. If he sells it now, then his income for the year will be the interest on its initial value. If he retains it, he receives rent and capital gains (the difference between its value at the end of the year and its initial value). He should choose the option which yields the higher value, and if he decides to retain the land, then this places a lower bound on the amount of rent it must produce. In equilibrium, the rent to land will equate these returns. The cost of producing crops (assuming the land is retained for agriculture) will be the production cost plus this rent, or, equivalently, the rent will be the value of the crops less the production cost. The current equilibrium can only be solved once the value of land at the end of the period is determined, and since its value then depends on its ability to yield future rents and capital gains, this will require a full intertemporal solution. The standard way in which problems of land valuation are simplified is to suppose that the future looks exactly like the present, in which case the value of the land is just the present discounted value of its ability to produce rental income, and the rent on a particular piece of land is determined (in a static economy) by its cost advantage over marginal land which has zero rent, or, alternatively, by the rent which equates supply and demand for the fixed amount of land.

In the land valuation problem, described above, the simplifying assumption that the future is exactly like the present is reasonable, since the assumption is correct in a deterministic, stationary equilibrium. The nonrenewable (or exhaustible) nature of many resources leads to a fundamentally different problem. Current extraction alters future possibilities, so in equilibrium the future cannot be identical to the present. It is not reasonable to model the market as if agents believed that such would be the case. An attractive alternative is to assume that agents have rational expectations. This requires that agents form expectations about how other agents will behave in the future, and that these expectations are confirmed in equilibrium. Most of the literature on dynamic consistency assumes perfect certainty, and in that case the assumption of rational expectations implies either that agents have perfect foresight, or that all future prices are revealed on futures markets. (In the case of market power, agents need to know future demand and supply schedules, not just future prices.)

The problem of dynamic inconsistency (DI) arises when agents with market power would like to make promises which they would subsequently want to break. Unless there exists some mechanism to bind agents to their promises, such as a legal system that enforces contracts, such promises are not credible, and thus cannot form the basis for a rational expectations equilibrium. This chapter discusses situations in depletable resource models where dynamic inconsistency plays an important role. In each of these situations there are three key ingredients:
(i) The future affects the present.
(ii) At least one agent has market power and can influence the future.
(iii) The agent with market power cannot credibly commit himself to future actions.

The next section provides a framework for the discussion of the problem of DI and considers the relation between that problem and dynamic games. The rest of the chapter reviews specific models where DI is important. We first consider situations where purchasers of a depletable resource have market power and then examine cases where market power resides with the sellers (producers). The next section considers how the equilibrium is affected by the length of the period of commitment. A conclusion summarizes the principal points.

2. A framework

This section reviews several examples of dynamic inconsistency and defines two types of consistent equilibria. We emphasize the contributions that control theory and dynamic game theory make in understanding such problems. An equilibrium is dynamically inconsistent if it involves an agent with market power making promises from which he would later like to renge, in circumstances where he is unable to bind himself to those promises. Although there is no ambiguity about what constitutes dynamic inconsistency, the same cannot be said for the meaning of dynamic consistency. Dynamic consistency is, arguably, not simply the absence of dynamic inconsistency.

Simaan and Cruz (1973) were among the first to recognize formally the possibility of equilibria being dynamically inconsistent. Kydland and Prescott (1977) demonstrated the importance of this in economic modelling. The best-known examples of DI arise where governments attempt to influence the behavior of nonstrategic but forward looking agents. For example, suppose that the government chooses a sequence of taxes which affect the behavior of resource owners. As we pointed out in the Introduction, the future affects the current rent of the resource and therefore effects current behavior. At time zero the government would like to choose the tax employed at a future time, t, in such a way as to balance the effect of the future tax on resource extraction at time zero, and the effect of that tax on future revenue; the optimal tax takes into account these possibly conflicting goals. At time t the extraction at time zero is taken as given, and cannot be altered by changing the tax. Therefore the problem faced by the government at t differs from that at time zero. Consequently, the government at t would like to change the tax announced by its predecessor. The policy that is optimal from the standpoint of the government at zero is time-inconsistent. As this example emphasizes, the time-inconsistency

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1 See Chari, Kehoe and Prescott (1989) for a review of this literature.
problem arises because agents in the model are forward looking: desired current extraction depends on future producer prices and thus on future taxes.

2.1. Time consistency and perfection

If strategic agents are unable to sign binding contracts, i.e. to ‘precommit’, the time-inconsistent equilibrium is implausible. The non-strategic but forward looking agents will not base their decisions on announcements which they do not expect to be carried out. A sensible equilibrium (in the absence of commitment) must be dynamically consistent (DC). It is very rare that the requirement of consistency leads to a unique equilibrium. Nevertheless, it is useful to distinguish between two types of dynamically consistent paths: ‘time-consistent’ and ‘perfect’ equilibria.

The weakest notion of dynamic consistency simply requires that the equilibrium be dynamically inconsistent. We describe such an equilibrium as time-consistent. Suppose that there are one or more agents who behave strategically, and a ‘follower’ who is nonstrategic but forward looking. For example, the follower may be a proxy for a continuum of competitive agents, each of whom acts as if he is unable to affect the outcome. If there is more than one strategic agent, we assume that they play a non-cooperative game where the equilibrium concept is Nash. A Nash equilibrium is one in which the agent takes the strategies of other agents as given and, given the choices that he predicts other agents will make, chooses the best available strategy. If there is only one strategic agent this game is degenerate. Consider a feasible strategy for the strategic agent(s). This may be either a sequence of actions or of behavioral rules (one for each strategic agent). We refer to a particular strategy as a ‘reference strategy’. That strategy induces a sequence of decisions by the follower and a trajectory for the state variable. We denote the reference strategy and the associated trajectory of the state as the reference path. We return to the definition of the state below; it may, for example, consist only of current stock of a depletable resource. The reference path is said to be time-consistent if the continuation of the reference strategy represents an equilibrium of the (possibly degenerate) game whose initial condition is any point on the reference path. For example, suppose that all agents had adhered to the reference path until time \( t \) so that the current state (e.g., resource stocks) lay on the reference trajectory. If that path is time-consistent, it must be the case that for arbitrary \( t \) the continuation is an equilibrium for the game starting at \( t \) and the associated state. That is, given that no agent has reneged in the past, nor expects any other agent to renge in the future, no agent has any incentive to renge unilaterally at any point.

The definition of perfection is stronger. Suppose that some agent had reneged in the past, so that at \( t \) the state does not lie on the reference trajectory. Perfection requires that the continuation of the reference strategies be equilibrium strategies
in the game which begins at $t$ with the initial state given at that time. Moreover, this must hold for all $t$ and for all values of the state which could possibly be reached (i.e., for all possible deviations).

An example helps to clarify the distinction. Suppose that two large importers use tariffs to extract rent from the competitive supplier of a resource. The supply at any point in time depends on the current stock level and current and future prices, and thus on current and future tariffs. For this example we define the state to be the level of remaining stock. Consider a pair of reference strategies in which each importer announces a time profile of tariffs. The competitive supplier takes these profiles as given and chooses a time profile of extraction. The reference path then consists of the tariff trajectories and the associated trajectories of price and extraction. The reference path is time-consistent if, for arbitrary time $t$, no agent would like to deviate unilaterally from the reference strategies, given that no agent has deviated in the past, nor expects any other agent to deviate in the future. The question of what would happen if some agent had deviated is not relevant to determining whether the reference path is time-consistent. That question is, however, central in determining whether the reference path is perfect. Suppose that at some period one of the buyers deviated by imposing a lower tariff than the level indicated by the reference strategy and that this causes the price and the rate of extraction to be higher than the indicated level during that period. Consequently the stock remaining in the next period is lower than the indicated level; that is, the state is off the equilibrium trajectory in the next period. If no agent wants to deviate unilaterally from the reference strategies, despite the fact that the stock is not at the expected level, and if this is true for all levels of stock that could have been reached by a series of deviations, then the reference strategies are perfect.

This example indicates why, in the language of dynamic game theory, perfect strategies (typically) cannot be open-loop, where an open-loop strategy is one in which agents announce at the beginning of the game their trajectory of controls for the entire game as a function of time and the initial state. For the above example, the equilibrium open-loop tariffs depend on the initial condition, i.e. the stock level. If one buyer were to deviate, causing the stock to be greater or lower than expected, the equilibrium open-loop strategies to the ensuing game would differ from the continuation of the reference (open-loop) strategies. The example suggests that the reference strategies might be perfect if they determine the tariffs as functions of the state and (possibly) time, i.e. were closed-loop, in the language of dynamic game theory. This is because such strategies may take into account what would happen if some agent were to deviate. (Below we provide a method of supporting open-loop equilibria as functions of the state – as ‘feedback forms of the open-loop strategies’ which may or may not be perfect.) It is important to recognize that a closed-loop strategy may be time-consistent without being perfect. It may be possible to make a particular path (such as a time-inconsistent open-loop equilibrium) time-consistent using closed-loop rules. For example, consider decision rules which, evaluated on
the equilibrium path of the state, give the values of the open-loop tariffs; off the equi
librium path the rules may imply tariff levels which damage (perhaps) all agents
to such an extent that no agent wants to deviate. Time consistency does not require
that the rules be credible (i.e., equilibrium strategies) off the equilibrium path, since
the test for time consistency does not involve consideration of off-the-equilibrium-
path behavior. Perfection does require that the decision rules be credible both on
and off the equilibrium path. To summarize, perfect strategies typically involve
state-dependent decision rules; time-consistent strategies may involve either open-
loop or closed-loop strategies. Perfection implies time consistency, but the converse
is not true.

The previous discussion obscures an important issue by being vague about what
constitutes the state variable. In the example we assumed that the state consisted
solely of the level of remaining stock. In this case, the strategic agents’ current
decisions influence the future of the game only via their influence on the stock.
It is possible, however, to define the state more broadly, to include not only the
current stock level, but also the history of tariffs. This typically enlarges the set of
perfect equilibria.

In discussing perfect strategies, we shall assume that all agents have the same
information set. In this situation, we make no distinction between the state and
the information set. The state summarizes all the information upon which agents
base their decisions. Agents’ action affect different elements of the state in different
ways. For example, suppose that the state (information set) consists of the level of
the stock of resource and the entire history of each agent’s controls (e.g., tariffs).
In an arbitrary period an agent may be able to affect indirectly the evolution of the
stock of the resource by means of his tariff; and he directly chooses one element of
the future state, the history of his own control. The decision of which variables to
include in the state is a modeling choice, and one which leads to equilibria with very
different characteristics. The choice is not arbitrary, and it is worth distinguishing
between two types of states: those which have a direct physical or technological
impact on the game (e.g., stock levels) and those which affect the psychology of
agents and thus affect their behavior (e.g., histories of controls). For example,
the stock level determines feasible levels of future consumption, and possibly also
extraction costs. Given the current stock level, the history of agents’ tariffs (for
example) is in one sense irrelevant: from a technological perspective it makes
no difference what quantities agents consumed in the past, and since the game
is one of perfect information (by assumption) an agent’s previous behavior does
not provide additional information about what ‘type’ he is. However, the history

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2 In open-loop equilibria agents choose their entire trajectory of controls in the initial period, so the
information set consists of only what they know at that time. It is still convenient in this situation to refer
to the trajectory of the state, e.g., the evolution of the stock of the resource. But in this case the state is
not synonymous with the agents’ information set.
of an agent’s behavior may affect the psychology of other agents, if, for example, they believe that non-cooperative behavior in the past indicates that the agent will also behave non-cooperatively in the future. The next subsection considers Markov equilibria, in which the state consists only of variables which have a direct physical or technological impact on the game. The remainder of this subsection discusses the situation where the state also consists of variables which have a psychological impact on behavior.

It is well known that in supergames there typically exist a multiplicity of perfect (and therefore dynamically consistent) equilibria if agents have access to the history of play [Fudenberg and Maskin (1986)]. This is known as the Folk Theorem. In the simplest example, agents agree to cooperate provided that no agent has cheated in the past; if any agent cheats, all agents expect to play non-cooperatively in all future periods. This is a credible threat, since the non-cooperative equilibrium is individually rational; if agents have a sufficiently small discount rate, so that their future pay-offs are important to them, this threat sustains cooperation as a non-cooperative strategy. Although the Folk Theorem refers to supergames, it is widely recognized that similar results hold for dynamic games [Haurie and Pohjola (1987), Ausubel and Deneckere (1989), Gul, Sonnenschein and Wilson (1986)]. The equilibrium that arises depends on what agents expect would happen if some agent were to deviate from equilibrium. The requirement of perfection imposes very little restriction on these beliefs, so a great variety of equilibria can be supported. Since it is beliefs about out-of-equilibrium behavior, rather than strategic interactions, that form the basis for the multiplicity of equilibria, results similar to the Folk Theorem can be obtained even if there is a single strategic agent who confronts a continuum of non-strategic but forward looking agents. [See Ausubel and Deneckere (1989) and Stokey (1981).]

2.2. Markov equilibria

These remarks have important, but mostly negative, implications for the modelling of resource problems. It is usually straightforward to determine whether a proposed equilibrium is dynamically inconsistent, and in many cases such equilibria are implausible and therefore uninteresting (except perhaps as benchmarks). However,

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3 A supergame is an infinitely repeated one-shot game. Agents' current actions may affect future behavior, as occurs when the strategies are history dependent, but current actions do not directly affect the structure of the game. In a dynamic game, on the other hand, current actions (e.g., extraction of a resource) do directly affect the future game (e.g., the amount of resource remaining).

4 Our use of the term 'perfect' includes situations where there is a single strategic agent. This term usually appears in the context of a game, which necessarily involves at least two strategic agents. In the light of the previous discussion, and in the context of the models studied below, our use of the term should not cause confusion.
since the requirement of perfection does not significantly narrow the range of candidates for equilibrium, additional assumptions are needed if we are to move beyond vacuous statements. A further refinement which is frequently employed, and which often gives rise to a unique equilibrium, is to restrict agents’ information sets (i.e., the variables upon which they can condition their strategies) to variables which directly affect their current and future pay-offs. The resulting equilibria are referred to as Markov. In resource problems the current stock of the resource provides a natural state variable. Whether the restriction to Markov equilibria is reasonable depends on the specific market under study. Although there can be no general answer to this question, Markov equilibria still merit study, since they (often) provide a simple and plausible way of achieving uniqueness.

The following is an example of a situation where the Markov equilibrium need not be unique. Suppose that a group of countries exploit a common property resource, such as the Antarctic, and their welfare depends on the amount of the resource they extract and also on environmental quality. One non-cooperative equilibrium involves all countries ignoring the externality of their actions as regards the environment — that is, they ignore the fact that degradation of the environment lowers not only their own utility, but that of their rivals as well. This equilibrium involves excessive environmental degradation. An alternate non-cooperative equilibrium has each country believing that their rivals will behave as above if the environmental quality falls below a certain level, but will behave responsibly if the quality is maintained at a higher level. This credible threat may be enough to sustain responsible behavior. The equilibrium is Markov, since agents’ actions depend only on variables that have intrinsic importance (e.g., environmental quality) and are currently observed. In this equilibrium, the actions are discontinuous in the state, since there is a sudden regime change if quality falls below a certain level. Such discontinuities are typical of equilibria that involve threats.

2.3. Consistency and the principle of optimality

When attention is restricted to Markov equilibria the decision problems facing the agents become much simpler to describe. Each agent chooses a (possibly

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5 On this interpretation an open-loop strategy is the only feasible choice if the information set is restricted to calendar time.

6 These equilibria are sometimes called ‘Strong Markov’ to emphasize that current decisions are conditioned only on the current state, and not (directly) on actions that occurred even in the previous period.

7 It is more difficult to punish deviations from equilibrium when these deviations cannot be directly observed, as is the case when the state consists of only the current resource stock. As the threat of punishment decreases, it becomes harder to enforce ‘good’ behavior, and the range of possible equilibria thus narrows.
nonstationary) function of the state to maximize its objective, taking as given the
decision rules of other agents. Just as a static game can be thought of as a system
of static optimization problems, so a dynamic game can be thought of as a system
of control problems. Except where otherwise indicated, we consider deterministic
environments, and confine the discussion to pure strategies. It is well known that the
open-loop and feedback solutions to deterministic control problems (as opposed
to dynamic games) are the same; that is, in such a problem it does not matter if the
decisionmaker chooses a time path for his control (an open-loop policy) or a state-
contingent (feedback) control. However, as the previous discussion emphasized,
each agent's control problem differs depending on whether he takes as given
his rivals' actions or their decision rules. Therefore the open-loop and feedback
equilibria in dynamic games typically differ.

The feedback equilibrium is, by construction, perfect (and thus dynamically
consistent)\(^8\), since each agent’s decision rule is required to be individually rational
(optimal) from every possible state, and not simply the states that occur in
equilibrium. The open-loop equilibrium is unlikely to be perfect, but it may be
time-consistent. A simple observation, based on control theory, provides a sufficient
condition for the time consistency of the open-loop Nash equilibrium in a dynamic
non-cooperative game. We now turn to a discussion of this condition, which is used
in subsequent sections.

Define a ‘fixed initial state’ control problem as one in which the initial condition
of the state variable is exogenous. For deterministic fixed initial state control
problems, Bellman's Principle of Optimality implies that the continuation of an
optimal program is optimal from states along the trajectory that arises from optimal
behavior in the past. It is tempting to conclude from this Principle that the open-
loop Nash equilibrium in a non-cooperative dynamic game is time-consistent. The
reasoning is as follows: Consider matters from the vantage point of agent \(i\), who
solves a control problem in which he takes as given the future actions of his
rivals (since the equilibrium is open-loop). If no-one has deviated in the past, and
\(i\) does not expect any agent to deviate in the future, then the Principle of Optimality
suggests that \(i\) will also not want to deviate from the program that was optimal at the
initial time. Since this holds for all agents, the open-loop equilibrium appears to be
time-consistent. The problem with this argument is that often the control problem
that agent \(i\) faces is not a fixed initial state problem, and the Principle of Optimality
is not applicable. This is likely to be the case even if agent \(i\) is the only strategic
player in the game, provided that there are other agents who are forward looking.

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\(^8\) Note that the ‘feedback equilibrium’, which we also refer to as the Markov equilibrium or the perfect
(Markov) equilibrium, is not the same concept as the ‘feedback form of the open-loop equilibrium’,
which we discuss below. In the latter case, ‘feedback’ means that decisions are represented as a function
of the current state of the system, but ‘open-loop’ implies that those decisions were drawn up at time
zero on the assumption that the agent would be committed to them throughout the game.
The Principle was enunciated, and is valid, for control problems in which the initial condition of the state variable is exogenous. In many economic problems this is not the case. A variable whose position at arbitrary time \( t \) is determined by future events (and possibly current events) is referred to as a 'jump state'. The initial condition of a jump state is endogenous. The Principle of Optimality is not valid in problems with jump states; in these situations the argument sketched above, which purports to show the time consistency of open-loop Nash equilibria to non-cooperative games, is also invalid. In macroeconomics, the standard example of a jump state is the exchange rate, which is affected by current interest rates and agents' expectations of future monetary supply. In many resource models with dominant agents, the price of a resource is a jump state, since the current price depends in part on agents' expectations of future policies, such as tariffs by large importers or sales by large suppliers.

The following example illustrates why the open-loop policy is typically not time-consistent if there is a jump state; the reasoning is the same whether there is a single strategic agent or two or more agents who play a game. Let there be two states: \( x \) is a normal state with given initial condition, and \( p \) is a jump state. For concreteness, we can think of \( x \) as the stock of the resource and \( p \) as the sales price. Their laws of motion are

\[
\frac{dx}{dt} = F(x, p, u, t), \quad x_0 \text{ given},
\]

\[
\frac{dp}{dt} = G(x, p, u, t),
\]  

(2.1)

(2.2)

where \( u \) is a vector whose components are the controls of \( n \) strategic agents.

For the specific example we can think of the strategic agents as large importers of a depletable resource; \( u(t) \) can be regarded as the vector of tariffs at time \( t \).

The equation of motion of \( p \) is the equilibrium condition obtained from the profit-maximizing behavior of competitive resource owners with rational (point) expectations. Agent \( i \) wants to maximize

\[
\int_0^T L_i(x, p, u, t) \, dt.
\]  

(2.3)

We can interpret \( L_i(\cdot) \) as agent \( i \)'s discounted sum of consumer surplus and tariff revenue. If there is a single importer with market power (\( n = 1 \)), that agent solves a control problem with a jump state. If there is more than one strategic importer, those agents play a non-cooperative game, in which each agent takes the time path of his rivals' tariffs as given; from agent \( i \)'s perspective, he still solves a control problem with a jump state. Agent \( i \)'s Hamiltonian is

\[ H_i = L_i + \mu_i F(\cdot) + \lambda_i G(\cdot), \]  

(2.4)
where $\mu_i$ and $\lambda_i$ are, respectively, $i$'s co-state variables associated with the ordinary state $x$ and the jump state $p$. If there is an interior solution to $i$'s problem he chooses his trajectory of controls in such a way as to result in an optimal value of the initial value of the jump state, $p_0$. Optimality of $i$'s program requires that $\lambda_i(0) = 0$. [See Simaan and Cruz (1973) for a statement of this result.] For problems in which $i$'s value function is differentiable in $p$, the economic interpretation of the co-state variable is as the shadow value of the state; the last equality says that at an interior optimum agent $i$ chooses the initial condition of the jump state so that its shadow value is zero. The equation of motion for $\lambda_i$ is

$$\frac{d\lambda_i}{dt} = -\frac{\partial H_i}{\partial p_i}. \quad (2.5)$$

In general, this expression does not equal zero at all points along the equilibrium path. Therefore, for some time $\tau > 0$, one has $\lambda(\tau) \neq 0$. If agent $i$ had the opportunity to revise the remaining part of his program he would (typically) wish to do so, even if no agent had deviated in the past and agent $i$ did not expect any other agent to deviate in the future. Agent $i$ would like to revise the remaining part of his trajectory in order to alter the value of $p(\tau)$, which, since $\lambda_i \neq 0$, is not optimal for $i$ at $\tau$. In this case, the open-loop Nash equilibrium is not time-consistent; if there is a single strategic player, his open-loop policy is not time-consistent.

2.4. Concluding comments

Before proceeding with the specific situations where the possibility of dynamic inconsistency arises, we summarize the main points of this section. We began by describing the source of dynamic inconsistency, and then discussed the difference between time-consistent and perfect equilibria. Perfect equilibria must be credible even if some agent deviates; time-consistent equilibria are defined without regard to out-of-equilibrium behavior. 'Folk Theorem'-type arguments imply that there typically exist many perfect equilibria; the restriction to Markov strategies can often be reasonably motivated and may imply uniqueness. If a problem contains no jump states, the Principle of Optimality implies that open-loop equilibria are time-consistent, although they generally fail to be perfect. If a problem contains jump states then the open-loop equilibrium, in which agents announce at the beginning of the game their trajectory of controls for the entire game, will usually not even be time-consistent. There are exceptions to this rule of thumb. The following sections show by example that the open-loop equilibrium may be time-consistent when there are jump states, and market power resides with either the buyers or the sellers of a depletable resource.
3. Strategic buyers, competitive sellers

The previous section alluded to the situation in which strategic buyers attempt to extract rent from competitive suppliers of a non-renewable resource. This section considers the problem in greater detail. The simplest situation, reviewed first, occurs where there is a single buyer of a depletable resource, i.e. a pure monopsonist. Next we consider the situation where a dominant buyer confronts competitive sellers and competing, but non-strategic buyers, and finally we examine the case of symmetric strategic buyers and competitive sellers.

3.1. A pure monopsonist

The situation in which a pure monopsonist attempts to exercise market power when confronted with competitive sellers of a depletable resource provides a particularly simple illustration of the problem of DI, studied by Karp (1984). It is helpful to begin by contrasting this case to a static model in which a monopsonist faces competitive producers of a reproducible good. To make the two situations comparable, suppose that there is a capacity constraint for the reproducible good so that in both cases aggregate production is limited. If the reproducible good is produced at constant cost (up to the capacity constraint) the monopsonist can drive price down to average cost. In this case the sellers obtain no rent (producer surplus), and consumption occurs at the socially optimal (competitive) level. If the reproducible good is produced at increasing costs, so that the monopsonist faces an upward-sloping supply schedule, consumption occurs where the monopsonist’s marginal utility of consumption intersects the schedule which is marginal to the supply schedule. In this case the competitive producers obtain some rent and consumption is less than the socially optimal level. These two possibilities are analogous to two cases that arise in the depletable resource model. If the resource is extracted at constant cost (until exhaustion occurs), the monopsonist is able to extract all rent, and consumes at the efficient rate. Extraction costs may, however, be stock-dependent, if mines are of different qualities or if it is more expensive to withdraw oil as the reservoir is depleted. The case of stock-dependent extraction costs corresponds to the case of an upward-sloping supply schedule in the static model: the monopsonist is unable to extract all of the rent, and the consumption

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9 This description assumes that the monopsonist is unable to discriminate. If he can, then he could extract all rent by paying different prices for different units of the good; alternatively, he could make an ‘all or nothing’ offer.

10 The analogy we have described is not the only possibility. One could also think of the situation where extraction costs depend on the rate of extraction, and compare this to the case of an upward-sloping supply curve in the static model. This analogy is less useful, since the problem of DI is caused by stock-dependent costs, but not by costs which depend only on the rate of extraction.
path is not socially efficient. However, there is an additional complication in the depletable resource model, and this gives rise to the problem of DI. With the depletable resource, consumption occurs over time, rather than at a point in time as in the static model. The rent that the monopsonist pays at any point in time depends on future rents, as discussed above, and these rents depend on future prices and future tariff rates. The best that the monopsonist could do would be to drive these rents to zero; in which case he would have no incentive to depart from the trajectory he announced at time zero. If, however, it is not feasible to force the rent down to zero, the monopsonist has an incentive to announce future tariffs to influence the rent he pays. As the discussion in Section 2.1 indicated, these incentives differ at different points in time, and this leads to the problem of DI.

To demonstrate these results, and to aid intuition, we use the following formal model. Let $S$ be the stock of a resource, $q$ be the flow of extraction, $c(S)q$ the total cost of extracting at rate $q$, and $r$ the interest rate. Under the assumption of no storage, $q$ equals the monopsonist’s rate of consumption. The cost function implies that extraction costs (may) depend on the remaining stock level, but not the rate of extraction (see footnote 10). Define $\rho$ as the competitive seller’s current value of rent on the resource and $p$ as the market price of the resource. The variables $S$, $q$, $p$ and $\rho$ change over time, but we suppress this dependence. When the non-negativity constraint on extraction is not binding, the following conditions must hold:

$$p - c(S) - \rho = 0, \quad \frac{d\rho}{dt} = r\rho + c'(S)q.$$ (3.1a,b)

Equations (3.1a,b) imply the following intertemporal arbitrage condition

$$\frac{dp}{dt} = r[p - c(S)].$$ (3.2)

We can rewrite eq. (3.2) in integral form as

$$p(t) = p(T)e^{r(t-T)} + r \int_t^T e^{r(t-\tau)} c[S(\tau)] \, d\tau,$$

where $T$ is an arbitrary time greater than $t$ at which extraction is positive. If $c$ is independent of $S$ this equation can be simplified to state that the present value of rent (price minus cost) must be equal at any two points, $t$ and $T$, when extraction is positive. This is the standard Hotelling intertemporal price arbitrage equation. If $c$ depends on $S$, there is an additional component to rent. When selling in the current period, the producer must consider not only the opportunity cost of not being able to sell that unit at a later period, but also the fact that current sales increase the cost of future extraction, because they lower the future stock. The
difference in the present values of current and future price must equal the interest payments of the present value of the stream of average extraction costs, given that extraction is chosen optimally from \( t \) to \( T \). [See Heal (1976) and Solow and Wan (1976).]

The monopsonist wants to maximize the discounted flow of consumer utility of consumption, \( U(q) \), minus payments to the seller, \( pq \):

\[
\int_0^\infty e^{-rt} [U(q) - pq] \, dt,
\]

subject to condition (3.2) and the stock depletion equation: \( \dot{S} = -q \). This is the type of control problem discussed in the previous section: there is a state with fixed initial condition, \( S \) (the stock), and one jump state, \( p \) (the price received by the seller). The current-value Hamiltonian of the monopsonist's problem is

\[
H = U(q) - pq - \mu q + \lambda r [p - c(S)],
\]

where \( \mu \) and \( \lambda \) are the co-states associated with \( S \) and \( p \), respectively. We can think of the buyer as choosing a profile of consumption or a profile of tariffs to support a consumption profile; the first interpretation is more straightforward. The necessary conditions for an interior solution to the buyer's problem include

\[
\begin{align*}
U'(q) - p &= \mu, \\
\frac{d\mu}{dt} &= r\mu + r\lambda c'(S), \\
\frac{d\lambda}{dt} &= q.
\end{align*}
\]

\( (3.3a-c) \)

Since \( U'(q) \) gives the domestic price for the monopsonist, eq. \( (3.3a) \) states that the unit tariff at time \( t \) equals the monopsonist's shadow value of the stock, \( \mu \). Equation \( (3.3c) \) and the boundary condition \( \lambda(0) = 0 \) imply that \( \lambda(t) = S_0 - S(t) \), where \( S_0 \) is the stock of the resource at time zero. (We discuss the interpretation of \( \lambda \) below.) We can use this relationship to eliminate \( \lambda \) from the system of necessary conditions. If we totally differentiate eq. \( (3.3a) \) with respect to time, using the remaining necessary conditions, we obtain the following second-order differential equation in \( S \):

\[
U''(q) \frac{dq}{dt} - r \left[ U'(q) - c(S) + (S_0 - S)c'(S) \right] = 0,
\]

\( (3.3d) \)

where \( \frac{dq}{dt} = -d^2 S/dt^2 \). The solution requires two boundary conditions. One of these is supplied by the initial condition \( S_0 \) and the other is supplied by the transversality condition to the buyer's control problem.

There are three noteworthy features about eq. \( (3.3d) \). The first of these is that the term \( S_0 \) appears in the equation only where it is multiplied by \( c'(S) \). Therefore the time derivative of consumption depends directly on the initial condition if and only
if $c'(S) \neq 0$. If the importer were able to re-solve the problem at some $t > 0$, i.e., renege on the initial promise, then the initial condition of his new problem would be $S(t)$ rather than $S_0$. If $c'(S) = 0$, the dynamics of equilibrium consumption in the new problem are identical to those of the old; in this case the seller has no incentive to change the path of consumption (or tariffs) that he announced at the initial time, so the open-loop policy is time-consistent. If, however, $c'(S) \neq 0$, the equilibrium dynamics are different for the new problem, and the initial policy is not dynamically consistent.

The intuition behind this result was given above. If $c'(S) = 0$, the seller is able to choose a path of consumption so that the initial price is $p_0 = c$. Using eq. (3.2) and $c(S) = 0$, this implies that $p(t) = c$ for all $t$. That is, the use of an open-loop tariff enables the monopolist to expropriate the stock. He is then able to consume optimally, so he has no incentive to alter his plans. If, however, $c'(S) \neq 0$, the price arbitrage equation (3.2) is binding (i.e., reduces the importer's pay-off), and at time zero the importer has to trade off current and future benefits. This trade-off changes over time, as 'the present becomes the past'.

The fact that the arbitrage condition is, in most cases, binding, provides one explanation why competitive owners of a depletable resource may be reluctant to permit foreign ownership of their stock of the resource, and instead prefer to sell a flow. If resource owners were willing to sell the stock and importers were monopolistic, the latter could make the resource owners a 'take it or leave it' offer, and essentially expropriate the resource [as Dasgupta and Heal (1979) point out (pp. 335–336)]. If, however, the exporters are committed to retain ownership of the stock and the importers can only use tariffs, the exporters obtain some rent (provided that $c'(S) \neq 0$). This holds even under the somewhat unrealistic assumption that the importer could commit himself to an open-loop policy. If the importer cannot commit, the exporters would generally do even better. Maskin and Newbery (1990) point out that a two-part tariff, in which the importer charges a time-varying fixed fee for an import licence, as well as a unit or ad valorem tariff, would permit the importer to extract all the rent.

The second and third features of the extraction path are not surprising. Whenever $c'(S) \neq 0$, extraction is more conservative under the monopolist who uses the open-loop policy than under perfect competition (a zero tariff). That is, at any time after the initial instant, there is more stock remaining under the monopolistic than under the competitive regime. Finally, if $c'(S)$ is not identically zero, aggregate consumption may be lower under the monopolist.

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This is an example of the general principle that enlarging the dominant agent's set of control variables typically gives him more power. It may increase his power to such an extent that he is able to exactly control the followers' (in this case, the sellers') behavior, so that the problem of DI does not arise. Hillier and Malcolmson (1984) provide an example of this in a two-period game and Papavassilopoulos and Cruz (1980) provide an example in a Stackelberg differential game.
It is worth spending a moment on the interpretation of the co-state variable \( \lambda \). If the buyer's value function is differentiable in \( p \), then \( \lambda \) can be interpreted as the shadow value of price. That is, suppose that at time zero the buyer announces a profile of consumption (or tariffs), which induce a price at every point in time, \( p(t) \), and an associated value of the co-state variable, \( \lambda(t) \), \( t > 0 \). If the value function is differentiable, \( \lambda(t) \) gives the value to the buyer of a marginal increase in \( p(t) \).

If \( c'(S) \neq 0 \), we can interpret the co-state variable \( \lambda \) as the buyer's shadow value of price. Given a value of \( p(t) \), the buyer at \( t \) faces a standard control problem which induces a value function defined over \( S \) and \( p \). If \( c'(S) \neq 0 \) it can be shown that the seller's price is strictly greater than extraction costs at every point along the open-loop trajectory (except at the last instant, \( T \), when this is finite). Therefore, for all prices along the open-loop equilibrium (for \( t < T \)) a small positive or negative change in the current price leads to a small change in the buyer's welfare. His value function is continuous in price. If \( U(q) \) and \( c(S) \) are smooth and have the right curvature properties, the buyer's value function is also differentiable in price. [See Benveniste and Scheinkman (1979) and Seierstadt and Sydsæter (1987) for a discussion of differentiability of the value function.]

The boundary condition \( \lambda(0) = 0 \) and eq. (3.3c) imply that \( \lambda(t) \) is positive for \( t < 0 \). In the situation where \( \lambda(t) \) can be interpreted as the buyer's shadow value of the price, this implies that if the buyer at time \( t \) were able to renege on the open-loop path announced at time zero, he would alter the program in a way which increases the price received by the seller. This observation should not be surprising in light of the two previous remarks that the open-loop extraction path is more conservative than the efficient path, and that the open-loop path need not exhaust the resource (i.e., cumulative extraction may be less than under the efficient regime). If we think of the buyer choosing a tariff rather than a consumption profile, we can interpret the

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12 This interpretation is not valid if extraction costs are constant since in that case the value function is not differentiable in price at \( p = c \). A small increase in the current price lowers the monopolist's pay-off by a small amount, since this transfers rent to the seller without altering the extraction path. However, an arbitrarily small decrease in \( p \) below \( c \) causes the buyer's pay-off to fall to zero, since the resource owner will not sell at a price below extraction costs. (So \( p = c \) must be the optimal price path for the buyer.) Since the value function is not continuous in \( p \) at \( p = c \), it is clearly not differentiable; in this case \( \lambda \) cannot be interpreted as a shadow value. In fact, when \( c'(S) \equiv 0 \) we can think of the monopolist choosing a constant price rather than a function of time; the state variable \( p \) is replaced by a constant. This approach has the advantage of not causing confusion over the interpretation of the co-state variable \( \lambda \), but it impedes a unified treatment of the problem.

13 If the buyer at \( t \) regarded \( p(t) \) and eq. (3.2) as given, he would have no incentive to depart from the open-loop trajectory. The problem of DI arises precisely because he does not take \( p(t) \) as given, unless he is in some way committed to continuing on the open-loop path announced at \( t = 0 \).

14 This is the condition that fails when \( c'(S) \equiv 0 \).

15 One way to think about the different cases \( c'(S) \equiv 0 \) and \( c'(S) \neq 0 \) is to note that in the former the open-loop equilibrium price path is on the boundary of the set of feasible price paths (those which induce positive extraction), whereas in the latter case the open-loop equilibrium is in the interior of that set.
open-loop path as a threat. The buyer threatens to use a high tariff in the future in order to obtain the resource for a low price in the current period. The buyer would prefer not having to carry out the threat: he would like to defect to a program with a lower tariff and a higher price trajectory for the seller.

It is clear that the open-loop policy is not perfect (since it is not even time-consistent) whenever \( c'(S) \neq 0 \). The open-loop tariff is not perfect even if \( c'(S) = 0 \), but a perfect equilibrium can sustain the same result. From eq. (3.3), the open-loop unit tariff rises at the rate of interest when \( c'(S) \equiv 0 \). Suppose that over some interval the importer deviates from the open-loop tariff, so that at the end of that interval there is either more or less remaining stock than would have been the case along the open-loop trajectory. If the importer were then allowed to resolve his problem, he would not want to return to the open-loop policy, which, we emphasize, gives the tariff as a function of time and the initial condition at \( t = 0 \). The reason is that that policy would not result in the optimal profile of consumption, given that the state is off the equilibrium path. [To see this, notice that the time profile of the co-state variable \( \mu(t) \) implied by the original open-loop policy does not solve the importer's problem at \( t \) given an off-the-equilibrium value of \( S(t) \); recall that \( \mu \) gives the value of the open-loop unit tariff.]

Thus, the open-loop policy is not perfect even if \( c'(S) \equiv 0 \). However, the open-loop path can be sustained as a (Markov) perfect equilibrium. In order to do this we can write the open-loop policy in 'feedback form'. By this we mean that we solve for the open-loop tariff, \( \mu \), as a function of the state, \( S \). In principle this is easy to do. We have already discussed how to obtain a differential equation for consumption in the open-loop equilibrium. We can solve this to obtain the future trajectory of \( S \) as a function of time in the future and an arbitrary value of the state at time \( t \), \( S(t) \). By substituting this function into the integral form of eq. (3.3b) we obtain \( \mu(t) \) as a function only of \( S(t) \). This gives the open-loop tariff as a function of the stock: it gives the 'feedback form of the open-loop policy'. This equilibrium is perfect. If the importer were to deviate over some interval, causing the stock to be off the equilibrium path, and if the importer were then given the option of resolving the problem, taking as given the level of the stock, he would want to use the open-loop trajectory that is optimal from the given stock level. That is, he would want to use the feedback form of the open-loop policy.

The following example may help to clarify this concept. Suppose that demand is \( 1/p \) for \( p \leq \bar{p} \) and zero for \( p > \bar{p} \). (The buyer has access to a backstop technology, and has a unit elastic demand.) Suppose \( c = 0 \), and let the tariff \( \tau(t) = \bar{p} e^{-r(t-T)} \), where \( T \) is such that the oil is just exhausted by that date. The equation for the stock remaining at date \( t \) is

\[
S(t) = \frac{e^{rT}}{\bar{p}} \left( e^{-rt} - e^{-rT} \right),
\]  

(3.4)
so the feedback form of the open-loop tariff can be written as

\[ \tau(t) = \frac{\dot{p}}{1 + r\beta S(t)} \]  

(3.5)

Any deviation or exogenous shock (an unexpected discovery) which changes \( S(t) \) can be promptly corrected and the tariff remains optimal given remaining stocks.

3.2. Monopsonist with fringe buyers

The situation where a monopsonist attempts to capture rents when resource owners are competitive and there are fringe (non-strategic) buyers was studied by Newbery (1976), Maskin and Newbery (1978, 1990) and Kemp and Long (1981). These papers assume that extraction costs are constant. We saw in the previous subsection that this assumption in a pure monopsony model leads to a very simple equilibrium. The introduction of competing buyers leads to the same sorts of complications as do stock-dependent extraction costs.

We begin by showing how the formal model of the previous section is altered by including competing buyers when their aggregate demand is given by the stationary function \( y(p) \); we maintain the assumption that \( c'(S) \equiv 0 \). The producer’s problem is unaltered. The stock dynamics facing the dominant importer are now given by \( \dot{S} = -[q + y(p)] \), where \( q \) now represents imports by the dominant buyer and \( q + y \) gives aggregate consumption. The buyer’s Hamiltonian is altered by including the term \( -\mu y(p) \). The necessary conditions (3.3a,b) must still hold, except that \( c'(S) \equiv 0 \) by assumption. The additional term \( \mu y'(p) \) must be included in eq. (3.3c); however, the rate of change of the unit tariff, \( \mu \), is independent of \( \lambda \) in this case.

The necessary conditions therefore imply that the dominant buyer’s unit tariff increases at the rate of interest; a constant present-value tariff is optimal. This was also true for the pure monopsonist, as noted in the previous subsection. To understand this result, imagine that at time zero oil importers bid for oil fields and then extract them as they wish. The dominant importer will bid for fewer than his domestic consumers collectively would bid for, because this enables him to drive down the price of the fields. Once the oil field is purchased, it pays the importer to extract it efficiently, which requires that the rent rise at the rate of interest; the price facing consumers is the extraction cost plus this rent. This result is duplicated by a tariff if the price paid by consumers is the same in both cases. This requires that the time path of rent for the imported oil (received by the oil producers) plus the import tariff should be the same as the time path of rent for the purchased and domestically exploited oil field. Only if the tariff rises at the rate of interest will this condition hold.

In general, the open-loop tariff is DI if there are competing buyers, even if extraction costs are constant. The clearest example of this occurs where both
the dominant importer and the fringe have the same choke price (the price at which demand falls to zero), \( \bar{p} \). At a finite time \( T \) the world price rises to \( \bar{p} \), so there is some interval of time before \( T \) when the world price is strictly less than \( \bar{p} \) but the price in the dominant country is no less than \( \bar{p} \). That is, there is an interval of time when the dominant importer has ceased importing even though world price is less than its choke price. During that interval the dominant country would certainly want to renege from its open-loop tariff, which is therefore DI. Newbery (1976) demonstrated that if both the dominant importer and the fringe had the same constant elasticity demand functions (in which there is no choke price), and extraction costs were zero, the open-loop policy is time-consistent. We consider a slightly more general version of this result in the next subsection.

As in the case without a fringe, the open-loop tariff is a threat: the importer is able to import at a low price in the current period by threatening a high tariff, and a resulting low world price, in the future. Distinguishing open-loop policies on the basis of whether they constitute threats or promises is sometimes useful.

Arguably, it is harder to precommit to threats than to promises, since in the former case both the threatener and the threatened would like to permit a defection at a later time; with promises, it is in the interest of one of the parties to make sure that the open-loop plan is followed.

The open-loop tariff is the unique optimal program from the standpoint of the dominant importer at the initial time, and it is (typically) not time-consistent. Therefore, it is generically true that requiring the importer to use a perfect strategy would reduce the discounted stream of his pay-off (below the level obtained in the open-loop equilibrium). Maskin and Newbery (1978, 1990) demonstrate a stronger result; they show that in a perfect equilibrium the dominant importer's total discounted pay-off may be lower than the level he would have achieved had he been able to commit to a zero tariff in every period, i.e. behave competitively. Such a commitment is not feasible in a perfect equilibrium. This result implies that market power may be disadvantageous.\(^{16}\)

It is easiest to obtain a perfect equilibrium in a discrete time setting. We first consider the case were there are two periods, and then discuss how the problem generalizes to an arbitrary number of periods.\(^{17}\) A two-period model means that the resource is worthless after the end of the second period; one justification for such an assumption is the anticipated introduction of a substitute which is cheaper to produce than the extraction costs. The solution method uses dynamic programming. In the second period the dominant importer is faced with a standard optimal tariff problem. The dominant importer chooses the optimal point to

\(^{16}\) There is a growing body of literature that recognizes that increased market power, or, in an international policy framework, increased cooperation, does not necessarily improve welfare.

\(^{17}\) If there are a finite number of periods the perfect equilibrium obtained using the dynamic programming approach is often unique. The types of punishment strategies discussed in Section 2 cannot be used to support other outcomes, since the punishments always 'unravel'.
consume on the excess supply curve which, for prices greater than the extraction cost, is given by \( S_2 - y_2(p) \). Here \( S_2 \) is the remaining supply at the beginning of period 2, and the subscript on the fringe’s demand function indicates that it may be non-stationary. The solution to this problem induces an equilibrium price function, \( p_2(S_2) \). Competitive producers have rational expectations and therefore understand that price in the second period will be given by this function, even though they do not behave strategically. If the resource is extracted in both periods, competitive arbitrage requires that \( p_1 = \beta \left[ p_2(S_2) - c \right] + c \), where \( \beta \) is the discount factor. Rational expectations and competitive arbitrage may make it very costly, and perhaps impossible, for the dominant importer to influence the time path of extraction; this is the reason why market power may be disadvantageous.

An extreme example illustrates this. Suppose that \( y_2(p) \) is arbitrarily close to zero and \( y_1(p) \) is large; that is, the fringe nearly disappears in the second period but has high demand in the first period. In this case the dominant importer can and will essentially expropriate whatever of the resource remains in the second period. Competitive arbitrage then implies that virtually all of the resource is extracted in the first period: producers have no incentive to delay extraction when they know that they will obtain a very low price in the second period. If the dominant importer’s utility of consumption is low in the first period, relative to his utility in the second period, his total discounted utility under the perfect equilibrium is lower than under the equilibrium in which he behaves competitively (uses no tariffs). (To verify this, think of the limiting case where he has no utility of consumption in the first period and high utility in the second period – the opposite of the fringe’s situation.)

This argument does not rely on there being only two, or even a finite number of periods. Consider a continuous-time infinite-horizon model in which the fringe’s choke price is finite but its demand is very elastic below this price. In this case (under a perfect equilibrium) price never rises above the fringe’s choke price, and exhaustion occurs relatively quickly (due to the assumption of elastic fringe demand). Suppose also that the dominant importer’s demand is positive at the fringe’s choke price and is very inelastic at all prices. With this pair of demand functions, the dominant importer’s total utility is likely to be lower in a perfect equilibrium than in the perfectly competitive regime. As in the two-period example, the reason for this result is that in the perfect equilibrium the dominant importer consumes much less than he would have under perfect competition.

In a perfect equilibrium the possibility of disadvantageous monopsony power arises because of the competing importers: the dominant importer is not a pure monopsonist. In the absence of a fringe, monopsony power cannot be disadvantageous. This is easy to see in the two-period model. In the last period the monopsonist has a static problem, so market power can not be disadvantageous. For any stock level the monopsonist will charge a tariff, resulting in a lower price than would his competitive counterpart. In the second to the last period the monopsonist
could charge a zero tariff; if he were to do so the level of sales would be greater than under the competitive regime, due to the arbitrage equation and sellers' rational anticipation of a tariff in the second period. By charging a positive tariff in the penultimate period the monopsonist can induce the same level of sales as occur in the competitive regime. That is, it is feasible for the monopsonist to duplicate the competitive extraction path but pay lower prices in both periods: so market power cannot be disadvantageous. This argument can be extended to an arbitrary number of periods.

Maskin and Newbery (1990) show that forward trading or storage can prevent disadvantageous outcomes. Futures markets allow the dominant importer to sell oil forward at an agreed price, which means that he will make losses on these futures sales if the second-period price is driven down, thereby effectively committing himself to the open-loop equilibrium. In the case of storage, negative stocks are infeasible, but the only case in which market power is disadvantageous is when the importer's future demands are relatively high -- and if he stores oil he can meet this demand from his own stocks. Of course, futures markets typically have a limited future coverage, and storage is expensive relative to leaving oil in the ground, so that these options may not alleviate the problem of disadvantageous market power in practice.

In summary, both the requirement of perfection and the presence of competing buyers are necessary for market power to be disadvantageous. The dominant agent's power is 'incomplete' or 'imperfect' in two senses: he cannot control the behavior of other buyers (since he is not a pure monopsonist), and he cannot make credible promises about his own behavior in the future (since he is restricted to using perfect strategies). The removal of either of these 'imperfections' is sufficient to ensure that market power is advantageous.

3.3. Oligopsonistic buyers

We saw in the previous subsections that the problem of dynamic inconsistency arises in the cases where a single buyer faces competitive sellers with stock-dependent costs, or a dominant buyer faces a fringe of competitive buyers. The situation where more than one strategic buyer uses open-loop tariffs presents a minor variation of the latter case. Since each buyer takes the tariff trajectories of his rivals as exogenous functions of time, he behaves as if he were facing competition represented by a nonstationary rest-of-world demand, \( y(p_t) \). The fact that the form of the non-stationarity is caused by strategic behavior is of no concern to any individual buyer. Therefore the results of the previous subsection are not altered: the open-loop unit tariff rises at the rate of interest, and in most cases is time-inconsistent.
Bergstrom (1982) points out that if extraction is costless and importers impose a constant *ad valorem* tax on consumption, the entire incidence of the tax falls on producers. This result holds because of two equilibrium conditions which must hold for any consumption tax. First, the intertemporal arbitrage condition requires that the producers’ rent, which equals price (since extraction is costless) must rise at the rate of interest. The price at time \( t \) is therefore \( p_0 e^{rt} \), where \( p_0 \) is the producers’ initial price. The (constant) *ad valorem* tariff can affect the price trajectory only by affecting the initial price. The second equilibrium condition is that cumulative consumption equals the initial stock level; this is the ‘exhaustion condition’. Now compare two regimes with arbitrary but different (constant) *ad valorem* tariffs. Due to the arbitrage condition and the assumption of zero extraction costs, the ratio of producer prices in the two regimes is a constant (given by the ratio of the initial prices). We now use the exhaustion condition to show that the consumer prices in the two regimes are the same. By the previous remark, the ratio of consumer prices is a constant, given by the product of the ratio of the initial producer price and the ratio of 1 plus the tariff. This means that the two consumer price trajectories are either identical or one lies strictly above the other. In the latter case, the exhaustion condition cannot be satisfied in both regimes. Therefore it must be the case that the trajectories of consumer price are the same for both regimes: the tariffs affect producer prices but leave consumer prices unchanged. The entire incidence of the tax falls on producers when extraction costs are zero.  

Bergstrom studies a one-shot non-cooperative game amongst importers in which they each optimally choose a constant *ad valorem* tariff, and each takes the tariffs of other importers as given. By the previous argument, producers bear the entire incidence of the tax (for zero extraction costs). He points out that in general a constant tariff is not an optimal response, except for the special circumstance where all importers have constant elasticity of demand. In that case, the open-loop Nash equilibrium does involve a constant *ad valorem* tariff.

Karp and Newbery (1992) show that if the importers in the non-cooperative game are identical and if extraction costs are a constant, \( c \), then the open-loop Nash equilibrium to the game in which importers choose unit tariffs is time-consistent if and only if demand in country \( i \) is \( q^i(p) = \alpha_i k(\sigma_i - c)^{-\gamma} \), where \( \sigma_i \) is the domestic,  

\[ q^i(p) = \alpha_i k(\sigma_i - c)^{-\gamma} \]

18 If extraction costs are constant but positive, a constant *ad valorem* tax does alter the price trajectory that consumers face, so consumers bear some of the tax burden. In this case a constant tax based on the difference between the producer price and cost shifts the entire burden to the producer and is equivalent to a rent tax.

19 When there is a dominant agent using an open-loop policy, we can think of him as choosing either a consumption trajectory or a tariff trajectory to support that consumption trajectory. In the non-cooperative game each agent takes his rivals’ tariffs as given. The individual is indifferent between using price and quantity controls; but it does matter what each agent takes as given. That is, importer \( i \)’s optimization problem differs depending on whether he takes as given his rivals’ import trajectories or their tariff trajectories, and in the latter case it matters whether he takes as given unit or *ad valorem* tariffs.
tariff-ridden price. If the scaling parameter \( \alpha \) is country-specific, this form is sufficient for the open-loop Nash equilibrium to be time-consistent. The condition for consistency is stronger, when there is more than one buyer, than in the pure monopsony case. In the latter case constant costs are necessary and sufficient, whereas the former requires constant costs, a particular demand function, and a particular relation between the parameters of the demand function and cost.

The conditions given above for time consistency of the open-loop Nash equilibrium hold if importers choose unit tariffs. If, however, importers choose ad valorem tariffs and the demand functions are as shown, the additional restriction that \( c = 0 \) is required for the open-loop equilibrium to be time-consistent. In general the open-loop Nash equilibria differ depending on whether unit or ad valorem tariffs are used; constant elasticity of demand and zero extraction costs provide an exception. Static games also predict that Nash equilibria differ depending on whether agents use ad valorem or unit tariffs, so it is not surprising that the same result carries over to dynamic games. Recall that for any level of constant extraction cost the open-loop unit tariff rises at the rate of interest. Therefore the ad valorem equivalent is a constant if and only if world price also rises at the rate of interest, which requires that extraction costs be zero.\(^{20}\)

In the case where the pure monopsonist confronts competitive producers with constant extraction costs, we saw that although the open-loop tariff is not perfect, the same result could be sustained as a perfect equilibrium using the feedback form of the open-loop tariff. In general, this procedure does not work if there is more than one strategic agent. Consider the case of constant extraction costs and the demand function \( q^*(p) = \alpha' k (\sigma^t - c)^{-\varepsilon} \) so that the open-loop unit tariff is time-consistent. In principle it is straightforward to find the feedback form of this function; that is, to write \( i \)'s (open-loop) equilibrium unit tariff as a function of the current stock. If, however, \( j \) takes the function rather than the tariff level as given, his decision problem is altered; he then know that he will be able to affect the trajectory of \( i \)'s future tariffs by deviating from his own open-loop tariff level, and thus altering the rate of extraction. This result holds even if \( c = 0 \). If, however, \( c = 0 \) and the importers (with constant elasticity of demand) choose ad valorem tariffs, then the open-loop tariff can be sustained using the feedback form of the open-loop policy. The reason here is that the (open-loop) equilibrium involves a constant ad valorem tariff, so the feedback form is simply the trivial function: the equilibrium tariff is independent of the stock.\(^{21}\) Therefore importers have no incentive to deviate from their equilibrium tariff in the hope of inducing their rivals to deviate.

\(^{20}\) It is likely that this restriction could be relaxed by, for example, making the ad valorem tax depend on profit (rent) rather than price.

\(^{21}\) The proof of independence follows from the fact that the constant ad valorem tariff is time-consistent in this case. The range of possible stock levels is from zero to the initial stock. By the fact of consistency, the constant tariff chosen at the initial time represents an equilibrium at all initial times, and therefore from all initial stocks.
From these remarks it should be clear that open-loop strategies are unlikely to result in a perfect equilibrium. It is worth considering how to construct such an equilibrium. For reasons discussed above, we restrict attention to Markov equilibria. For discrete-time models with general functions, dynamic programming can be used; this approach becomes unwieldy if there are more than two periods. The infinite-horizon continuous-time model is often easier to work with. Markov equilibria in these models involve decision rules for agents; these give the current control (e.g. tariff) as a function of the current stock level. In a stationary model it is reasonable to look for stationary decision rules. These decision rules and the optimizing but non-strategic behavior of producers induce a function which gives the current price as a function of the current stock level. Although the price function is endogenous to the game, each importer (even where there is a single importer) takes it as given. This price function is a sufficient statistic for any single player. He does not have to know his rivals' decision rules; he is able to predict how he himself will behave in the future, but is unable to commit himself to future actions which will be suboptimal given the stock level at the time the actions occur. (Importers are unable to make binding commitments.) The problem then is to find a price function such that when all importers take this function as given, their optimal decision rules and producers' behavior induce that price function.

It is easy to solve this equilibrium in the case where all agents have linear demand functions with the same choke price, \( \bar{p} \), and the cost of extracting at rate \( Q \) is \( (k_0 - k_1S)Q \), provided that \( \bar{p} < k_0 \). A linear price function and linear decision rules provide a stationary equilibrium for this model. The importance of the last inequality is that it insures that the non-negativity constraint on the stock is satisfied. If that inequality is not satisfied, there is no linear equilibrium. The problem with linear demand and constant extraction costs is consequently more difficult. An analytic characterization of the equilibrium for that case has not been obtained. Karp and Newbery (1991) show that it is given by the solution to a system of ordinary differential equations; this can be solved numerically, and their solutions are discussed in the reference.

There are several features of the Markov equilibrium that are apparent from its definition, and other features which can be determined from numerical solutions to particular problems. Since the equilibrium price must be increasing over time and the stock level must be decreasing over time, given any initial stock level greater than zero, it must be the case that the equilibrium price function decreases in the stock level. Since each importer takes the current stock and the price function as given, he takes as given the current price. The incentive to use the tariff remains, however. Current tariffs affect the rate of extraction and thus the evolution of the stock, and thereby affect future prices and future tariff revenues.

Numerical solutions provide further insights into the perfect tariff. For the case of \( n \) identical importers with linear demand, and constant cost of extraction, the perfect (unit) tariff is non-monotonic in the stock, as Figure 1 shows. [This figure,
taken from Karp and Newbery (1991), gives the tariff levels when the terminal price $\hat{p} = 10$ and $c = 1$. The figure shows two other trajectories, labelled 'Nash' and 'reneged'. These are described below. For low levels of stock, the equilibrium tariff is increasing in the stock. For large stock levels the tariff is decreasing. Since there is an inverse relation between the remaining stock and calendar time, this means that for large initial stock levels the tariff initially increases and later decreases over time. In the Markov equilibrium (as in the open-loop equilibrium) player $i$'s unit tariff equals $i$'s shadow value of the stock. The previous remarks imply that $i$'s value function is concave in stock at large stock levels and convex at low stock levels. To understand why, consider the limiting cases as the stock approaches either infinity or zero. In the former case, the fact that $i$'s value function is bounded above (because of discounting) and non-decreasing in stock, implies that it must be concave (under the maintained hypothesis that it is differentiable). The value function is also bounded below by zero, and is equal to zero when the stock is exhausted. Moreover, the shadow value of the stock (for player $i$) approaches zero as the stock approaches zero: as the importers compete for an arbitrarily small remaining stock they drive the price to their choke price. These remarks imply that the value function must be convex near the origin.

The economic interpretation is as follows. When the stock is arbitrarily large the situation of the buyers approximates that of importers of an ordinary good that
can be produced at constant costs. In this case the excess supply curve facing any importer is perfectly elastic, and it is optimal to impose a zero tariff. As the stock becomes very small competition amongst the importers for the remaining supply intensifies, as suggested above. This is because the supply (a flow) must approach zero as the stock approaches zero (as can be shown using a proof by contradiction), so the excess supply facing any importer is again infinitely elastic and a zero tariff is optimal. For intermediate stock levels a positive tariff is optimal; hence the non-monotonicity.

Karp and Newbery (1989) compare the perfect equilibrium to a 'reneged or naive open-loop equilibrium'. In the latter, at each moment importers choose a trajectory of open-loop tariffs. However, only the tariff in the first period (instant) is used. The importers are able to continually revise their policies. This is an implausible description of a market (why would anyone continue to believe the announcements?) but it is useful as a benchmark. The reneged open-loop tariff lies above, but has the same shape as, and appears to provide a good approximation to the perfect tariff, as Figures 1 and 2 show. The approximation improves with the number of importers, \( n \), and is very close for three or more. The reneged open-loop tariff is useful because its simplicity makes it easy to compute even for asymmetric games – thus Karp and Newbery (1989) compute it for the case of the single dominant importer. To the extent that it approximates the perfect equilibrium
it is an appealing alternative to a difficult computational problem. Figure 1 also illustrates that the naive open-loop tariff tends to be more 'pro-competitive' than the Markov tariff. When agents use Markov strategies they have an incentive to 'preempt' their rivals by using a relatively low tariff in order to consume more of the stock, and thus influencing rivals' future tariffs. This preemptive incentive is absent with open-loop strategies.

We conclude this section with an alternative description of buyer and seller interaction. The model discussed above assumes that at each moment (i.e., each period) importers choose their tariffs knowing how sellers will arbitrage supply intertemporally. Importers thus make their decisions before sellers in each period while sellers have rational point expectations of how importers will behave in the future. In this case, importers view aggregate supply at each moment as perfectly elastic. An alternative is to assume that buyers and sellers make their decisions simultaneously within a period, in which case the outcome is the same if sellers move first. Each agent's decision is conditioned on the current level of the stock. As above, importers are strategic and sellers are price takers with rational expectations. If importers take sellers' decision rules as given, and take the current stock level as given, then at each instant they take the current aggregate flow of supply as fixed. In contrast to the previous model, importers here act as if current aggregate supply is completely inelastic; they play a succession of static tariff setting games. In this case the equilibrium tariff for country \( i \) is

\[
\tau_i = \frac{q^i}{\sum_{j \neq i} q^j / \partial p}.
\]  

(3.6)

where \( \tau_i \) is the tariff imposed by country \( i \) and \( q^j \) is demand by country \( j \). When the demand schedule is linear, so that demand by \( i \) is \( q^i = \alpha_i \beta(\bar{p} - p - \tau_i) \), and \( \alpha_i \) is the market share of country \( i \) when the aggregate (untaxed) demand schedule is \( Q = \beta(\bar{p} - p) \), then the formula for the tariff is

\[
\tau_i = \alpha_i (\bar{p} - p).
\]  

(3.7)

The resulting equilibrium, referred to simply as 'Nash', is graphed in Figures 1 and 2.

---

22 The papers by Fershtman and Kamien (1987), Reynolds (1987), Karp and Perloff (1988) and Van der Ploeg (1987) provide other examples where Markov strategies are more pro-competitive than open-loop strategies. Fudenberg and Tirole (1986), discuss situations where Markov strategies are either more or less pro-competitive than open-loop strategies.

23 This terminology is not entirely satisfactory, since both tariffs are Nash and both are perfect. In Karp and Newbery (1991) we use the inexact but more accurate terminology PIMF and PEMF – 'perfect, importers move first' and 'perfect, exporters move first'. The former is shortened to 'perfect' and the latter to 'Nash', purely because perfect sounds as though it involves a more complex determination, and Nash sounds as though it is simpler, both of which are the case here.
4. Strategic sellers, competitive buyers

We have seen in the previous section that if buyers are strategic, and sellers both competitive and able to arbitrage instantaneously prices by switching their sales between different time periods, then open-loop import tariff plans are DI, except for special cases. There are a number of papers characterizing the intertemporal equilibrium of an exhaustible resource where there is market power on the sellers' side, in contrast to the rather small number which look at the buyers' or importers' side. The consensus is that the open-loop Nash–Cournot equilibrium in which the sellers each take the extraction profile of the other sellers as given, is time-consistent. The obvious question to ask is "wherein lies the difference in the two kinds of problems which explains the difference in the dynamic consistency of the open-loop equilibria?"

Agents that wield market power do so subject to the constraints imposed by optimizing behavior of non-strategic agents. For exhaustible resource problems it is standard to model competitive sellers as solving dynamic problems, while competitive buyers typically solve a sequence of static problems – given the current price of the exhaustible resource, how much should the country buy? There are exceptions – if buyers are selecting durable goods or investment goods, then they must decide on the time path of investment or purchase by solving an intertemporal problem, and it is notable that the open-loop solution to such problems [for example, the durable good monopolist described by Bulow (1982), Coase 1972, Kahn (1987)] frequently exhibit DI. It is also the case as argued in the previous section that one can restore the symmetry between the two types of problem by modelling the sellers as Nash suppliers choosing a time profile of extraction plans. Once the link between different time periods has been severed on the supply side of the market, the open-loop Nash equilibrium is again dynamically consistent.

It is perhaps natural to model oligopolistic supply models as open-loop Nash equilibria, but this is only one of the possible market structures. Pure monopoly on the supply side is a straightforward generalization of the classic Hotelling (1931) problem, except that marginal revenue instead of price is arbitraged. [Stiglitz (1976), Sweeney (1977), Hoel (1978), Dasgupta and Heal (1979)]. Given the absence of any other strategic agents making dynamic decisions, there is no problem of DI. The other main type of market structure which has a natural appeal for modelling the oil market is that of a dominant producer (or cartel, such as OPEC) facing a competitive fringe of suppliers (the non-OPEC oil producers). The natural equilibrium concept is that of von Stackelberg (1952), in which the cartel takes the supply behavior of the fringe as given, and optimizes against this.

Several papers [Gilbert (1978), Ulph and Folie (1980, 1981), Newbery (1981, 1984), Ulph (1982), Ulph and Ulph (1989), Groot, Withagen and de Zeeuw (1989)] explore this equilibrium concept and show that the 'Binding Contract' or open-loop Stackelberg equilibrium, in which the cartel announces that it will follow its best open-loop plan, is potentially DI.

What is interesting about these results is that the open-loop Stackelberg equilibrium is not necessarily DI, in contrast to the case of a dominant buyer, where, except for a knife-edge case, the open-loop equilibrium is always DI. The reason is that although there is a jump state in these problems, in which the costate variable can be freely chosen to be zero at time zero, it may be that the costate then remains at this zero value – that is, there is nothing in the problem to change its value. In practice, the papers cited have not used this diagnostic technique to identify cases of DI – instead they have checked the open-loop Stackelberg equilibrium directly to see whether the cartel would have an incentive to deviate subsequently. This raises the question of whether the time-consistent open-loop Stackelberg equilibrium is perfect. The answer, as before, is no, for if the fringe deviates or sells a non-optimal amount, then fringe stocks will differ from that expected. If all agents were to recompute their optimal open-loop strategies from the present stocks, the paths would not coincide with those previously computed, and in that sense the cartel's open-loop plans are not perfect. However, if the cartel's original open-loop extraction plans are transformed into a function of remaining stocks instead of time, then there would be no need to recompute a new plan, and the original strategy would be Markov perfect. This was described in the previous section as the 'feedback form of the open-loop policy'. With that modification, if the open-loop Stackelberg equilibrium is time-consistent, it can be supported as a Markov perfect equilibrium, since it reflects all the available market power (at least, in the Markov setting in which the fringe have perfect foresight). For the simple market structure of a single dominant producer one does not have to worry about the vexing problems of oligopoly (which, of course, are not peculiar to dynamic games). The next section surveys this literature.

4.1. Dominant seller facing competitive fringe: Open-loop equilibria

There are two different solution techniques and two alternative ways of modelling fringe supply. The standard approach is to model the problem as a single-agent intertemporal maximization problem using Pontryagin's Maximum Principle, where the behavior of the fringe producers is taken as a constraint on the actions of the cartel, and is described by an arbitrage equation. The problem with this approach is that it is often opaque and inaccessible to all but the most mathematically literate. Because the emphasis is on the formalism of problem, early practitioners were slow to identify issues of DI. The alternative, advocated by
Newbery (1981), is to develop an intuitive, quasi-graphical approach which makes the analysis accessible to mathematically unsophisticated economists, drawing on principles with which they are familiar. Given the emphasis on visualizing the result and understanding the interaction of objectives and constraints over time, it is easier to see whether the proposed equilibrium is dynamically consistent. There are limitations — it is necessary to simplify the problem (constant unit extraction costs, static demand), though the same is typically true for deriving analytically tractable solutions using Pontryagin\(^{25}\). There is a more serious problem with both techniques, and that lies in piecing together the successive phases.

With intertemporal control problems such as these, there are a sequence of constraints, not all of which will bind at each moment. Successive phases are distinguished by the set of binding constraints, and much of the skill in deriving the full solution lies in determining the correct sequence and in piecing together the successive phases. Provided the co-state variables are continuous across the boundaries of successive phases it is relatively simple to piece together the solution path, and, acting on this assumption, earlier papers have proposed solutions to this problem. However, Groot, Withagen and de Zeeuw (1989) have pointed out that this assumption of continuity of the co-state variables is unwarranted, and argued that as a result, the equilibrium price path may also be discontinuous.

Their approach also illustrates the second of the two alternative methods of modelling fringe supply. The standard method is to describe the outcome of competitive arbitrage in terms of a competitive price path which constrains the sales of the cartel. The other method, which is equivalent, but suggests a different set of control variables, is to take fringe supply as a function of the price and cartel supply (essentially computing the residual demand facing the fringe). Groot, Withagen and de Zeeuw (1989) adopt this approach which enables them to model the cartel’s problem as one in which the choice variables are production levels. The cartel’s maximization problem is

\[
\text{Max} \int_0^\infty \left[ p - c^e - x^e(t) - X^e(t) \right] x^e(t) e^{-rt} \, dt, \tag{4.1}
\]

subject to

\[
\frac{dS^e}{dr} = -x^e(t), \quad S^e(0) = S^e_0, \tag{4.2a}
\]

\[
\frac{dS^t}{dr} = -X^e(t), \quad S^t(0) = S^t_0, \tag{4.2b}
\]

\(^{25}\) Ulph and Folie (1981) assume time-invariant linear demand and constant unit extraction costs, and follow a more informal analytical approach to piecing together the phases without recourse to the Maximum Principle.
\[
\frac{d\lambda^i(t)}{dt} = 0, \quad (4.2c)
\]
\[
x^c(t) + x^i(t) + c^i + \lambda^i(t) e^i - \beta \geq 0 \quad (4.3)
\]
where \(x^c\) is cartel extraction, \(X^i\) is total fringe extraction (all members being identical), \(c^i\) is unit extraction cost \((i = c, f)\), the demand schedule is linear: \(p = \hat{p} - Q\) \((Q\ being\ total\ demand)\), \(\lambda^i\) is the co-state variable for the typical fringe producer's stock depletion decision, and \(S^i\) is remaining stock. The control variables are \(x^c(t)\) and \(X^i(t)\), and the state variables are \(S^c, S^f,\) and \(\lambda^i\). For this problem the constraint qualification (required for the normal application of the Maximum Principle) does not hold at some points, and the authors rely on an alternative set of necessary conditions, due to Neustadt (1976), and set out in Seierstad and Sydsaeter (1987). These alternative conditions do not require the co-state variables to be continuous, and in this problem they can indeed be discontinuous.

With this qualification in mind, we can still approach the derivation of the solution geometrically, using the principles enunciated by Newbery (1981). These require a number of simplifying assumptions. The first is that the maximum feasible sales price exceeds the unit extraction costs from all stocks or fields (and this can be taken as the defining characteristic for inclusion of a field among the set of economically viable resources). Next, extraction costs are stock-independent
by field, and finally, that demand is stationary\textsuperscript{26}. The first principle, already mentioned, is the Hotelling arbitrage principle, which requires the equality of the present value of the marginal rent at each moment of positive production. For a competitive producer, the marginal rent is equal to the rent, or the price less the marginal cost, but for an agent with market power it is equal to the marginal revenue less the marginal cost. The second principle is that the price path can be computed as a function of 'time to go' (to exhaustion), working back from the terminal price, which is the maximum feasible sales price for the resource (equal to \( \bar{p} \), the choke price, above). Over the period to this terminal date all stocks of oil must be exhausted (if they are competitively supplied), or the stocks of oil that the cartel finds it advantageous to sell must be exhausted. This principle can be described as the exhaustion principle.

Finally, the ability of the fringe to arbitrage prices presents the cartel with a truncated or kinked instantaneous net demand schedule, as shown in Figure 3. The cartel can sell an amount up to \( q(p(t)) \) at a price infinitesimally below \( p(t) \), and its marginal revenue will there be equal to the sales price as it faces a perfectly elastic net demand. If it wishes to sell more than \( q(p(t)) \), then it faces the original demand schedule with the old marginal revenue lying below it. The marginal revenue will thus be discontinuous at point B, dropping from B to A. This shows that the cartel

\textsuperscript{26} This assumption of stationarity can be relaxed, but then the location of the terminal date will require iterative calculations.
will be willing to sell at \( p(t) \) or at point B if the opportunity cost of the oil (i.e. its present discounted value if sold at some other date) lies between A and B.

Figure 4 shows the application of these principles when the fringe extraction cost is substantially above the cartel’s unit cost. The location of the various points is found as follows. The point E is fixed by the choke price, \( \bar{p} \), and the competitive price path EDC is found by the arbitrage principle. At a date \( z \) years before exhaustion, the price is

\[
p^c_z = c^f + (\bar{p} - c^f) e^{-rz},
\]

where \( p^c_z \) is the competitive price at time-to-go \( z \). The length of ED is such that cumulative demand is equal to fringe stocks. At point D the cartel’s marginal rent must be equal to the competitive price less cartel cost (since the fringe imposes a limit price as in Figure 3). Along AD the marginal rent rises at the rate of interest, and this locates the monopoly price path CB. For the linear demand schedule is given by

\[
p^m(t) = \frac{1}{2} [m(t) + \bar{p}],
\]

where \( p^m(t) \) is the monopoly price and \( m(t) \) is marginal revenue at time \( t \). The starting date, initial price, and time taken before the exhaustion of all oil is determined by the requirement that along the cartel’s extraction path BCD, cartel
oil is exhausted at the moment the price reaches D.\textsuperscript{27} The open-loop equilibrium is dynamically consistent, for the same solution would be derived starting from any point during the cartel's extraction phase BCD. The fact that the fringe does not deplete any of its oil until all the cartel oil is exhausted means that the constraints facing the cartel do not change over time and cannot be affected by the cartel's actions, so the conditions under which DI is a problem never materialize.

One other simple case occurs when the cartel's extraction costs are, implausibly, above that of the fringe: $c^e > c^f$. This implies that the fringe extracts before the cartel starts to produce, as in Figure 5. The monopoly price path ABM is located by the marginal revenue path ADF, and BCDE is a competitive price trajectory, located so that the cartel exhausts along CBA and the fringe exhausts along EDC. The exact location of point B is found by balancing two offsetting tendencies. Along BC the cartel is constrained by the limit price of the fringe for the following reason. Although the fringe does not plan to sell after C, if the cartel announced in advance that it planned to sell at any higher price than those along the continuation of the competitive price path beyond DC, then the fringe would not choose to sell along ED, and the proposed solution would not be an equilibrium. Consequently, the cartel cannot announce sales on a price trajectory higher than CB, but, other things being equal, it is willing to supply provided the market price lies between the unconstrained monopoly price and the marginal revenue trajectory (which gives the value of delaying extraction until some point on the unconstrained trajectory BA). However, other things are not equal, as the length of the constrained extraction phase, BC, is a choice variable. The longer is the constrained phase, the lower is the price at C, and the faster is the cartel oil and the fringe oil extracted. This advances the date at which oil is first sold by the cartel (at C) (and is therefore desirable), but lowers its price (which is undesirable). Balancing these two factors determines the location of point C.

This open-loop equilibrium is clearly DI, for once the fringe has extracted all its oil at C, the cartel is free to raise the price to the unconstrained monopoly level along ABM. Knowing this, the fringe would retain some oil to benefit from the capital gain as the price is raised from C to some point on BM. If the cartel is unable to make binding commitments, the fringe will not cease extracting until the price has reached the unconstrained monopoly price trajectory.

One solution that is dynamically consistent is the open-loop Nash-Cournot equilibrium, in which the cartel takes the fringe's extraction trajectory at the initial date as given (and similarly each fringe member chooses its extraction plan given that of all other agents; as argued above, in the limit as each fringe member is a vanishingly small fraction of the total fringe, they behave competitively, but

\textsuperscript{27} The sequencing of the cartel and fringe extraction phases is determined by the relative steepness of the price paths at their point of intersection C. The cartel price path is steeper, encouraging the fringe to delay extraction until after the cartel is exhausted.
aggregate fringe supply is still given at each date). The equilibrium is shown in Figure 5 as ABMNFG. Along GF the fringe is the sole seller, along MBA the cartel is sole seller, and along FNM both sell, with the fringe share gradually moving from 100% at F to 0% at M. The point F is found from the intersection of the marginal revenue trajectory ADF with the competitive price trajectory MNFG. The sales level of the cartel is given by \( x'' = (p - m)/b \), where \( b \) is the slope coefficient of the linear inverse demand schedule. Again, the location of this trajectory is determined by working back from the terminal price, and its location is independent of the fringe stocks. The Nash–Cournot equilibrium is one in which everyone takes the others’ extraction rates as given, so there are no jump states and the equilibrium is dynamically consistent.

All these equilibria have continuous price paths. In the first case, the cartel cannot sell at a price above the competitive price path, for the fringe would then flood the market. He will either sell at the unconstrained monopoly price or at the highest price limited by the fringe – and so the price path will be continuous at points such as C and D. This is not the case if the fringe has higher extraction costs than the cartel, but not so much higher that the fringe extracts in the final phase.

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28 The location of M and MF depends only on total cartel stocks, though if the fringe does not have enough stocks to supply along the whole of FM, then that part of the path will be truncated, and extraction will start somewhere along FM.
Figure 6 shows a possible sequence in which the cartel sells in the final phase along an unconstrained monopoly price trajectory AB, and is constrained by the competitive price trajectory BCD in the early phase. Given that the cartel is constrained by the competitive price path, it would prefer to sell earlier rather than later on that path, i.e. along DC, as its costs are lower than those of the fringe (and hence its present discounted rent is falling with the passage of time along DCB).

Again, there is a problem of deciding the location of the point B, but once that is done, the length of BC is such as to exhaust the fringe, and the initial price and time until exhaustion is such as to exhaust the cartel along DC and BA taken together.

Now consider raising the competitive price path an infinitesimal amount to FGH. The rate of supply will be lower at each point (i.e. at each point equally distant from the date of exhaustion) as the price is higher, and so the period of sales along the competitive path will be slightly extended, as shown. This will be a small-order effect. The price at which the cartel sells along HG will be higher, which will be more profitable. Finally, the downward jump in the price at date F creates no problems, as the fringe plans to exhaust over GF, and will not find it attractive to delay until after F and face a capital loss. As BA is the unconstrained monopoly price path, the cartel will not wish to sell at the higher price F, and so this is a satisfactory equilibrium for the cartel.

The argument is that it would be desirable to raise the competitive price path, starting from a continuous price path such as DCBA, as this has a first-order positive effect on cartel profits during HG, and only a small effect in delaying the date of the monopoly sales along BA. As the competitive price path is raised further, this delay becomes increasingly costly, until at some point the balance of initial advantage is offset by the extra delay of subsequent profits. Groot, Withagen and de Zeeuw (1990) give the formal derivation using the Maximum Principle, and show how to locate the various phases and the initial price. An alternative approach is to see the problem as a choice of two variables: the level of the competitive price at the moment of fringe exhaustion and the date at which the final cartel phase begins, at B. Further examples of discontinuous price paths are given by Groot, Withagen and de Zeeuw (1992) and are discussed by Newbery (1992b).

It may not be so obvious that this open-loop equilibrium is DI, but the location of the competitive phase FG depends on balancing the advantages of higher cartel prices along GH with deferred profits along BA. With the passage of time, the cartel would like to change the location of this competitive segment, and to raise and defer it, to further raise the current cartel price. This problem would arise even if there were no discontinuity in the price path at F, and the discontinuity itself creates no problems of DI, at least on the assumption that consumers are passive and only concerned with the current price. In a world in which consumers held precautionary stocks, a prediction that the price would move sharply down in the near future would be accompanied by a fall in stockholding, though once consumer stocks had fallen to zero, nothing further could be done to undermine
the equilibrium. As for the cartel, B is the unconstrained profit maximizing price, and F, being above B, is therefore not an attractive alternative.

If the fringe stocks are relatively small, then the competitive price path during which the fringe sells may be above the unconstrained monopoly price at a points such as G where the fringe begins to sell. The competitive price path may still intersect the monopoly path, and the sequence would then be: a first phase in which the cartel is constrained by this competitive phase, a second phase in which the competitive price exceeds the monopoly price and the cartel is unconstrained, a jump up in price to the competitive level when the fringe begins to sell, followed by a jump down at a point such as F to B, and the final unconstrained cartel phase. Discontinuities can only occur between the monopoly phase and the competitive phase in which the fringe is selling, and only when the competitive price is above the monopoly price. Of course, if consumers can store oil and have rational expectations, then upward jumps in the price cannot occur — consumers would buy in anticipation, and then consume from stock until the point at which the cost of purchasing and holding the stock (price plus the accumulated interest) were equal to the market price.

Newbery (1981) has pointed out that other cases of DI may arise if the cartel delays extraction not because fringe costs are relatively low compared to cartel costs, but because the cartel discounts future oil rents at a lower discount rate than the fringe. This is quite likely because issues of sovereignty which are the source of the inability to enforce international oil supply contracts will also lead to a fragmented capital market. Capital surplus oil exporters, whose marginal product of investment is below that in oil-importing countries, but who have nationalized the overseas concessions of the oil-importing countries during the OPEC crisis, will be understandably wary of locating immobile capital hostages within those oil-importing countries, for fear they be expropriated in due course, if not explicitly, then by taxation. Lower OPEC interest rates will encourage the cartel to delay, inducing fringe producers to extract first. As a result, OPEC will be left with relatively greater market power in the future — and, indeed, this seems to be the current pattern, with the high-cost fringe producers like the UK, Norway and the US extracting as rapidly as possible, despite low Reserve/Production ratios, and the low-cost Gulf States extracting at far less than capacity, while their Reserve/Production ratios remain high. It would therefore be rational to expect OPEC’s market power to increase over time unless non-OPEC discoveries proceed more rapidly, or alternative sources of fuel become available.

4.2. Perfect Stackelberg equilibria

If the open-loop Stackelberg equilibrium is dynamically consistent, then it can be supported as a perfect equilibrium, by constructing the feedback form of the open-
loop strategy, as explained in Section 3.1. This involves solving for time or time to
go as a function of fringe and cartel stocks, and then replacing time by this function
of stocks in the expression for the cartel's extraction plan – making the cartel's
decisions depend on current stocks. Provided there are no errors or disturbances to
stocks, the feedback solution will appear identical to the open-loop solution. If the
open-loop Stackelberg equilibrium is DI, then the perfect equilibrium will differ
from the open-loop equilibrium, and it will not be possible to derive it from the
open-loop solution. Newbery (1980, 1992a) presents a model in which the open-
loop Stackelberg equilibrium is DI, and derives an analytical expression for the
perfect equilibrium. The model is set up in discrete time, and there is a date \( T \) after
which oil is valueless. Demand in each period is given by the same unit elastic
demand schedule, truncated above by a 'backstop' price, \( \bar{p} \), sufficiently high not to
affect the open-loop or competitive equilibria. Extraction costs of fringe producers
are zero and those of the cartel are constant at \( c \). The model thus captures the idea
that the cartel will have a comparative advantage in delaying sales. The (truncated)
unit elastic demand means that the cartel's price would be set at the backstop price
in the absence of the fringe, dramatically illustrating its potential market power.
The open-loop or binding-contract equilibrium is easy to characterize, for the cartel
will be constrained by the competitive price path over the whole \( T \) periods, and the
competitive price is given by

\[
p(t) = p_0 \beta^{-t},
\]

where \( \beta < 1 \) is the discount factor, or \( 1/(1 + r) \), if \( r \) is the rate of interest. The cartel
will not plan to sell more than \( S^c < S_0^c \), and if the fringe stock is \( S_0^c \), then the initial
price is given by

\[
p_0 = \frac{1 - \beta^{T+1}}{(1 - \beta)(S^c + S_0^c)},
\]

which comes from the fact that demand in any period is \( 1/p(t) \). The fringe sells
first, the cartel sells last, and the cartel chooses \( S^c \) to determine a profit-maximizing
initial price. Clearly, this equilibrium is DI. The cartel will wait until the fringe has
exhausted, and then raise its price to \( \bar{p} \) for the remaining time periods.

The perfect equilibrium is found recursively. Let \( n \) denote the number of time
periods remaining before oil becomes valueless (\( n = 0, 1, \ldots, T \)). Let the terminal
price be \( p \), so that \( p^n = \beta^n p \) is the price \( n \) periods before the terminal date. Let
supply by the fringe be \( y^n \), supply by the cartel be \( x_n \), and remaining stocks be \( Y_n, X_n \),
all in period \( n \). In the final period, or period 0, the cartel chooses \( x_0 \) to maximize:

\[
W_0 = x_0(p - c),
\]
subject to demand equals supply:

\[
\frac{1}{p} = x_0 + Y_0, \quad x_0 \leq X_0.
\]  

(4.8b)

The solution is

\[
x_0 = \frac{p - c}{p^2}, \quad y_0 = \frac{c}{p^2}.
\]  

(4.9)

The cartel’s credible supply is a function of the state of the system, and can be written as

\[
x_0 = \psi_0(X_0, Y_0) = \text{Max} \left( \sqrt{\frac{Y_0}{c}} - Y_0, X_0 \right).
\]  

(4.10)

Fringe supply in the final period is particularly simple:

\[
y_0 = \gamma_0(X_0, Y_0) = Y_0.
\]  

(4.11)

In the penultimate period the cartel announces \(x_1\), and the fringe chooses \(y_1\), and hence \(y_0 = Y_1 - y_1\), knowing that the cartel will sell an amount \(x_0 = \psi_0(X_0, Y_1 - y_1)\), and hence knowing what the price will be in the final period. Define the critical credible stock as the maximum amount that will be sold by the cartel in future periods, then, provided at any date the current cartel stock exceeds this critical level, the future sales of the cartel will be solely determined by fringe stocks. In that case

\[
x_0 = \psi_0(Y_0), \quad p_1 = \beta p = \pi [Y_1 - y_1(x_1, Y_1)].
\]

The cartel chooses \(x_1\) to maximize the two period discounted profits, where prices in each period are ultimately a function of \(Y_1\), given, and \(x_1\), a choice variable. The resulting solution gives a value for \(x_1, x_0\), and hence the critical credible stock in period 1, \(X_1(Y_1)\), a function of fringe stock.

This technique works recursively, and analytical recursive relations can be found for the sales of the fringe in each period, the price, and hence the sales of the cartel. Cartel sales will be positive, working back from the terminal date, until the critical credible stock in period \(n\) first exceeds the initial cartel stock. If this does not happen before reaching the initial date, the cartel will sell in every period, not just the final \(n\) periods. Newbery (1980, 1992a) gives the recursive formulas and solves these numerically for a range of values of the cartel extraction cost, \(c\). It is easy to derive the open-loop Nash–Cournot equilibrium (OLNCE), and the competitive equilibrium (CE), and to compare the cartel’s profits in each of these equilibria,
and in the open-loop Stackelberg (OLSE) and feedback Stackelberg (FBSE) or perfect equilibria. The comparisons are given in Figure 7. It is clear that the OLSE must yield the highest level of cartel profits, as any of the other equilibria were feasible OLSE but were not chosen. None of the other equilibria can be ordered, and examples of each ordering are visible in Figure 7. In particular, as with the perfect import tariff of Maskin and Newbery (1976, 1990), the agent with market power may be disadvantaged by his market power in the absence of the ability to commit, relative to the competitive equilibrium in which he exercises no market power at all.

Given the difficulty of computing the perfect equilibrium it is worth asking whether there are any plausible candidates which can be computed and are not too different from the perfect equilibrium. Karp and Newbery (1989) show that the reneged or naive open-loop Nash equilibrium is a reasonable approximation to the perfect Nash equilibrium in a dynamic game of symmetric importers each with market power facing a competitive supply, as illustrated above in Figures 1 and 2.

29 The parameters are: discount rate = 5%; \( T = 30 \); Fringe stock = 30; cartel stock = 10.

30 This is found by solving for the optimal open-loop decision at date \( t \) as a function of the remaining stock at that date, and then characterizing the decisions of the agent as functions of his remaining stock. There is of course an implausibility inherent in computing the current open-loop decision on the assumption that the whole time path of future decisions will be adhered to, when at each successive moment they will in fact be recomputed. But the concept is useful only in so far as it approximates the perfect equilibrium, not as a plausible equilibrium concept in its own right.
Would the same approach work in the current problem, bearing in mind that there is an essential difference between an oligopolistic equilibrium, and a dominant agent or Stackelberg equilibrium? It is, however, clear that the reneged open-loop Stackelberg equilibrium is not a candidate in the above example, for until the fringe exhausts its stocks, the cartel makes no sales. It is true that the total amount that the cartel plans to sell in the future (i.e. the critical credible stock in the language above) depends on the amount of fringe stock remaining, but if the fringe continues to believe that the cartel is committed to its original plan, the fringe will observe nothing to persuade it that the plan has changed, until it finally exhausts and the cartel reneges.

A more promising alternative is the feedback Nash–Cournot equilibrium. This is computed in the same way as the perfect equilibrium, working back from the terminal period, but differs in that in each period the cartel and the fringe each choose their supplies without observing the plans of the other. In the Stackelberg equilibrium the fringe decides its output in each period after observing the cartel’s supply decision, and the cartel takes this response into account in choosing its supply. The two equilibria coincide in the final period, since at that date the fringe must sell all its remaining oil. Newbery (1992a) shows that in the above model, if the cartel has enough oil, then the feedback Nash–Cournot equilibrium coincides with the open-loop Nash–Cournot equilibrium. (In Figure 7 this holds for $c \geq 0.6$.) The reason is that if the cartel would not wish to sell all its oil, then cartel sales in later periods will not affect sales decisions in earlier periods, since they do not affect the effective availability of oil then – a situation corresponding to the open-loop Nash–Cournot equilibrium with excess oil.

The two equilibria differ in that the cartel appears to sell more oil in the feedback Nash–Cournot equilibrium than in the feedback Stackelberg equilibrium (when oil is in excess supply, i.e. when $c \geq 0.6$), and earns higher profits. Whether this is a general result is not clear, but it could result from the inability of the fringe to condition their decisions on the cartel’s output in the feedback Nash–Cournot case, allowing the cartel to sell more oil in the current period than might otherwise be prudent. For lower cartel costs, and lower cartel oil stocks, when oil has a scarcity value in the open-loop Nash–Cournot equilibrium, the feedback Nash–Cournot equilibrium will differ from the open-loop Nash–Cournot equilibrium, and the latter may yield lower profits than the feedback Stackelberg equilibrium. One tentative conclusion is that the feedback form of the open-loop Nash–Cournot equilibrium, though in general not perfect, may be a defensible approximation, increasingly so if there are several suppliers with market power.
4.3. *Endogenous market structure*

Ulph and Ulph (1989) studied a two-period model in which there are two identical large suppliers competing with a fringe of competitive producers. They assume zero extraction costs and different discount rates for the fringe and dominant suppliers. They study the consequences of two different kinds of commitment. The dominant agents may be able to commit to cooperate and behave as a cartel in both periods. Each agent (acting separately or collusively, depending on the market structure) may also be able to sign binding long-term supply contracts with its customers, thus committing itself to the open-loop Stackelberg equilibrium. Whether either form of commitment is feasible will depend on the institutional structure – long-term contracts may not be enforceable across national boundaries, and collusive behavior may be illegal, or unsustainable for other reasons. There are thus four possible market configurations:

(i) Collusion and long-term contracts.
(ii) Collusion without long-term contracts.
(iii) Non-cooperative production with long-term supply contracts.
(iv) No collusion and no long-term contracts.

The first case is the best for the dominant suppliers. In (ii) and (iv) the perfect equilibrium differs from the open-loop equilibrium.

The interesting finding is that case (iv) may yield higher profits than case (ii), suggesting that it is not in the interest of the large producers to collude if they are unable to bind themselves to the open-loop supply plan. If the large suppliers cannot collude, they will be too competitive in the first period. If the large suppliers discount the future more heavily than the fringe, they will also wish to sell more in the first period, and so their inability to make either commitment reinforces each other, and their joint absence is doubly harmful. If the large suppliers discount the future less heavily than the fringe, then the two forms of precommitment pull in opposite directions, and it will not be clear which is stronger, and either one by itself may be worse than when both are absent. If suppliers cannot precommit their supply plans, and if the open-loop equilibrium is time-inconsistent, then cartels may be less likely to form than if precommitment were possible.

5. *The period of commitment*

If there is a single dominant agent, it is obvious that that agent prefers the open-loop equilibrium to the Markov equilibrium, unless the two result in the same outcome. The open-loop equilibrium allows the dominant agent to make a commitment for the entire horizon of the problem, so the outcome in the Markov equilibrium is feasible. If there are two or more strategic players, the open-loop and Markov equilibria are (again) typically not equal. However, it is no longer unambiguous
which equilibrium gives the dominant agents higher pay-offs. Nevertheless, there are several known cases where symmetric agents do better under the open-loop equilibrium. The reason, as discussed in a previous section, is that with Markov equilibria agents have incentives to alter their rivals' behavior, and this tends to cause them to behave more competitively – i.e., to move further away from a cooperative solution; with open-loop strategies there is no such incentive. Whether there are one or many strategic players, the open-loop equilibrium corresponds to an infinite period of commitment, and the Markov equilibrium corresponds to a finite period of commitment.

We denote the period of commitment by $\varepsilon$ which can take any non-negative value. We can view the value of $\varepsilon$ as exogenous, and ask how the equilibrium depends on that value. There are two ways to model this issue. The first method begins by fixing the times at which agents' controls can be changed. For example, it may be possible to have a tariff that changes every day or every year. We often want to know what happens as the period of commitment becomes arbitrarily small. This suggests that it is convenient to assume that decisions are made in continuous time, e.g., the tariff can be adjusted continuously. In this case, altering $\varepsilon$ changes only the period of commitment. Letting $\varepsilon$ equal infinity (or whatever the horizon of the problem is) reproduces the continuous time open-loop equilibrium. Letting $\varepsilon$ approach zero reproduces the continuous-time Markov equilibrium. Obviously, it makes no sense to have a period of commitment shorter than the period between the moments in time when controls can be changed. That is, if it is possible to change the tariff at most once a year, the period of commitment cannot be shorter than a year, and this may provide a natural restriction. Alternatively, if futures contracts can be enforced, and if contracts extend for one year forward, then again the period of commitment may have a natural length.

A second alternative is to identify $\varepsilon$ with the amount of time during which the control must be constant. The disadvantage of this approach is that with it, changing $\varepsilon$ involves not only a change in the period of commitment, but also a change in the degree of flexibility that the agents possess. If, however, one is primarily interested in the limiting behavior as $\varepsilon$ approaches zero, this disadvantage is not likely to be serious. The reason is that the continuous time trajectory can be approximated to any degree of accuracy by choosing the length of the intervals between changes in the control to be sufficiently small. This means that the loss resulting from being required to choose a constant control over a period of length $\varepsilon$, rather than being allowed to choose a path over that period, is of a smaller order of magnitude than $\varepsilon$. (This assumes that the path that would have been chosen is continuous over that interval, so that any two points on that path are close – an assumption that holds for most economic models.)

The advantage of the second method, relative to the first, is that it provides a simple way of motivating the continuous-time Markov equilibrium. We can do this by setting up a discrete-time dynamic programming equation which is standard
except for the dependence of the value function(s) and the decision rule(s) on $\varepsilon$. A first-order Taylor expansion around $\varepsilon = 0$ yields the dynamic programming equation for the continuous-time Markov equilibrium.\footnote{The validity of this expansion requires that the value functions and future decision rules be analytic in $\varepsilon$. This implies that the equilibrium is a continuous function of $\varepsilon$ near $\varepsilon = 0$. Since $\varepsilon = 0$ is of interest for its role as a limiting case, rather than its empirical plausibility, it seems reasonable to require continuity, as a defining characteristic. If the equilibrium was not continuous at $\varepsilon = 0$, that limiting case would not be of interest, since it would convey no information about equilibria for small positive values of $\varepsilon$.}

Reinganum and Stokey (1985) illustrate the first approach. They model the problem of duopolists who extract a resource in the absence of property rights. Demand has constant elasticity and extraction costs are zero. In the open-loop equilibrium ($\varepsilon = \infty$) the duopolists choose a trajectory of extraction and receive positive profits. As $\varepsilon$ approaches zero, the duopolists extract essentially the entire stock in the first instant. This drives price arbitrarily close to zero, so profits are negligible. The inability to precommit means that the duopoly equilibrium reproduces the open-access competitive equilibrium (in the limit as $\varepsilon$ approaches zero). For intermediate values of $\varepsilon$, the equilibrium lies between the two extremes.

The intuition is as follows. When $\varepsilon \to \infty$ each duopolist takes the other’s extraction trajectory as given, and consequently takes its total extraction as given. It therefore takes the stock available to itself as given, and chooses the profit-maximizing sales strategy. Such a strategy clearly does not entail instantaneous exhaustion. If $\varepsilon$ is finite, each duopolist takes its rival’s current actions and future decision rules as given. These rules imply that equilibrium sales by one’s competitor is a decreasing function of the stock available. This gives each duopolist two reasons to increase its own current extraction: (i) by doing so it increases the aggregate amount that it will sell over the remaining program, and (ii) it causes the residual demand that it will face in the future to shift out. Neither of these incentives are present under open-loop strategies. As $\varepsilon$ becomes arbitrarily small the incentives become overwhelming.

This result is in the same spirit as the Coase Conjecture, which maintains that as the period of commitment of a durable goods monopolist (who produces under constant costs with no capacity constraint) approaches zero, the monopolist looses all market power and behaves competitively. We know that in a Markov equilibrium for the durable goods monopolist, eliminating either the assumption of constant cost or the assumption of no capacity constraint is enough to overturn the Coase Conjecture. The analogous result holds in Reinganum and Stokey’s model: either convex extraction costs or an upper limit on the rate of extraction implies that exhaustion does not occur instantaneously, and also implies that the duopoly equilibrium does not approach the competitive equilibrium as the period of commitment approaches zero.
Karp and Newbery (1991) illustrate the second approach. This paper, described above, considers the continuous-time perfect equilibrium in which strategic buyers choose state contingent consumption rules or, equivalently, tariff rules that support that level of consumption. The tariff trajectory in the continuous-time Markov equilibrium (infinitesimal period of commitment) lies below the tariff trajectory in the open-loop equilibrium (infinite period of commitment). Here too, shrinking the period of commitment causes the strategic agents' behavior to move toward competitive behavior – a zero tariff. However, the Markov equilibrium does not converge to the competitive equilibrium, as was the case in Reinganum and Stokey's model. (Recall that in that case convergence to competitive behavior occurs because of restrictive assumptions of their model, and is not robust.)

If there is a single strategic agent, the Markov equilibrium pay-off to that agent is monotonic in the period of commitment; decreasing the period of commitment reduces the feasible set of options and therefore cannot increase the agent's pay-off. One would expect the same result to hold in a game with more than a single strategic agent. Indeed, this is true in Reinganum and Stokey's model, but it has not been shown more generally. It is worth emphasizing that this result is due to the assumption that agents are restricted to Markov strategies. Consider the case of a single strategic agent. The smaller is the period of commitment, the worse is the outcome for that agent in a Markov equilibrium. However, in non-Markov games, bad outcomes are useful because they provide 'threats' that can support good outcomes. This is easiest to see in the durable goods monopolist model with constant production costs and no capacity constraints. The following argument is taken from Ausubel and Deneckere (1989). If the period of commitment is infinite, the monopolist can achieve the first best outcome. If the period of commitment is infinitesimal, the monopolist achieves zero profits in the Markov equilibrium. In this case it is very important for the monopolist to preserve its reputation that it will not use a Markov strategy. The threat of losing its reputation enables it to obtain a pay-off arbitrarily close to the infinite-commitment pay-off. Therefore, if the period of commitment is either zero or infinite, the monopolist's pay-off can be essentially the same.

Since the set of perfect equilibria is not independent of $\varepsilon$, it follows that there is some value of $\varepsilon$ between zero and infinity which minimizes the supremum of the set of perfect equilibria.

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32 The expression 'can be' is used advisedly, since there are many other perfect equilibria with very different characteristics.

33 More concisely, let $E(\varepsilon)$ be the set of equilibrium payoffs for the monopolist, given $\varepsilon$, and let $\varepsilon^*$ be the supremum of $E(\varepsilon)$. Then there is an $\varepsilon^*$ that minimizes $\varepsilon(\varepsilon)$. 
6. Conclusions

Problems of dynamic inconsistency are likely to arise when the constraints of an agent's intertemporal optimization problem involve forward-looking behavior by other, nonstrategic agents. The current actions of those agents depend on what they expect to happen in the future. This gives the dominant agent an incentive to make promises or threats that yield benefits in early periods, but impose costs in later periods. Over time, the future costs outweigh the future benefits of maintaining the original program, and the dominant agent would like to change his mind. The same type of situation arises if there is more than one dominant agent, i.e. if the optimization problem is replaced by a non-cooperative game.

Modelling rational behavior in depletable resource markets requires dynamic models. Sellers base current sales on expectations of future prices as well as the current price. Whether dominant buyers attempt to extract rent from competitive sellers, or dominant sellers attempt to manipulate fringe sellers, the problem of dynamic inconsistency is likely to arise. If dominant agents in resource markets are unable to make binding commitments, a dynamically inconsistent equilibrium is implausible. It makes little sense to think of non-strategic agents as being sufficiently rational to solve their intertemporal optimization problems, but being sufficiently naive to believe promises or threats which will not be carried out. Binding commitments typically require an institutional structure, such as an international judicial system, which does not exist or is very weak. Dynamically inconsistent equilibria are therefore poor candidates for studying imperfectly competitive markets. Such equilibria do, however, have their uses. They are obtained as solutions to standard control problems or non-cooperative games with open-loop strategies, and therefore can be studied using the Maximum Principle. The advantage of this is that it is often quite easy to detect special cases where DI does not arise. In these situations dominant agents are permitted to make promises or threats but no enforcement mechanism is required, since the original plans continue to be optimal at subsequent stages. We discussed such special cases in contexts where either buyers or sellers are dominant. Even where the open-loop equilibria are dynamically inconsistent, they sometimes provide a useful benchmark against which to compare other equilibria.

We emphasized the distinction between an equilibrium which is simply time-consistent, and one which is perfect. The former type can be obtained by solving a standard control problem or non-cooperative game with open-loop strategies, provided that there are no jump states. The resulting equilibrium is not perfect. If there is a single strategic agent, the same realization can often be supported as a perfect equilibrium if the open-loop strategy is replaced by the feedback form of that strategy. This approach does not work if there is more than one strategic agent; that is, the feedback form of open-loop strategies does not constitute a
perfect equilibrium in a non-cooperative game. We have illustrated the approach in situations where there is a single dominant agent, on either side of the market.

We discussed the fact that there generically exist a multiplicity of perfect equilibria. The intuition provided by the Folk Theorems of infinitely repeated non-cooperative games carries over in a straightforward manner to dynamic/differential games (even if the game is degenerate, in the sense that there is only one strategic agent). There is an emerging literature which suggests that not all of those perfect equilibria are reasonable, since they may fail to be ‘re-negotiation proof’; the putatively credible punishment strategies that support a particular realization may not be credible after all. If this new literature succeeds in calling into question some of the strong conclusions of the Folk Theorems, we can expect a corresponding tendency in the theory of dynamic games. We may decide that not all of the perfect equilibria to such games are plausible, despite their perfection. These comments are speculative, since the issues they raise have not been resolved.

We chose instead to concentrate on a particular type of perfect equilibrium, in which agents’ actions and expectations are conditioned on the current value of the state variable. The state could be defined so generally as to make this restriction empty, and to avoid this we require the state to consist only of variables that have intrinsic economic meaning. The result is a Markov equilibrium. For depleatable resource models, the (only) obvious candidate for the state is the remaining stock of the resource (or the vector of stocks held by the various resource owners).

The resulting equilibrium can then be obtained using dynamic programming. We illustrated this technique with a two-period model and an infinite-horizon continuous-time model, and showed that market power can be disadvantageous. The models also indicate why equilibrium decision rules need not be monotonic functions of the state, and they suggest alternative ways of approximating perfect equilibria, for instance as the continuously reneged open-loop equilibrium for the case of the optimal import tariff.

The multiplicity of perfect equilibria emphasizes the extent to which the analysis of a market structure depends on the assumptions of the model. Although this dependence is true of modeling in general, it has special force when an attempt is made to model imperfectly competitive dynamic markets. In these cases there is no widely accepted definition that is likely to lead to a unique equilibrium. We have defended the Markov assumption on the grounds of plausibility. However, one can think of markets that appear to provide counter-examples. Major diamond producers have apparently operated a cartel which relies in large part on the type of reputational considerations that are excluded by the Markov assumption.

Even the requirement of perfection is not beyond criticism. We discussed a time-consistent but not perfect equilibrium in which only buyers exercise market power. This was obtained by beginning with a game in which buyers and sellers choose open-loop strategies, and letting each seller become individually insignificant. Whether such an equilibrium is more or less plausible than the
Markov equilibrium depends on the degree of sophistication of agents, the cost of processing information, and the ease with which plans can be altered. The answer to these types of questions must necessarily be found in the facts of the case rather than theory.

The problem of dynamic inconsistency is fundamental to the study of imperfectly competitive resource markets, and especially to problems of resource taxation. It is fundamental to a number of other problems in economics, and the insights gained from a careful modelling of solution concepts in the depleteable resource case may throw light on these other areas of enquiry, and conversely. Resource economists have been slow to take account of this (and the earlier volumes in this series have no entry on the subject). Practical men have long been sensitive to the problem of credibility and commitment. Companies deciding on large irrecoverable investments in foreign countries have long had to decide whether the announced tax policies of that country were likely to remain in force, or be changed opportunistically, and have made their forecasts taking into account what, ex post, it would be in the interest of the host country to do. The problem of the ‘obsolescing bargain’ which arises because the balance of advantage shifts from the investor to the host country after the investment has been made, has vexed multinational companies for years. [See the interesting papers in the book edited by Pearce, Siebert and Walter (1984).] Oil companies contemplating exploration have had to take a view of the likely rate of future oil taxation, and have generally been rather cautious as a result. More naive economists in international organizations have often criticized them for a bias against investment in developing countries, failing to appreciate the nature of the problems they face, though the UK oil tax regime may have the record for the frequency with which it has been adjusted to extract rent.

We do not wish to imply that it is a simple matter to characterize perfect equilibria – the fact that we still lack a satisfactory theory of oligopoly shows how difficult the task is. Many of the most difficult problems in understanding resource markets arise because these markets are oligopolistic. So far, most of the theoretical progress has been in models which greatly simplify the market structure, and abstract from issues of learning, asymmetric information, and straightforward uncertainty and ignorance. Nevertheless, we have made considerable progress in understanding at least some of the simpler forms of market structure, and taking due account of problems of dynamic inconsistency.

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