Solutions to Problem Set 6

ARE 261

November 8, 2001

Question 2

1. For the monopolist the current value Hamiltonian for the exhautible resource problem is

$$H = aq - bq^2 - \frac{c}{2}q^2 - \lambda q \tag{1}$$

The necessary conditions are

$$a - 2bq - cq - \lambda = 0 \tag{2}$$
$$-2b - c \leq 0 \tag{3}$$
$$\dot{\lambda} = \delta\lambda \tag{4}$$
$$\dot{R} = -q \tag{5}$$

We know that the stock is exhausted at T so the only variable that is free is T. We need a transversality condition for this which is given by $H(T) = 0$.

2. We know that the resource is exhausted at T and so $q(T) = 0$. From the first necessary condition we know that $\lambda(T) = a$. Furthermore the third necessary condition implies that $\lambda(t) = k\exp(\delta t)$ where k is a constant. We can determine k using the boundary condition $\lambda(T) = a$. This implies that $k = ae^{-\delta T}$ and so $\lambda(t) = ae^{\delta(t-T)}$. We can use the first order condition and the expression for $\lambda(t)$ to write the optimal policy function $q(t)$ as a function of T

$$q(t) = \frac{a}{2b+c}(1 - e^{\delta(t-T)}) \tag{6}$$

The only unknown is T.

3. The resource is exhausted at T and so the condition that determines T is

$$\int_{t=0}^{T} q(t)dt = R_0 \tag{7}$$

where R_0 is the initial reserve of the stock.

The Planner's problem is solved in the same manner.