Problem Set 3: Euler Equation

ARE 261

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Question 1

Find the Euler equation and its solution for

$$\int_1^2 [x + tx' - (x')^2] dt \tag{1}$$

subject to $x(1) = 3$ and $x(2) = 4$.

Question 2

Find the Euler equation and its solution for

$$\int_a^b F(t, x, x') dt \tag{2}$$

subject to $x(a) = A$ and $x(b) = B$, where

1. $F(t, x, x') = \frac{(x')^2}{t^3}$,

2. $F(t, x, x') = (x')^2 - 8xt + t$

Do not evaluate constants of integration.

Question 3

Define x as inventory and c_2 as the cost of holding inventory; $g(x')$ is an increasing convex cost of adding new inventory. (x' is the change in inventory.) The initial inventory is $x(0) = 0$ and the terminal constraint is $x(T) = B$. Show that at an optimal solution to the problem

$$\min \quad \int_0^T e^{-rt}[g(x') + c_2 x]dt \tag{3}$$

the firm is indifferent to producing a marginal unit at t or producing it at $t + \Delta$, since the sum of the marginal discounted cost of producing at t and discounted holding cost from t to $t + \Delta$ is equal to the marginal discounted production cost at $t + \Delta$.

Question 4

Interpret the Euler condition for the problem in question 3 for the case $r = 0$.

Question 5

Find candidates to maximize or minimize

$$\int_{t_0}^{t_1} \left\{t\left[1+(x')^2\right]\right\}^{\frac{1}{2}} dt \tag{4}$$

subject to $x(t_0) = x_0$ and $x(t_1) = x_1$. You need not find the constants of integration.

Question 6

Find candidates to maximize or minimize

$$\int_{t_0}^{t_1} F(t, x, x')dt \tag{5}$$

subject to $x(t_0) = x_0$ and $x(t_1) = x_1$, where

1. $F(t, x, x') = x^2 + 4xx' + 2(x')^2$,
2. $F(t, x, x') = x^2 - 3xx' - 2(x')^2$,
3. $F(t, x, x') = x'(\ln x')^2$,
4. $F(t, x, x') = -x^2 + 3xx' + 2(x')^2$,
5. $F(t, x, x') = t \exp^{x'}$

Do not find the constants of integration.