Problem Set 2: Predator-Prey Problem

ARE 261

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The purpose of this problem set is to teach you phase diagram analysis for a system of differential equations. Begin by ftping two MATLAB files called predator-prey.m and prey.m into your local directory from the ftp site /pub/classes/are261/. Then start MATLAB and load the file predator_prey.m (type predator_prey at the MATLAB prompt). This file contains MATLAB code to analyse a system of differential equations, namely the Lotka-Volterra predator-prey model, graphically.

$$\dot{y}_1 = (1 - \alpha y_2)y_1 \tag{1}$$
$$\dot{y}_2 = (-1 + \beta y_1)y_2 \tag{2}$$

where, variables y_1 and y_2 measure the sizes of the prey and predator populations, respectively. The quadratic cross terms account for the interaction between the species. All parameters are positive and strictly less than one.

The MATLAB file is written to enable you to play with different parameter values and to see how these changes effect the results. To begin with try the following parameters values (but re-run the file with different parameter values)

- Time interval - $T = 8$
- Initial predator population size - 20
- Initial prey population size - 20
- α - 0.01
- β - 0.02

At the MATLAB prompt, where you are asked to specify values for any one of these parameters, enter the value and press the ENTER key.

The file will show you how to plot species as a function of time, graph isoclines, to determine the direction of trajectories in each isosector, and to plot the system in phase space. The program will pause in places to give you time to look at the results. To continue just press the SPACE BAR.

Next consider a different system of differential equations where there is a natural ceiling on the prey, in the absence of the predator, and predators compete for prey.

$$\dot{y}_1 = (1 - \alpha y_2 - \gamma y_1) y_1 \tag{3}$$
$$\dot{y}_2 = (-1 + \beta y_1 - \rho y_2) y_2 \tag{4}$$

where all the parameters are positive and strictly less than 1.

Write new MATLAB code to solve the following:

- Plot species as a function of time.
- Graph isoclines $\dot{y}_1 = 0$ and $\dot{y}_2 = 0$ and determine the direction of trajectories in each of the isosectors.
- Plot the system in phase space (y_1 on the x-axis and y_2 on the y-axis).