Problem Set 1

ARE 261

August 27, 2001

Question 1

Given the ordinary differential equation (ODE)

$$\ddot{y} + a\dot{y} + by = 0 \tag{1}$$

- Write the general solution under the assumption that there are distinct real roots and write the formula for the roots.
- Given the initial conditions $y(0) = 0$ and $\dot{y}(0) = 1$, show how to find the complete solution to the ODE.
- Given the boundary conditions $y(0) = 1$ and $\dot{y}(1) = 0$, show how to find the complete solution to the ODE.

Question 2

Evaluate the derivative

$$\frac{d}{dx}\int_{7x}^{5+3x}(x\log t + x^2 sint)dt \tag{2}$$

Question 3

For the following system of first-order linear differential equation

$$\dot{y}_1 = -5y_1 + 3y_2 \tag{3}$$

$$\dot{y}_2 = -2y_1 + 10y_2 \tag{4}$$

- Find the isoclines (where slope of the trajectory is either 0 or ∞) and the steady state(s).
- Find the separatrix (HINT: see page 1.7 of notes).
- Draw the directional arrows and the trajectories.
- Graphically analyse the stability properties of the steady state(s).

Question 4

The point of this exercise is to solve an ODE. During a season of length T, the stock of fish $x(t)$ changes according to $\dot{x} = -\big(m(t) + f(t)\big)x(t)$, where $m(t)$ is the natural mortality rate and $f(t)x(t)$ is the harvest at time t (Harvest is a flow variable, x is a stock variable.). The initial stock $x(0)$ is given.

- Obtain an expression for $x(T)$, the final stock, and $y(T)$, the season's *cummulative* harvest ($\dot{y} = f(t)x(t)$).
- Simplify these expressions for the case where $m(t)$ and $f(t)$ are constants, m and f.