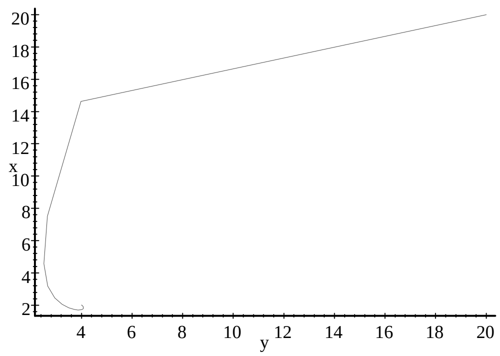
These notes contain an example of how Scientific Workplace can be used. This system integrates word processing and mathematical packages (both numerical and analytic). Below are two examples of phase portraits of the predator prey system. I also show two series approximations of the solution to a system of equations, and the exact solution to a single equation. Scientific workplace makes it possible to enter equations in your text, solve these (or operate on them in some fashion) and have the results in the same text. It takes seconds to obtain a phase portrait. Graphing the functions as functions of time takes a couple of minutes.

$\alpha = .4$
$\gamma = .8$
$\beta = .3$
$\rho = .5$

$$\begin{array}{l} \frac{dy}{dt} = (1 - \alpha x - \gamma y)y \\ \frac{dx}{dt} = (-1 + \beta y - \rho x)x \\ x(0) = 20 \\ y(0) = 20 \end{array}, \text{ Functions defined: } y, x$$



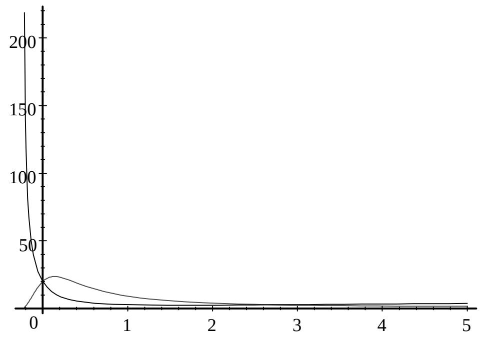
The Phase Portait

Here is another example:
$\alpha = .1$

$\gamma = .2$
$\beta = .3$
$\rho = .1$

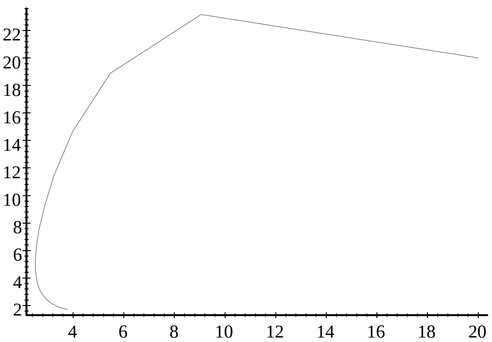
$$\begin{array}{l} \frac{dy}{dt} = (1 - \alpha x - \gamma y)y \\ \frac{dx}{dt} = (-1 + \beta y - \rho x)x \\ x(0) = 20 \\ y(0) = 20 \end{array}, \text{ Functions defined: } y, x$$

Here is a graph of x and y as functions of time.



x and y as functions of time

Here is the phase portrait



Phase portrait for second example

Below are two "series solutions" (a polynomial approximation of the solution) of the above equations.

Series solution (15th order approximation) :

$$y(t) = 20 - 100.0t + 390.0t^2 - 1456.7t^3 + 5932.5t^4 - 25581.t^5 + 1.0922 \times 10^5 t^6 - 4.5411 \times 10^5 t^7 + 1.87 \times 10^6 t^8 - 7.7464 \times 10^6 t^9 + 3.2308 \times 10^7 t^{10} - 1.3485 \times 10^8 t^{11} + 5.615 \times 10^8 t^{12} - 2.3343 \times 10^9 t^{13} + 9.7071 \times 10^9 t^{14} + O(t^{15})$$

,

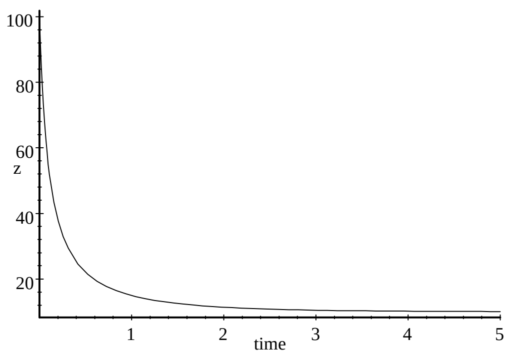
$$x(t) = 20 + 60.0t - 270.0t^2 - 30.0t^3 + 2397.5t^4 - 5169.5t^5 - 6617.4t^6 + 49690.0t^7 - 41210.0t^8 - 3.2334 \times 10^5 t^9 + 1.0617 \times 10^6 t^{10} + 14473.t^{11} - 7.2084 \times 10^6 t^{12} + 1.1691 \times 10^7 t^{13} + 4.0686 \times 10^7 t^{14} + O(t^{15})$$

Series solution (9'th order approximation):

$$y(t) = 20 - 100.0t + 390.0t^2 - 1456.7t^3 + 5932.5t^4 - 25581.t^5 + 1.0922 \times 10^5 t^6$$
$$-4.5411 \times 10^5 t^7 + 1.87 \times 10^6 t^8 + O(t^9)$$
$$x(t) = 20 + 60.0t - 270.0t^2 - 30.0t^3 + 2397.5t^4 - 5169.5t^5 - 6617.4t^6$$
$$+49690.0t^7 - 41210.0t^8 + O(t^9)$$
,

Here is an example of an exact solution to an ODE:

$$\frac{dz}{dt} = (1 - \alpha z)z \qquad z(0) = 100$$
, Exact solution is : $z(t) = \frac{10.0}{1.0 - .9e^{-1.0t}}$



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z as a function of time