Section 2 of "Fundamentals Versus Beliefs under Almost Common Knowledge" The full text is available at http://are.Berkeley.EDU/~karp/

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1 A Simple Migration Model

The small, open, competitive economy[1] consists of two sectors. The stock of (domestically mobile) labor is normalized to 1. The amount of labor in the manufacturing sector is L, and the amount of labor in the agricultural sector is $1 - L$. The manufacturing sector has increasing returns to scale. The wage in manufacturing is $a + bL$. The agricultural sector has constant returns to scale, and the wage there is c, a constant. In this simple model with certainty, the parameters satisfy $c > a > c - b$. Thus, the wage in agriculture exceeds the wage in manufacturing if and only if $L < \theta \equiv \frac{c-a}{b}$. Other things equal, *a larger value of θ makes it more attractive to be in the agricultural sector.*

The *net* number of migrants into manufacturing is u. With common knowledge, there is never two-way migration in equilibrium, so $| u |$ equals the total (the gross) number of migrants. Given the initial allocation L, the number of workers in manufacturing in the next (and final) period is $L + u$. In order to change sectors, a worker needs to pay for "migrations services", such as education. The price he pays depends on the aggregate amount of

[1] For the small open economy commodity prices are fixed, so workers' real income equals their nominal income, which equals their wage. Therefore, their migration decision depends on the comparison of wages in the two sectors. Labor is not internationally mobile.

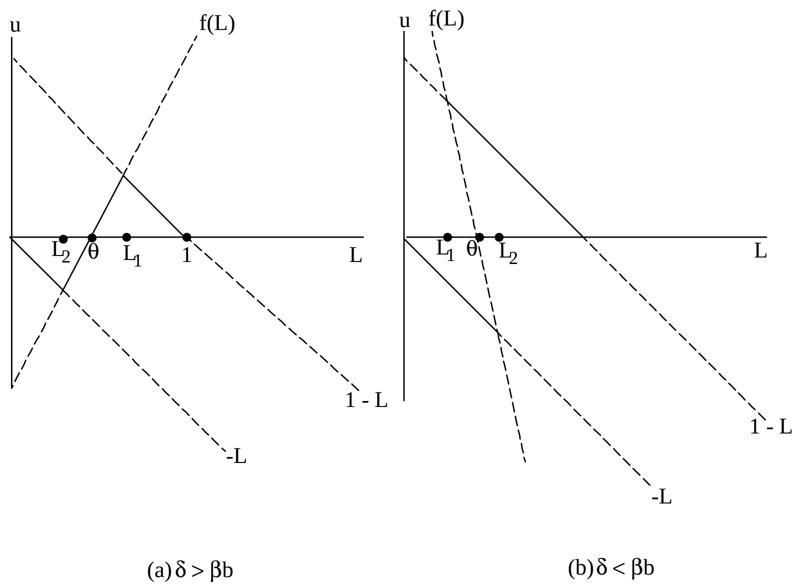

Figure 1: The Equilibrium with Perfect Information

migration, and equals $\delta \mid u \mid$, $\delta > 0$.[2] The total amount of migration is constrained by $-L \leq u \leq 1 - L$.

The discount factor is β. Each worker decides whether to migrate (i.e., he makes a 0-1 decision). An agent who migrates pays the price of migration in the current period, and benefits from the wage differential in the next period. If an interior equilibrium involves migration into manufacturing ($1-L > u \geq 0$) the equilibrium satisfies $\beta\left[a + b(L+u) - c\right] - \delta u = 0$. In this case, the private cost of migration equals the present value of the wage differential. If an interior equilibrium involves migration into agriculture ($-L < u \leq 0$), the equilibrium satisfies $\beta\left[c - a - b(L+u)\right] + \delta u = 0$. If neither of these conditions hold, the equilibrium is on the boundary ($u = -L$ or $u = 1 - L$).

[2]Section 4 discusses the manufacturing production function and the cost function for migration services, needed for welfare analysis. In order to determine the equilibrium we need only the wage function, $a + bL$, and inverse supply function for migration services, $\delta \mid u \mid$.

To describe the equilibrium, define

$$f(L;\theta) \equiv \frac{\beta(a-c+bL)}{\delta-\beta b} = \frac{\beta b}{\delta-\beta b}\left(L-\theta\right),$$

the value of u at an interior equilibrium. Figure 1 graphs the two constraints $u \geq -L$ and $u \leq 1-L$ and the function $f(L)$ for the two cases where $\delta > \beta b$ (Figure 1a) and $\delta < \beta b$ (Figure 1b). L_1 denotes the intersection of $f(L)$ and the constraint $u = 1-L$, and L_2 denotes the intersection of $f(L)$ and the constraint $u = -L$. The values of L_i are

$$L_1 = \frac{1}{\delta}\left(\beta b\theta + \delta - \beta b\right); \qquad L_2 = \frac{\beta b}{\delta}\theta. \tag{1}$$

The graph of $f(L)$ shows levels of u at which the present value of the wage differential equals the price of adjustment. If the constraint $-L \leq u \leq 1-L$ is not binding, $u = f(L)$ is an equilibrium, but not necessarily a *stable* equilibrium. The solid lines in the two panels show the set of stable equilibria as a function of the initial allocation, L.

Using the definition of $f(L)$, the net benefit of migrating to the manufacturing sector, i.e. the present value of the wage differential minus the price of migration, given that migration is u, equals $(\delta-\beta b)\left(f(L)-u\right)$.

The equilibrium is unique if $\delta > \beta b$ (Figure 1a). In this case all labor moves to manufacturing if $L \geq L_1$ and all labor moves to agriculture if $L \leq L_2$. For $L \in (L_2, L_1)$ the interior equilibrium $u = f(L)$ is stable. In order to verify stability, consider the effect of a deviation from the interior equilibrium. That is, suppose that $L \in (L_2, L_1)$ and migration is $u = f(L) + x$ rather than the equilibrium level $u = f(L)$. In this case, the net benefit of migrating to manufacturing is $(\delta-\beta b)\left(f(L)-u\right) = (\delta-\beta b)\left(f(L)-f(L)-x\right) = -\left(\delta-\beta b\right)x$. If $x > 0$ (i.e., in the deviation migration exceeds the equilibrium level), then the net benefit of migration is negative; consequently, fewer people would want to migrate, restoring the equilibrium to $u = f(L)$.

If $\delta < \beta b$ (Figure 1b) the equilibrium is either unique or indeterminate, depending on the value of L. All labor moves to manufacturing if $L \geq L_2$ and all labor moves to agriculture if $L \leq L_1$. For $L \in (L_1, L_2)$ there are two stable equilibria ($u = -L$ and $u = 1-L$) and an unstable equilibrium $f(L)$. In the stable equilibria all labor ends up in either manufacturing or in agriculture. To confirm that $u = f(L)$ is an unstable equilibrium for

$L \in (L_1, L_2)$ consider the effect of a deviation, as above. Again, suppose that migration is $u = f(L) + x$ rather than the equilibrium level $u = f(L)$. The net benefit from migrating to the manufacturing sector is, once again, $(\delta - \beta b)(f(L) - u) = (\delta - \beta b)(f(L) - f(L) - x) = -(\delta - \beta b)x$. In this case, however (since $\delta < \beta b$) the sign of the net benefit is the same as the sign of x. Thus, for example, if $x > 0$ (the number of migrants exceeds the equilibrium level), the net benefit of migrating to the manufacturing sector is positive. In this case, more agricultural workers want to migrate. In other words, the initial deviation promotes a further deviation in the same direction, moving the outcome away from the (unstable) equilibrium toward the boundary $u = 1 - L$. Similarly, if the initial deviation is $x < 0$ (the number of migrants falls short of the equilibrium level), the outcome moves toward the boundary $u = -L$.

The interesting case is $\delta < \beta b$, where the equilibrium is indeterminate for $\{L : L_1(\theta) < L < L_2(\theta)\}$. The length of the set of L at which the equilibrium is indeterminate is $L_2 - L_1 = \frac{\beta b}{\delta} - 1$. This set of indeterminacy increases with $\frac{\beta b}{\delta}$ and collapses to the null set as $\frac{\beta b}{\delta} \to 1$. A larger value of b implies a greater degree of increasing returns to scale ("more non-convexity"), and a larger value of β makes this non-convexity more important to agents' decisions. A larger value of δ increases the slope of the inverse supply function for migration services and dampens workers' incentive to imitate other migrants. The measure of the set of indeterminacy depends on the relative strengths of the two forces that increase or moderate the model's non-convexity.