Final exam, AREP 263
Fall 2006

You have three hours to work on this exam – closed book, closed notes. Please return the exam to me or to my mailbox in 207 Giannini by 5 PM on Friday.

Answer all questions. You cannot avoid using mathematics to answer these questions. However, DO NOT GET BOGGED DOWN IN ALGEBRA. It is sufficient to provide a clear statement of the algebraic manipulations that you would perform, and the nature of the result that you would obtain.

I hope that there are no typos or ambiguities in the following questions. If you find a problem of this sort, just be sure to let me know what assumptions you have made.

Question 1. Consider the following two-sector model, with total stock of labor = 1. The amount of labor in the manufacturing sector at time t is equal to $L(t)$. The value of output in the manufacturing sector is $Lf(L)$ with $f'(L) > 0$ The increasing returns to scale in the sector are external to the firm, so each worker receives the wage $w = f(L)$. The value of output in the agricultural sector is $(1 - L)a$; the agricultural wage is the constant a. Assume that $f(0) < a < f(1)$. The flow of labor into the manufacturing sector is m. The social cost of migration is $\gamma m^2/2$, and each worker who changes sectors pays the migration cost $\gamma |m|$. The constant discount rate is r.

a) (Economic interpretation) (i) How would the model change if the increasing returns to scale were internal to the firm? (ii) What is the implication of the assumption that $f(0) < a < f(1)$?

b) Discuss the qualitative solution to the optimization problem of the social planner who is able to directly choose migration in order to maximize the present discounted value of output minus adjustment costs.

c) Discuss the qualitative solution to the equilibrium problem of (non-strategic) workers with rational point expectations, each of whom decides (at each point in time) whether to migrate. A worker wants to maximize the present discounted value of wages minus adjustment costs. (Your answer should explain what you mean by "rational point expectations".)

d) Can you design a tax policy that causes the competitive equilibrium to produce the socially optimal program?

d) Describe an algorithm (or algorithms) for solving the optimization problem and the equilibrium problem. What (if any) differences between these two problems do you need to take into account?

Question 2 Consider the following two problems:

Problem A: You have a three-stage (two decision periods) cake eating model. In each period the utility from eating c units is $U(c)$. The total size of the cake is 1, so $c_1 + c_2 + c_3 \leq 1$. The one-step ahead discount rate is r_1 and the two-step ahead discount rate is r_2. (These are known numbers.) The corresponding one-step and two-step discount factors are $\beta_i = \frac{1}{1+r_i}$. That is, the present value at time 1 of a unit of cake at time 3 is $\beta_1\beta_2$; the present value at time 1 of a unit of cake at time 2 is β_1; the present value at time 2 of a unit of cake at time 3 is β_1 (NOT β_2). The decision-maker at time 1 wants to maximize the present discounted value of the stream of utility.

Problem B: You have a three-stage (two decision periods) cake eating model. In each period the utility from eating c units is $U(c)$. The total size of the cake is 1, so $c_1 + c_2 + c_3 \leq 1$. The discount rate in period i is

$$r_i = r_{i-1}\left(1 + \varepsilon_i\right)$$

where ε_i are independently and identically distributed random variables with mean 0 and support $(-1, 1)$. In period i, when choosing how much cake to eat, the decisionmaker knows r_{1-1}. The present value at time 1 of a unit of cake at time 2 is β_1; the present value at time 1 of a unit of cake at time 3 is $\beta_1\beta_2$; the present value at time 2 of a unit of cake at time 3 is β_2 (NOT β_1). The decision-maker at time 1 wants to maximize the expected present discounted value of the stream of utility.

a) Discuss the differences in assumptions between these two models, and explain the economic significance of these differences. (I see two important differences in assumptions. I want you to tell me how these differences reflect differences in the underlying story that the models are intended to describe.)

b) Explain how you would find the decision rules, and the first period decision, in these two models. (Tell me what problems you need to solve; I'm not asking for an algorithm that produces a numerical solution.)

Question 3. Two fishers harvest from a common property resource. Agent i's harvest in period t is $h_i(t)$. The stock of the resource in period t is x_t. The stock changes according to

$$x_{t+1} = (x_t - h_1(t) - h_2(t))^{\alpha}.$$

Agent $i's$ single period utility in period t is $\ln(h_i(t))$, and both agents have the constant discount factor β. They each want to maximize the present discounted infinite stream of utility.

a) How would you find the cooperative equilibrium (the one that maximizes joint welfare)? What, if any, restriction do you need to impose on α for your proposed solution to be correct?

b) How would you find the non-cooperative Nash equilibrium?