Exam ARE 263
Fall 2003

Answer both questions 1 and 2 and either question 3 or 4. Each question is worth the same number of points.

1) When an agent harvests h_t fish in a period and the current stock is S_t, the stock the next period is

$$S_{t+1} = f(S_t - h_t).$$

The benefit of harvest in the current period is $U(h_t)$ and the discount factor is β. The growth function f is concave.

(a) Write down the optimization problem for the agent who maximizes the present value of the stream of benefits of harvest and write down the dynamic programming equation.

(b) Use the DPE to obtain the Euler equation.

(c) Give an intuitive interpretation of the Euler equation.

(d) Write down the steady state Euler equation. How does the steady state stock depend on (i) the discount factor β and (ii) the elasticity of marginal utility? [Hint: This question does not require any algebra.]

(e) Discuss an algorithm for solving this (discrete stage) control problem numerically.

2) Consider the continuous time problem

$$\begin{aligned} &\max \int_0^\infty e^{-rt} \left(u(c_t) - D(x_t)\right) \\ \text{s.t. } \dot{x} &= c - \delta x \quad x_0 \text{ given} \end{aligned}$$

Assume that u is concave and increasing and D is convex and increasing.

a) Using the current value Hamiltonian, write the necessary conditions for this problem.

b) Sketch the phase portrait.

c) Explain how to obtain the comparative statics of the steady state with respect to δ. (Describe the recipe in enough detail to convince me that you know what to do. It is not necessary to perform the calculations.)

d) Explain how to obtain a numerical solution for the optimal control rule, $c^*(x)$.

3) Consider the problem described in Question 1, but suppose that there are two agents harvesting the fish non-cooperatively. If agent i harvests h_t^i her utility in period t is $U(h_t^i)$.

(a) Obtain the Euler equation for a (differentiable) symmetric Markov Perfect Nash Equilibrium.

(b) Explain the source of the non-uniqueness for this class of equilibrium. How does the non-uniqueness problem change if the resource is non-renewable (i.e. if $f(S_t - h_t) = S_t - h_t$)?

4) The present value at time 0 of social utility (u) of consumption at time t, (c_t) is $\alpha(t)\, u(c_t)$ where $\alpha(t)$ is the discount factor.

(a) Using this notation, what is the pure rate of time preference and what is the social discount rate? Explain the meaning of these two terms.

(b) Explain the time consistency problem that arises if the pure rate of time preference is declining, in an optimization problem where the objective is to maximize $\int_0^\infty \alpha(t)\, u(c_t)\, dt$ subject to $\dot{k} = f(k_t) - c_t$.

(c) Explain how you would solve this maximization problem (or its discrete time analog) in the two cases where the planner at time t either can or cannot make commitments about future behavior.