# NOTES ON RISK SHARING

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Contents



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## 1. Introduction

We've seen that when a farm-household has to bear agricultural risk, then the separation property fails. The farm's productive decisions will be made in a way which takes into account a desire to shield the household from bad states of the world. In our example, the household will work harder,[1] earn more income on average, but actually be worse off in terms of its expected utility.

Adding risk to agricultural production is undoubtedly a step toward improved realism, but it may be easy to overstate the effects that this sort of risk will have on the farm-household. The reasons are social: risk is the perfect illustration of the maxim that shared burdens are lighter. Suppose that there are two or more farm-households, each with similar stochastic production functions. If the shocks they face (their separate $\epsilon$s) are less than perfectly correlated, then by pooling their resources and *sharing* their risks, the variation in expenditures that either party would face separately is reduced.

In fact, once we notice that sharing risk improves welfare, we might start to wonder why (or if?) *all* risk isn't shared. And if it is all shared, then we get a pretty amazing result: the separation result will hold *even though* farm-households may have to deal with risky agricultural technologies.

## 2. Measuring Risk

It will be useful to consider ways of measuring the risk that households bear *after* any actions they make take (such as decreasing their leisure) to attempt to mitigate the effects of variation in their income.

In the case of the farm-household we've worked with to this point, let's maintain our assumption that utility is separable between consumption and leisure, and let the function $u(c)$ denote utility from consumption (assuming for now that there's aggregation within the household).

Now, we've seen that when the farm-household operates a stochastic agricultural production function, then consumption depends on the realized value of the random shock $\epsilon$. Thus, the expected utility of consumption for the farm household is simply

$$\mathrm{E}U(c(\epsilon)).$$

[1] A research topic that might be worth pursuing: If various market 'reforms' have the effect of increasing the risk that households bear, then this may have the effect of causing them to work harder, and average income will increase. Studies that focus only on income or average consumption might conclude that the reform was as great success, even though the household is actually worse off.

We assume that $U$ is strictly increasing, weakly concave, and continuously differentiable, so that the farm household is weakly risk averse.

Now, define the risk faced by the farm household to be a quantity

$$R = U(\mathrm{E}c(\epsilon)) - \mathrm{E}U(c(\epsilon)).$$

This cardinal measure orders probability distributions in the same manner as Rothschild and Stiglitz [1970]. Or more precisely, what Rothschild and Stiglitz [1970] offer is a partial ordering of risks. Our measure is consistent with their ordering (but of course any monotone transformation of our measure would be similarly consistent).

## 3. Example of sharing between two households

Consider two people (or households) indexed by $i = 1, 2$. Let person $i$ have utility over consumption given by $U_i(c)$; we assume only that $U_i$ is increasing, continuously differentiable, and strictly concave.

Imagine that each person $i$ faces a stochastic distribution of consumption $c_i(\epsilon)$ that depends on the state of the world $\epsilon \in \Omega$, where $\Omega$ is assumed to be finite.

Here's the question: When can these two households improve their *ex ante* expected utility by sharing?

To answer the question, think of person one devising a sharing scheme which she then proposes to person two. Person two accepts the scheme if and only if adopting the scheme doesn't make him worse off.

Thus, we have

$$\max_{\{\tau(\epsilon)\}} \sum_{\epsilon\in\Omega} \Pr(\epsilon) U_1(c_1(\epsilon) + \tau(\epsilon))$$

such that

$$\sum_{\epsilon\in\Omega} \Pr(\epsilon) U_2(c_2(\epsilon) - \tau(\epsilon)) \geq \sum_{\epsilon\in\Omega} \Pr(\epsilon) U_2(c_2(\epsilon)).$$

The key first order condition for this expression implies that there's no scope for improvement (i.e., $\tau(\epsilon) = 0$ for all $\epsilon \in \Omega$) if and only if

$$U_1'(c_1(\epsilon)) = \lambda U_2'(c_2(\epsilon))$$

for all $\epsilon \in \Omega$ for some positive constant $\lambda$. This in turn implies that the marginal utilities of consumption of the two people must be perfectly correlated.

Observations:

(1) This requirement of perfect correlation doesn't depend on relative wealth, or other possible differences between the households—in particular, even a very poor household will benefit by striking a deal with a very wealthy household which calls for the poor

household to make transfers to the wealthy household in some states of the world.
(2) Schemes like this will hold even if one of the households is risk-neutral, or even somewhat risk-seeking.
(3) Notice that we've been completely agnostic about the source of shocks which may make $c_i(\epsilon)$ a random variable; thus, insights here don't depend on the source of the risk.
(4) Don't *have* to have separability between e.g., leisure and consumption for this story to work. A minor extension would involve showing that one would still equate the marginal utilities of consumption (up to a constant); it's just that this marginal utility of consumption would now also depend on leisure.[2]
(5) Work out example using CRRA preferences. Also CARA preferences.

## 4. Interpretation of sharing as insurance

One interpretation of the solution to the sharing problem above is that the two households are engaged in the exchange of *insurance.* To make this interpretation clearer, suppose that person one is a risk-neutral insurer, and that $U_1(c) = c$, and $U_1'(c) = 1$.

Then from the optimality condition we obtained earlier, we have

$$1 = \lambda U_2'(c_2(\epsilon) - \tau(\epsilon)),$$

which implies that $c_2(\epsilon) - \tau(\epsilon)$ is a constant $c_2$. In states in which $c_2(\epsilon)$ is greater than $c_2$, person two pays a net premium for insurance; in states when $c_2(\epsilon)$ is less than $c_2$ person two receives an "indemnity" transfer guaranteeing them constant consumption.

What's the value of the constant consumption? We can see how large a constant by using the participation constraint, which now can be read

$$U_2(c_2) = \sum_{\epsilon \in \Omega} \Pr(\epsilon) U_2(c_2(\epsilon)).$$

solving this equation for $c_2$ gives the result.

What about expected profits for the insurer? These are given by

$$\pi(c_2) = \sum_{\epsilon \in \Omega} \Pr(\epsilon)(c_2(\epsilon) - c_2).$$

Note that if there's competition among insurers, then expected profits should be equal to zero, implying that $c_2 = \mathrm{E} c_2(\epsilon)$.

[2] This might be good fodder for a research project. Could non-separabilities explain some empirical rejections of risk-sharing?

## 5. Welfare theorems

**Definition 1.** An production economy is a triple $\mathcal{E} = (\mathcal{U}, \mathcal{F}, \mathcal{X})$ of sets of consumer preferences $\mathcal{U} = \{U_i : X \to \mathbb{R}\}_{i=1}^n$, production technologies $\mathcal{F} = \{F_i : X \to X\}_{i=1}^n$, and endowments $\mathcal{X} = \{x_i \in X\}_{i=1}^n$.

**Definition 2.** A *competitive equilibrium* for an economy with endowments $x_i(\epsilon)$ is a set of prices $p(\epsilon)$ and allocations $c_i(\epsilon)$ such that

(1) Given prices and endowments, the allocation $c_i$ solves the household's problem of maximizing expected utility subject to its budget constraint $\sum_\epsilon p(\epsilon)c_i(\epsilon) \leq \sum_\epsilon p(\epsilon)x_i(\epsilon)$.
(2) Given prices, expected-profit maximizing firms produce enough of each good to make the allocation $\{c_i(\epsilon)\}_\epsilon$ feasible.
(3) Given the allocation $c_i$, prices clear markets for all goods (indexed by $\epsilon$).

**Theorem 1** (First welfare theorem)**.** *Any competitive equilibrium is Pareto optimal.*

**Theorem 2** (Second welfare theorem)**.** *For any Pareto optimal allocation, there exists a set of endowments and prices such that the prices and allocations form a competitive equilibrium.*

## 6. Formulation of the Planner's Problem

People consume in several periods indexed by $t = 1, \ldots, T$, with person $i$ discounting future expected utility using a discount factor $\beta_i$. Different states of the world are realized in each period, with the probability of state $\epsilon_t \in \Omega$ being realized in period $t$ allowed to depend on previous realizations of the state and on the period, given by $p_t(\epsilon_t|\epsilon_{t-1})$.

Now, the social planner chooses state-contingent consumption allocations to solve

$$\max_{\{(c_{it}(\epsilon))\}} \sum_{i=1}^n \lambda_i \sum_{t=1}^T \beta_i^{t-1} \sum_{\epsilon_t \in \Omega} p_t(\epsilon_t|\epsilon_{t-1}) U_i(c_{it}(\epsilon_t))$$

subject to the resource constraints

$$\sum_{i=1}^n c_{it}(\epsilon_t) \leq \sum_{i=1}^n x_{it}(\epsilon_t),$$

which must be satisfied at every period $t$ and state $\epsilon_t$; the planner takes as given the initial state $\epsilon_0$ and a set of positive weights $\{\lambda_i\}$. By varying these weights one can compute the entire set of interior Pareto efficient allocations [Townsend, 1987].

If we let $\mu_t(\epsilon_t)$ denote the Lagrange multiplier associated with the resource constraint for period $t$ in state $\epsilon_t$, then the first order conditions for the social planner's problem can be written as

$$\lambda_i \beta_i^{t-1} p_t(\epsilon_t|\epsilon_{t-1}) U_i'(c_{it}(\epsilon_t)) = \mu_t(\epsilon_t). \tag{1}$$

Since this condition must be satisfied in all periods and states for every agent, from this it immediately follows that

$$U_i'(c_{it}(\epsilon_t)) = \frac{\lambda_j}{\lambda_i}\left(\frac{\beta_j}{\beta_i}\right)^{t-1} U_j'(c_{it}(\epsilon_t)) \tag{2}$$

for any period $t$, any pair of agents $(i,j)$ and any state $\epsilon_t$.

Examine (2), interpreting the marginal utilities of consumption for $i$ and $j$ as random variables (via their dependence on the random shock $\epsilon$). Note that the factor involving the $\lambda$-weights and discount factors is *not* random—though it may vary over time, this variation is entirely deterministic. An immediate consequence of (2), then, is that all agents' marginal utilities of consumption are *perfectly* correlated. This is the hallmark of *full insurance*.

## 7. Solution to the Planner's Problem: Separation!

## 8. Pareto Optima as Competitive Equilibria

## 9. Testing Full Insurance

Early tests of the full insurance hypothesis were conducted using U.S. data by Mace [1991] and Cochrane [1991]. The first test using developing country data was due to Townsend.

The usual test of full insurance essentially proceeds from (1), and a simple parameterization of the utility function. One convenient parameterization is the so-called "Constant Elasticity of Substitution" (CES) or "Constant Relative Risk Aversion" (CRRA) specification:

$$U_i(c) = \frac{c^{1-\gamma}-1}{1-\gamma}.$$

Here the parameter $\gamma$ is equal to the Arrow-Pratt measure of "Relative Risk Aversion" (which can also be thought of as the elasticity of the marginal utility of consumption). The parameter $\gamma$ is assumed to be non-negative, and larger values of $\gamma$ imply more risk aversion. There are three notable special cases:

- **$\gamma = 0$:** In this case, the utility function is linear, and the agent risk-neutral.

- **$\gamma \to 1$:** As $\gamma$ approaches unity, the CRRA utility function approaches $\log c$. A mathematical subtlety: if $\gamma$ is less than one, then the utility function is bounded below, but not above, while if $\gamma$ is greater than one, the utility function is bounded above but not below.
- **$\gamma = 2$:** In this case, the CRRA utility function simplifies to $1-1/c$, so that utility becomes a rectangular hyperbola.

In any event, with the CRRA parameterization $U_i'(c) = c^{-\gamma}$. Combining this with (1) yields the relationship

$$\gamma \log c_{it}(\epsilon_t) = \log \frac{\mu_t(\epsilon_t)}{p_t(\epsilon_t|\epsilon_{t-1})} + \log \frac{\beta_i}{\lambda_i} - t \log \beta_i.$$

This is a simple consumption function, describing consumption demand for person (or household) $i$ as a function of their Frischian 'wealth' $1/\lambda_i$, the date and their patience, $\beta_i$, and on the aggregate shock $\epsilon_t$.

It's worth noting that the effect of the aggregate shock $\log \frac{\mu_t(\epsilon_t)}{p_t(\epsilon_t|\epsilon_{t-1})}$ on the households consumption demand can also be interpreted as the reciprocal of the state-contingent 'price' of $t$-period consumption at date $t-1$, so this really does resemble a standard demand function.

Now, let $\tilde{c}_{it}$ denote observed consumption for person $i$ at time $t$, but suppose that this observed consumption may be contaminated by error, so that $\tilde{c}_{it} = c_{it} e^{\epsilon_{it}}$. Substituting this into our previous expression allows us to write a 'reduced form' consumption demand function:

$$\log \tilde{c}_{it} = \eta_t + \alpha_i + \delta_i t + \epsilon_{it}. \tag{3}$$

Here the term $\eta_t$ can be thought of as minus the log price of consumption, or alternatively as capturing the effects of aggregate shocks on consumption for person $i$. The term $\alpha_i$ reflects the influence of indidividual $i$'s wealth on her log consumption, while $\delta_i t$ captures the influence of the passage of time on the consumption of more or less patient consumers (note that if there's no variation in patience, then this term will be subsumed into $\eta_t$).

Now, one could use panel data to estimate (3). In this case, the $\eta_t$ would play the role of "time effects"; the $\alpha_i$ the role of household or individual level "fixed effects", and so on. With estimates of the reduced form coefficients in hand, one could put some restrictions (if not completely identify) underlying structural parameters of the model, such as the preference parameters $\gamma$ and $\{\beta_i\}$.

However, any such attempt to interpret the reduced form coefficients rests on the maintained hypothesis that the utility function is correctly specified, and that allocations are in fact efficient. And so before leaping to interpret, it may be wise to first test.

A simple collection of tests is available. Under different alternative hypotheses regarding the allocation of consumption, it may be that other variables ought to matter in (3). For example, if we've misspecified the utility function, then adding *other* functions of individual or aggregate characteristics to the right-hand side of the regression can serve as a test. For example, Kurosaki [2001] shows that if individuals have CRRA preferences, but if $\gamma$ varies across households, then this alternative implies that an interaction between individual fixed effects and aggregate time effects should be expected to appear on the right-hand side of the expression above.

A mis-specification of the utility function can be regarded as a somewhat *shallow* reason for a rejection of the model. A deeper reason may be that in fact there *is not* full insurance. Many alternative models would then suggest that *individual* shocks to income or resources ought to influence current period consumption. One way to test the full insurance model, then, is to add functions of individual income to the regression above—under the null of full insurance, the coefficients associated with these functions of individual income ought to all be zero.

## 10. Policy Interventions as Insurable Shocks

The implementation of a policy which targets the welfare and consumption of particular households within a community may be regarded by the members of that community as simply another kind of shock, to be insured against *ex ante* [Ligon, 2004]. **?** construct a test of this, using the randomized policy intervention afforded by the *Progresa* program in Mexico, which transfers resources to particular poor households in moderately poor communities.

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