NOTES ON WELARE AND DISTRIBUTION

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Contents

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1. Introduction

As a normative matter, what is the ultimate goal sought by development economists? Broadly, we seek to improve the human condition by changing the allocation of resources, or (less directly) perhaps by changing the manner in which resources are allocated.

Since we wish to improve the human condition, it makes sense to look for places were we think that economic reasoning and analysis can make a difference. Perhaps it's for this reason that we focus on the institutions and allocations in developing countries—there's often more evidence of *mis*allocation and poorly functioning insitutions in these settings.

2. Social Welfare Functions

Harsanyi (1955) provides an axiomatic argument that any well-behaved social welfare function will be an additive function of individual utilities. His argument is very similar to development of von Neumann-Morgenstern's argument that utility under uncertainty should be additive in utility in different states of the world.

One can compare the two settings by adopting a sort of Rawlsian "veil of ignorance" (though note that Harsanyi's paper anticipates Rawls' by over two decades): imagine that everyone who will ever live is floating around in some sort of limbo, waiting to be born. The uncertainty they face is over who they'll be: in a good outcome, they'll become a person who happens to be very happy; in a bad, very unhappy.

So let's follow Harsanyi's advice and adopt such a linear function:

$$W = \sum_{i=1}^{n} V_i$$

where V_i is person i's discounted expected utility.

How are the V_i related to the allocation of real resources?

Let x_i be a measure of individual wealth (or more generally, control over resources). Assume $V_i = \lambda_i V(x_i)$, and that V is an increasing and strictly concave function of x_i.

How should one interpret the weights λ_i? Perhaps as *joie d'vive*? As in von Neumann-Morgenstern setup, utility functions can only be identified from *behavior* up to an affine transformation. So maybe some people are just happier than others.

It's very tempting to apply Ockham's razor here (which is what Harsanyi does), and simply assume that $\lambda_i = \lambda$. This amounts to assuming that one person with a set of resources x_i (post-instantiation) is just as happy as another person j with the same set of resources.

2.1. **Writing contracts behind the veil.** Suppose it was possible to write binding contracts while still situated in Limbo, specifying exchanges or actions to be taken after people are born, and learn their identity. One possible thing to contract over would be the allocation of resources across people, x_i.

If souls were able to collectively negotiate an efficient contract from behind the veil of ignorance, it would dictate an allocation which would solve the planning problem

$$\max_{\{c_i\}_{i=1}^n} \frac{1}{n} \sum_{i=1}^n \lambda_i V(c_i)$$

such that $\sum_{i=1}^n c_i \leq \sum_{i=1}^n x_i$.

The solution to this problem must solve the first order condition

$$\frac{1}{n} \lambda_i V'(c_i) = \mu.$$

Noting that μ is common to all the first order conditions which pin down individual resources, and assuming (without loss of generality) that $\sum_{i=1}^n \lambda_i = n$, we have

$$\lambda_i V'(c_i) = \sum_{j=1}^n V'(c_j).$$

Thus, marginal utilities move together in proportion across the population as aggregate resources change—or better put, if society were to receive a larger aggregate endowment of resources rather than a smaller, then the larger endowment would be shared across the population according to a rule which would depend only on the shape of the marginal utility function V' and on the individual "weights" λ_i. Since the function V is assumed to be strictly increasing, it follows that a larger aggregate endowment would yield not just a shared change in marginal utilities, but an actual increase in everyone's resources c_i.

2.2. **Interpersonal Utility Comparisons.** If all the weights λ_i were equal to one, then it would follow that $c_i = \bar{c}$ for all (i, j), where $\bar{c}$ is the per capita value of resources, which again makes it tempting to borrow Ockham's razor. But by pursuing our Rawlsian thought experiment, it's possible to offer an alternative rationale for having equal weights.

By an argument due to von Neumann and Morgenstern, we know that behavior on this side of the veil of ignorance can only reveal information regarding the shape of V up to an affine transformation—this is one way of thinkng of the fundamental difficulty of conducting interpersonal utility comparisons. Even after 'instantiation' a person can't learn even their *own* λ-weight, since even if their know their own level of utility, they can't separately identify the effects of having a particular λ_i from having a proportionally larger or smaller V. Accordingly, post-instantiation it will be impossible for an individual to assert a claim to additional resources based on having a larger or smaller λ_i. Instead they may be able to assert a claim based on the *probability* that they have a particularly large or small λ_i, but since there's a fundamental and utter ignorance regarding not only the distribution of the λ-weights but also regarding one's own position in this distribution even *ex post*, the only consistent position to take regarding these probabilities is that they're all equal.

3. Inequality

From Harsanyi, we have the result that the ideal distribution of resources is an egalitarian distribution. Atkinson (1970) measures inequality by comparing the value of a Harsanyi-type social welfare function where everyone has the same resources to the value of a social welfare function where they don't. Evaluated from behind the veil of ignorance, the welfare loss associated with inequality in x_i can be measured by

$$A^* = V(\bar{c}) - \frac{1}{n}\sum_{i=1}^{n} V(x_i). \tag{1}$$

Since V is concave (by assumption) and since addition preserves concavity, by Jensen's inequality it's clear that this expression must be non-negative. But the units of this expression are utils, and Atkinson wants a number he can interpret more easily, so he instead adopts a measure of inequality which is a kind of equivalent variation: how much per capita income would society be willing to sacrifice in order to implement an egalitarian allocation? This can be computed by finding what Atkinson calls "equity-sensitive income" y_e solving

$$V(y_e) = \frac{1}{n}\sum_{i=1}^{n} V(x_i);$$

thus, the amount society would be willing to sacrifice is $\bar{c} - y_e$; Atkinson then normalizes this by total per capita resources $\bar{c}$ in order to obtain his inequality index. This has the advantage of being dimensionless, and can be interpreted as the proportion of per capita resources people would be willing to sacrifice in order to implement a perfectly egalitarian distribution of income.

Atkinson still faces the practical problem that one can't compute y_e without knowing V. So he assumes a CES utility function; this gives the result that Atkinson's inequality measure can be written

$$I = 1 - \left[\frac{1}{n}\sum_{i=1}^{n}(x_i/\bar{c})^{1-\gamma}\right]^{1/(1-\gamma)},$$

where γ is the coefficient of relative risk aversion, or alternatively, the elasticity of marginal utility with respect to x_i.

As noted above, Atkinson's gives a measure of the welfare cost of inequality which can be expressed as a proportion of aggregate resources—how much of their aggregate resources (as a proportion of total) would society (still behind the veil of ignorance) be willing to sacrifice in order to obtain an egalitarian distribution?

It's sometimes asserted that a shortcoming of Atkinson's measure of inequality is that it's not additively *decomposable* into inequality in different sub-populations. Suppose, for example, that we were to partition the population into two parts, $\{1, 2, \ldots, n_1\}$ and $\{n_1+1, n_1+2, \ldots, n\}$ people respectively. For a number of reasons, it would be convenient to be able to assert that if Atkinson's measure for the first group was A_1 and for the second group A_2, then the measure of inequality for the entire population would be a weighted (by relative population sizes and relative aggregate consumptions) sum of the group inequalities.

It's true that the definition of A above doesn't generally allow this sort of decomposition. But if one wishes to have a decomposable measure of inequality, the original utilitarian approach taken by Atkinson offers an obvious candidate: after all, the original sum of CES functions that Atkinson uses for his social welfare function is additively decomposable (as is the measure of welfare cost (1) we've provided above). To develop the decomposition we're after, let

$$A = \frac{1}{n}\sum_{i=1}^{n}\left(\frac{x_i}{\bar{c}}\right)^{1-\gamma}.$$

Now, dividing the population into two groups G_1 and G_2, we can write

$$A = \frac{1}{n}\sum_{i\in G_1}^{n}\left(\frac{x_i}{\bar{c}}\right)^{1-\gamma} + \frac{1}{n}\sum_{i\in G_2}^{n}\left(\frac{x_i}{\bar{c}}\right)^{1-\gamma}.$$

Multiplying and dividing the two terms by the number of members of each group $(\#G_1, \#G_2)$ gives us

$$A = \frac{\#G_1}{n}\left[\frac{1}{\#G_1}\sum_{i\in G_1}^{n}\left(\frac{x_i}{\bar{c}}\right)^{1-\gamma}\right] + \frac{\#G_1}{n}\left[\frac{1}{\#G_1}\sum_{i\in G_2}^{n}\left(\frac{x_i}{\bar{c}}\right)^{1-\gamma}\right].$$

Next let ρ_j be the population fraction for group j, and multiply and divide each term by per capita resources for the group, $\bar{c}_j$, which yields

$$A = \sum_{j=1}^{2} rho_j \left(\frac{\bar{c}_j}{\bar{c}}\right)^{1-\gamma}\left[\frac{1}{\#G_j}\sum_{i\in G_j}^{n}\left(\frac{x_i}{\bar{c}_j}\right)^{1-\gamma}\right].$$

Finally, letting $A_j = \frac{1}{\#G_j}\sum_{i\in G_j}^{n}\left(\frac{x_i}{\bar{c}_j}\right)^{1-\gamma}$ be the obvious measure of within-group inequality, population inequality can be written as

$$A = \sum_{j=1}^{2} rho_j \left(\frac{\bar{c}_j}{\bar{c}}\right)^{1-\gamma} A_j. \tag{2}$$

To compute the contributions of "within group" and "across group" inequality to total inequality, compute "across group" inequality by

$$\frac{A - \sum_{j=1}^{2} rho_j \left(\frac{\bar{c}_j}{\bar{c}}\right)^{1-\gamma}}{A}$$

and "within group" inequality as one minus this quantity.

This measure of inequality is denominated in utils. However, by taking the parameter $\gamma = 2$, this approach yields a measure which is both decomposable and which *also* has an interpretation which relates the welfare cost of inequality proportional changes in aggregate resources. In particular, taking the expression for A^* in (1), normalizing resources by $\bar{c}$, and taking $\gamma = 2$, we obtain

$$A^* = \frac{1}{n}\sum_{i=1}^{n}\frac{\bar{c}}{x_i} - 1.$$

Relating A^* to Atkinson's equity-sensitive income y_e gives us

$$A^* = \frac{\bar{c}}{y_e} - 1.$$

This has a natural interpretation: it's a measure of how much greater social welfare *would be* under an egalitarian distribution, expressed as the percentage increase in resources relative to the current equity-sensitive income y_e.

3.1. **Examples of Other Complete Inequality Measures.** Consider two different allocations of resources across a population of n people, $x = \{x_1, \dots, x_n\}$ and $x' = \{x'_1, \dots, x'_n\}$. Assume that these two allocations have the property that total resources are the same in both; i.e., that $\sum_{i=1}^{n}(x_i - x'_i) = 0$.

The Atkinson inequality measures we've discussed above are examples of *complete* inequality measures, by which we mean that the Atkinson measure (given a fixed value of the parameter γ implies a complete ordering of all possible allocations of resources. That is, for *any* pair of allocations (x, x'), we can evaluate each allocation using Atkinson's measure and a fixed γ and arrive at a conclusion that either x is more unequal than x', that x' is more unequal than x, or that they're equally unequal.

Perhaps it's unnecessary to add that there are other complete inequality measures. These include

The Gini Index: This is simply $G = 1 - 2\int L(\rho)d\rho$, where $L(\rho)$ is the Lorenz curve.

The Robin Hood Index: $R = \max_\rho \rho - L(\rho)$; this is the maximum vertical distance between the 45-degree line and the Lorenz curve. An interpretation: this is approximately the share of total resources which would have to be transferred from poorer households to richer in order to achieve an egalitarian distribution of resources.

Theil's (Entropy) Measure: $T = \int \rho log(L'(\rho))d\rho - \int \rho \log \rho$ Has a peculiar interpretation: imagine randomly selecting two different dollars from all the resources owned by people in the economy. T can be thought of roughly as a measure proportional to one minus the probability that those two randomly selected dollars will be owned by different people.

Kuznets' Measure: Introduced by Kuznets in his pioneering study of cross-sectional inequality in income across nations. This is a family of measures along the lines of $K_\rho = (1 - L(\rho))/L(1-\rho)$. Interpretation: the share of resources controlled by the wealthiest (e.g., top 5 per cent) people divided by the share of resources controlled by the poorest (e.g., bottom 5 per cent) people.

All of these are complete measures—just as with the Atkinson index, they can rank any pair of allocations (x, x'). However, they will not generally give the *same* ranking—it's perfectly possible that according to the Atkinson measure x is more unequal than x', while by the Theil measure the x' allocation is the more unequal.

3.2. **Lorenz-Consistency.**

4. Inequality over Time

To this point we've developed a conceptual framework that involves some uncertainty (regarding one's wealth position after 'instantiation'), but not involving any time—it's as if after exiting Limbo one becomes a Rockefeller or a pauper for a single day, and then returns to Limbo, nameless once more.

A more realistic utilitarian treatment of the welfare consequences of inequality would take account of how initial resources influence utility over a person's entire lifetime.

We can adapt Atkinson's static analysis to the dynamic case by thinking of the function V not as a utility function, but as the *value* function solving a functional equation along the lines of

$$V(x) = \max_{(c,x')\in\Gamma(x)} (1-\beta)U(c) + \beta \mathrm{E}[V(x')|x].$$

This equation expresses the idea that people may derive utility from consumption in many periods, and that future variation or shocks can influence welfare, even if these future shocks leave current resources x unchanged.

But this raises an issue. Suppose that we observe two individuals, each with identical initial resources x, but in different environments. Person one is able to invest his initial stock of resources in an annuity, and receives a constant consumption c in every period, and bears no risk. Person two lacks access to annuity markets, and uses the same resources to try and make a living as a peasant farmer. The dynamic program above can capture these differences: for the person facing more risk, this risk will influence the shape and level of the value function V.

But the effects of this risk on the value function will be ignored if we assume, with Atkinson, that V is a CES function. It seems more appropriate to assume that the *utility* function is CES, and then to let this assumption along with variation in the environment (which will show up in the distribution over which we integrate $V(x')$ to compute expectations and in the constraint set $\Gamma(x)$) influence V in the appropriate way.

Another way to think of this issue: under what conditions will the value function be a CES function of initial resources?

Sufficient conditions:

- Full insurance
- Common CES utility functions U

- No aggregate shocks

Under these conditions, households will insure away all risk. Differences in time preferences may nonetheless cause people to have different time profiles of consumption.

These conditions may not be necessary (an opportunity!), but it's hard to see how the existence of *any* idiosyncratic risk is compatible with assuming a common CES form for V. This is true even if every one faces the *same* distribution of idiosyncratic shocks, save perhaps for some very special cases.

The moral: Atkinson's measure of inequality adequately captures the welfare costs of inequality in the case in which there's no risk, whether idiosyncratic or aggregate.

5. Measuring the welfare costs of risk

Assume that welfare in any period is just expected utility, scaled by an "impatience" measure $1-\beta$, and that the distribution of different states is stationary (and finite):

$$(1-\beta)\mathrm{E}U(c) = (1-\beta)\sum_{s=1}^{S}\pi_s U(c_s).$$

One approach: First, either assume (or estimate) the distribution $\{\pi_s\}$ of possible consumption outcomes for households with different resources (and other characteristics).

Second, assume not just that the distribution of s is stationary, but that consumption depends only on the current realization of s (and perhaps on fixed characteristics of the household). Then discounted, expected utility will be simply

$$\mathrm{E}U(c).$$

Ligon and Schechter (2003)

6. Estimating the Distribution of Consumption

7. Poverty

Foster et al. (1984) Ravallion (1994)

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