Efficient intra-household allocations and distribution factors: implications and identification.[1]

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Abstract

This paper provides an exhaustive characterization of testability and identifiability issues in the collective framework in the absence of price variation; it thus provides a theoretical underpinning for a number of empirical works that have been developed recently. We first provide a simple and general test of the Pareto efficiency hypothesis, which is consistent with all possible assumptions on the private or public nature of goods, all possible consumption externalities between household members, and all types of interdependent individual preferences and domestic production technology. The test is proved to be necessary and sufficient. We then provide conditions for the identification of the sharing rule and the Engel curves of individual household members for a variety of different observational schemes.

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1 Introduction.

That a household comprising several adult members with specific preferences does not necessarily behave as a single rational agent should not be an object of debate. We know, at least since Arrow's famous impossibility theorem, that groups do not usually behave as individuals. Yet, for decades most theoretical and applied micro-economic work on household consumption, labor supply, savings or fertility behavior has been based on the assumption that indeed household decisions could be analyzed as stemming from a unique, well behaved utility function; this **is** sometimes known as the *unitary assumption*. As well as being theoretically suspect**,** the unitary model is poorly supported**;** there is now a considerable body of empirical evidence that critical implications of the unitary model are rejected when we test for them. These implications fall into two classes. First we have the Slutsky conditions. For example, a widely cited result on household demand data is that of Browning and Chiappori (1998) who find that Slutsky symmetry is rejected for a Canadian sample of couples but not for samples of single men and single women. This finding suggests that the rejection is due to having the wrong model for couples rather than, say, the choice of functional form, the particular goods modelled or the way heterogeneity is incorporated since these are the same across the three samples. The second set of results that are problematic for a unitary model is the widespread finding that there are variables that affect household decisions even though they do not impact on preferences nor on budgets directly. Such variables are usually known as *distribution factors*. Examples of distribution factors that have been suggested in the literature include relative incomes, relative wages, the 'marriage market' environment and the control of land. The household decisions considered range from expenditures on clothing and children through labour supply and time allocation to outcomes such as child health variables and fertility decisions. A convincing empirical example is the empirical analysis of Lundberg, Pollak and Wales (1997) who consider the effect of a change in the payment of child benefits in the UK in the 1980's. This change effectively shifted the payment from fathers to mothers without a major impact on household income. In a unitary model such a shift in 'who gets what' should not lead to changes in household decisions ('income pooling'). Lundberg *et al* show that, contrary to this, the policy changes caused significant increases in the demands for children and women's clothing as compared to men's clothing.

Recognizing that households might not behave according to the single rational agent model does not mean that there cannot be any restriction on their aggregate consumption or joint labor supply behavior. Rationality may still be present under one form or another at the household level. The problem is precisely to know under what form. The research program in this area thus consists of investigating alternative hypothesis about decision making in the household and testing them against each other on the basis of the restrictions they may imply for the household demand and labor supply functions. If some of these hypotheses appear to hold against empirical evidence, it may be expected that the corresponding restrictions on household demand behavior will permit the identification, at least partially, of the intrahousehold allocation mechanism and then the welfare of individual household members. One widely used hypothesis is that, however household decisions are made, the outcomes are Pareto efficient; this is known as the *collective model*. This was first proposed by Chiappori (1988, 1992) and Apps and Rees (1988) and has subsequently been elaborated by Browning, Bourguignon, Chiappori and Lechene (1994), Browning and Chiappori (1998) and Chiappori and Ekeland (2006). One of the obvious advantages of this approach is its generality. For example, any axiomatic bargaining approach that takes efficiency as an axiom is nested within the collective framework. A second advantage of the collective model is that it is almost as easy to work with as the unitary model (see Browning and Chiappori (1998)).

The question, of course, is whether this approach is not simply too general; that is, does it generate any testable restrictions at all? Surprisingly enough, several contributions have shown that the collective model, even in its most general version, generates strong testable restrictions on observed behavior. Two families of tests have been distinguished, depending on whether we observe price variations in the data. In this paper we shall be exclusively concerned with data environments in which we do not observe price variation.[1] This is typically the case for the standard cross-sectional analysis of consumption patterns, where it is assumed that individuals in the population from which the sample is drawn face identical prices. In this context, testing relies exclusively upon the effect of income and distribution factors. Although simple versions of such tests have been used in various contexts (see Bourguignon *et al* (1993); Browning *et al* (1994) and Thomas *et al* (1997)), no comprehensive theoretical analysis have been provided so far. The first goal of the present paper is to provide such an analysis. We find that there are surprisingly general and powerful tests. First, a simple general test of the Pareto efficiency hypothesis is presented which is consistent with all possible assumptions on the private or public nature of goods, all possible consumption externalities between household members, and all types of interdependent individual preferences and domestic production technology.[2] Moreover, the test is proved to be necessary and sufficient: if it is satisfied, then it is always possible to interpret observed behavior as if it was stemming from a collective framework with well-chosen preferences. Second, a test is provided of some separability properties in the preceding framework which are equivalent to considering private goods and egotistic or 'caring' agents.

If we do not reject the restrictions for a collective model then the second major issue concerns identification: when and to what extent is it possible to recover the underlying structure - preferences and the decision process - from observed behavior? With price variations, an identification result was first derived in the labor supply case by Chiappori (1992), then extended by Blundell *et al* (2000), Chiappori, Fortin and Lacroix (2002) and Chiappori and Ekeland (2008). Without price variations, Browning *et al* (1994) show that it is possible, using a parametric approach, to identify the intrahousehold allocation process and individual Engel curves[3] under the Pareto efficiency hypothesis when the consumption by one household member of at least one good is observed. The second goal of the present paper is to extend these results. Specifically, we provide a series of assumptions under which it is possible to identify the decision process and individual Engel curves. These assumptions allow for different observational regimes. For example, we may not observe anything about allocation within the household or we may observe individual consumptions of a specific good.

In the second section we describe the general structure of the model used to represent consumption decisions in a 2-person household. We also introduce a novel type of demand function which is useful in the subsequent analysis. Section 3 considers testing of the collective model. The main result is that a form of proportionality of the effects of distribution factors is both necessary (a known result) and sufficient for the collective model under a very wide range of circumstances. This establishes that the proportionality property is the full empirical content in an environment with no price variation. Section 4 considers the special case in which some goods are known to

[1] See Chiappori (1992), Browning and Chiappori (1998) and Chiappori and Ekeland (2002a) and (2002b) for testing if we do have price variation.

[2] For a related work, see Dauphin and Fortin (2001)

[3] Without price variation it is clearly impossible to identify preferences, even in a unitary model. The best we can hope for is to identify individual Engel curves.

be private and household members have caring preferences; this case has been used extensively in the literature. The most important concept in this case is the *sharing rule* which assigns total expenditures (rather than goods) to each partner. We establish the empirical implications of this case. Section 5 continues with the caring assumption and considers what happens if we observe the assignment of one or more goods within the household. The most popular candidate for an assignable good is clothing but the assignment of other goods is potentially observable. For example, in household expenditure surveys we usually only observe household expenditures on tobacco, but it would be trivial to also record who is actually smoking. As well as considering additional tests we also provide some results on the identification of the sharing rule and individual Engel curves. In section 5 we dispense with assignability assumptions and show that the surprising result that, subject to some conditions, we can still recover individual Engel curves and the sharing rule. Even though the identification here is more fragile empirically than if we do observe some assignment, the identification is nonparametric in the sense that it exploits particular features of the collective model and does not rely on any functional form assumptions. The final section concludes.

2 The basic framework.

2.1 Private and public goods.

We consider a two adult household in which the two people are denoted A and B. We assume for the moment that there are n consumption goods and that they all are market goods which may be consumed either privately or publicly by the two agents. For example, 'food' is partly private (in the sense of of consumption being rival) and partly public, since some food preparation costs are shared by both partners. We denote the vector of private consumption by household member m $(= A, B)$ as $\mathbf{q}^m \in \mathbb{R}^n_+$ and the vector of public consumption by $\mathbf{Q} \in \mathbb{R}^n_+$. The household consumption vector of private goods $(\mathbf{q}^A + \mathbf{q}^B)$ is denoted by $\mathbf{q}$, and that of total consumption $(\mathbf{q} + \mathbf{Q})$ by $\mathbf{C}$. Since there is no price variation all prices can be normalized to unity so that the budget constraint is:

$$\mathbf{e}'(\mathbf{q}^A + \mathbf{q}^B + \mathbf{Q}) = \mathbf{e}'\mathbf{C} = x$$

where $\mathbf{e}$ is a $n-$vector of ones. Here, x can be considered either as total income or, as in standard cross-sectional analysis of consumption patterns, as total household expenditure.

Each person has preferences represented by $u^A(\mathbf{q}^A, \mathbf{q}^B, \mathbf{Q}; \mathbf{a})$ and $u^B(\mathbf{q}^A, \mathbf{q}^B, \mathbf{Q}; \mathbf{a})$ respectively, where $\mathbf{a}$ is a vector of characteristics that affects preferences directly. We refer to the $\mathbf{a}$ variables as *preference factors*. Thus $\mathbf{a}$ might include the age, race and education of the two agents; regional location; the number and age of children etc.. Examples of variables that would *not* usually be thought of as preference factors are the relative wages of the two partners, their relative physical attractiveness and the local sex ratio. We refer to this preference structure as *altruistic preferences* because the private consumption of each member enters the preferences of the other. Note though that this might simply reflect positive or negative consumption externalities rather than a true altruistic behavior. Also, this general formulation does not exclude the possibility that one person does not care about the other. Finally, we assume that utilities are three times continuously differentiable and strongly convex; as a consequence, demand functions are three times continuously differentiable. This assumption is rather mild, in the sense that it cannot be falsified on any finite data set.[4] In summary, no restriction is placed on preferences, beyond assuming that they can be represented by a 'smooth' utility function for each adult in the household.

Since individual preferences u^A and u^B generally differ we need to specify how households make decisions about what to buy and how to assign the private elements of the good; that is, how they choose $\mathbf{q}^A$, $\mathbf{q}^B$ and $\mathbf{Q}$, given the budget constraint.[5] We now introduce the important concept of a distribution factor:

Definition 1 *A variable z_k is a* distribution factor *if it does not enter individual preferences nor the overall household budget constraint but it does influence the decision process.*

Thus, distribution factors are variables that affect the choices of $(\mathbf{q}^A, \mathbf{q}^B, \mathbf{Q})$ directly and not through preferences or the budget constraint. Theoretical examples include those factors mentioned in the previous paragraph as *not* being preference factors. As discussed in the introduction, several examples of distribution factors can be found in the empirical literature. Distribution factors will play a key role in the following for three reasons. First, the existence of such variables is inconsistent with the traditional, unitary framework - as recognized by the papers cited in the introduction. Secondly, the influence of distribution factors upon behavior provide the *only* testable restrictions for the collective model (if we do not have any price variation); this is the first main theme of this paper. Finally, distribution factors are extremely helpful in recovering some features of the intrahousehold decision process; this is the second major theme of the paper.

Below we denote household demand function for good i by $\xi_i(x, \mathbf{a}, \mathbf{z})$ (where $\mathbf{z}$ is a $K-$vector of distribution factors). We use this notation when we do not want to distinguish between public goods (then $\xi_i \equiv Q_i$), aggregate consumption of a private commodity (then $\xi_i = q_i^A + q_i^B$), or even individual consumption (then $\xi_i = q_i^A$ or $\xi_i = q_i^B$). In particular, the general tests described in the next section are valid whatever the particular interpretation. In all that follows we assume that demand functions are continuously differentiable.

2.2 z-conditional demands.

In considering the restrictions implied by various assumptions below we have found it useful to use a novel type of 'conditional' demand function whereby the demand for one good is expressed as a function of the demand for another good as well as total expenditure and preference and distribution factors. *Conditional demand* functions are often used in demand analysis where we assume a single utility function. In that framework, the demand for one set of goods (the 'goods of interest') are conditioned on the price of these goods, total expenditures on these goods and the quantities of another set of goods (the 'conditioning goods'); see Browning and Meghir (1991) for further discussion.

In the extended rational setting considered here, we define a somewhat different type of conditional demand function that turns out to be useful. Consider the demand for good j, $\mathbf{C}_j = \xi_j(x, \mathbf{a}, \mathbf{z})$ where some of the elements of $\mathbf{z}$ may not be observed but at least one is. For example, $\mathbf{z}$ might include relative wages which are often available in survey data and the attractiveness of the wife relative to the husband, which may impact on choices but is never observed. We make the following assumption:

[4] Specifically, if one observes a finite number of realisations of income and consumption, there exists an infinitely differentiable Engel curve on which these points are located; see Chiappori and Rochet (1987).

[5] We here ignore that the determination of total expenditure (and consequently saving) is itself a household decision and the two partners may have different views on this.

Axiom 1 (Assumption) *There is at least one good j and one observable distribution factor z_k such that $\xi_j(x, \mathbf{a}, \mathbf{z})$ is strictly monotone in z_k.*

The observability is not required for this paragraph but it is for estimation, which is discussed in the next two paragraphs. Given strict monotonicity, and taking the observable factor to be z_1 the demand function for god j can be inverted on this factor:

$$z_1 = \zeta(x, \mathbf{a}, \mathbf{z}_{-1}, \mathbf{C}_j)$$

where $\mathbf{z}_{-1}$ is the vector of distribution factors without the first element. Now substitute this into the demand for good $i \neq j$:

$$\mathbf{C}_i = \xi_i(x, \mathbf{a}, z_1, \mathbf{z}_{-1}) = \xi_i[x, \mathbf{a}, \zeta(x, \mathbf{a}, \mathbf{z}_{-1}, \mathbf{C}_j), \mathbf{z}_{-1}] = \theta_i^j(x, \mathbf{a}, \mathbf{z}_{-1}, \mathbf{C}_j). \tag{1}$$

Thus the demand for good i can be written as a function of total expenditure, preference factors, all distribution factors but the first and the quantity of good j. To distinguish this conditioning from the more conventional conditional demands discussed above, we shall refer to them as *z-conditional* demands. Note that, in the unitary setting, there are no distribution factors, so that $z-$conditional demands are not defined in this case. Various contributions apply the $z-$conditional demand approach developed here to collective models; the reader is referred in particular to Dauphin and Fortin (2001), Dauphin, Fortin and Lacroix (2003), Donni (2006) and Donni and Moreau (2007).

When we come to estimate $z-$conditional demands we have to allow that the quantity variable for good j might be endogenous for the $z-$conditional demand for good i in equation (1). To overcome this, we note that z_1 is excluded from the latter equation and is hence a natural instrument for $\mathbf{C}_j$. We now discuss the econometrics which requires that we allow for unobservable sources of variation in demands. We show first that even if we start with a demand model with additive taste shifters, we end up with an inherently non-additive model. Without loss of generality take the first good as the conditioning good and suppose that demands take the form:

$$\mathbf{C}_i = \xi_i(x, \mathbf{a}, \mathbf{z}) + \varepsilon_i \tag{2}$$

where the ε_i's are unobservable taste shifters. Using the inversion $z_1 = \zeta(x, \mathbf{a}, \mathbf{z}_{-1}, \mathbf{C}_1 - \varepsilon_1)$ and substituting in an equation with $i \neq 1$ we have:

$$\mathbf{C}_i = \xi_i(x, \mathbf{a}, , \zeta(x, \mathbf{a}, \mathbf{z}_{-1}, \mathbf{C}_1 - \varepsilon_1), \mathbf{z}_{-1}) + \varepsilon_i \tag{3}$$

which is non-additive in $(\varepsilon_1, \varepsilon_i)$ unless we make very strong (linearity) assumptions. This non-separability rules out the use of nonparametric techniques for estimation with endogenous regressors and additive errors, such as Newey and Powell (2003).

Given non-additivity, we may as well consider directly the stochastically nonseparable model:

$$\mathbf{C}_i = \xi_i(x, \mathbf{a}, \mathbf{z}, \varepsilon_i) \tag{4}$$

The 'error' term ε_i is a conventional index that captures all of the missing preference and distribution variables, as well as unobserved heterogeneity. Inverting (4) for $i = 1$ gives $z_1 = \zeta(x, \mathbf{a}, \mathbf{z}_{-1}, \mathbf{C}_1, \varepsilon_1)$

and substituting, we have:

$$\begin{aligned}\mathbf{C}_i &= \xi_i(x,\mathbf{a},z_1,\mathbf{z}_{-1},\varepsilon_i) \\ &= \xi_i(x,\mathbf{a},\zeta(x,\mathbf{a},\mathbf{z}_{-1},\mathbf{C}_1,\varepsilon_1),\mathbf{z}_{-1},\varepsilon_i) \\ &= \theta_i(x,\mathbf{a},\mathbf{z}_{-1},\mathbf{C}_1,\varepsilon_1,\varepsilon_i) \end{aligned} \tag{5}$$

The econometric problem that arises in estimating the unrestricted equation of interest, (5), is that one of the regressors, $\mathbf{C}_1$, is not independent of the two 'error' terms, $(\varepsilon_1, \varepsilon_i)$. But, as discussed above, we do have a good potential instrument, z_1. This is correlated with $\mathbf{C}_1$ (through (4)) but can be excluded from the equation of interest, (5). Nonseparable models with endogeneity and a single source of stochastic variation are discussed in Chernuzhukov, Imbens and Newey (2007). Nonseparable models with exogenous regressors and multiple sources of stochastic variation are discussed in Appendix A of Matzkin (2003). To date, we do not have estimators for the general form in (5) which is nonseparable and has multiple sources of stochastic variation. Fortunately, for the case to hand, we have extra structure that we can exploit. Blundell and Powell (2003) discuss a control function approach which is perfectly suited to the current context. In the first stage, we estimate the first equation (4) using whatever nonparametric or semiparametric estimator is thought appropriate.[6] A $2SLS$ approach would then take the prediction from this and plug it into (5) and then estimate ignoring the endogeneity. This leads to inconsistent estimates. In contrast, a control function approach takes estimates of ε_1 from the regression (4) for $i = 1$ and replaces ε_1 in (5) with these estimates. Then the problem of estimating (5) reduces to the model with endogeneity and one source of stochastic variation (ε_i). The price to pay for this identification is that we have to assume that the instrument z_1 is observed and satisfies stronger independence assumptions than the usual mean independence assumption; see Blundell and Powell (2003) for exact statements.

3 Testing for the unitary and collective models.

3.1 The unitary model.

In this section we investigate the restrictions imposed on the demand functions, $\xi_i(x, \mathbf{a}, \mathbf{z})$, and their $z-$conditional counterparts, $\theta_i(x, \mathbf{a}, \mathbf{z}_{-1}, \mathbf{q}_1)$, by the properties that one may be willing to assume about the intrahousehold decision process or its outcome. We shall essentially consider three hypotheses: the 'unitary' model; the 'collective' approach, as characterized by Pareto efficiency of the allocation of goods; and an additional, bargaining-type condition. We begin with the unitary model, in which we assume that a unique utility function is maximized. Formally :

Definition 2 *Let* $(\mathbf{q}^A, \mathbf{q}^B, \mathbf{Q})$ *be given demand functions of* $(x, \mathbf{a}, z)$*. These are compatible with unitary rationality if there exists a utility function* $U(\mathbf{q}^A, \mathbf{q}^B, \mathbf{Q}; \mathbf{a})$ *such that, for every* $(x, \mathbf{a}, \mathbf{z})$*, the vector* $(\mathbf{q}^A, \mathbf{q}^B, \mathbf{Q})$ *maximizes* $U(.)$ *subject to the budget constraint.*

The restrictions implied by this framework are trivial. Indeed, it assumes that the household maximizes a single utility function, that represents the 'household preferences' in some sense. A consequence is that, by definition, the household demand functions should depend on total expenditure x and the preference factors $\mathbf{a}$, but not on the distribution factors, $\mathbf{z}$. Formally :

[6] In practice we would also want to instrument total expenditure, x. The conventional instrument to use is some measure of household income.

Proposition 1 *A given system of demand functions is compatible with unitary rationality if and only if it satisfies:*

$$\frac{\partial \xi_i(x, \mathbf{a}, \mathbf{z})}{\partial z_k} = 0 \qquad \forall i, k \tag{6}$$

This condition is an immediate generalization of the 'income pooling' hypothesis, which has been tested (and rejected) by, for example, Schultz (1990), Thomas (1990), Bourguignon *et al* (1993), Browning *et al* (1994), Fortin and Lacroix (1997), Phipps and Burton (1998), Lundberg *et al* (1997).

An important remark is that a model with individual utility functions and a weighted sum of these as the household utility function *is formally a unitary model so long as the weights do not depend on distribution factors.* This fact has two consequences. First, the key insight of collective models is not that the household does not maximize some common index, but rather that this common index, if it exists, will in general depend *directly* on distribution factors (as well as prices and incomes). It is well known, for instance, that the Nash bargaining solution can be expressed as maximizing the product of individual surpluses. The crucial point, however, is that each agent's surplus (and therefore the index that is maximized by the household) cannot be seen as a 'household utility' in the unitary sense because it involves the agent's status quo point, which typically varies with income and distribution factors.

A second and more surprising implication of this result is the following. Consider a model of collective decision making in which the household maximizes a weighted sum of individual utilities, the weights being functions of household income but *not* of distribution factors. Although this model does *not* belong to the unitary class (since the index maximized by the household is income-dependent), it is *observationally equivalent* to a unitary setting, in the sense that any demand function $\xi_j(x, \mathbf{a}, \mathbf{z})$ it generates could alternatively be generated by a unitary framework. This paradoxical conclusion is due to the specific nature of the problem, and more precisely to the absence of price variations.[7] This stresses the fact that on cross-sectional data without price variations, distribution factors are indispensable to distinguish between the unitary and the collective setting.

3.2 The collective approach.

We now consider the more general framework, in which we explicitly recognize that the household consists of two members with potentially different preferences. Our *only* assumption, at this stage, is that the intrahousehold decision process, whatever its particular features, always leads to a Pareto efficient outcome. This hypothesis characterizes what we call 'collective rationality'. Let us state it formally :

Definition 3 *Let $(\mathbf{q}^A, \mathbf{q}^B, \mathbf{Q})$ be given functions of $(x, \mathbf{a}, \mathbf{z})$. These are compatible with collective rationality if there exists two utility functions $u^A(\mathbf{q}^A, \mathbf{q}^B, \mathbf{Q}; \mathbf{a})$ and $u^B(\mathbf{q}^A, \mathbf{q}^B, \mathbf{Q}; \mathbf{a})$ such that, for every $(x, \mathbf{a}, \mathbf{z})$, the vector $(\mathbf{q}^A, \mathbf{q}^B, \mathbf{Q})$ is Pareto-efficient. That is, for any other bundle $(\tilde{\mathbf{q}}, \tilde{\mathbf{q}}^B, \tilde{\mathbf{Q}})$ such that*

$$u^m(\tilde{\mathbf{q}}^A, \tilde{\mathbf{q}}^B, \tilde{\mathbf{Q}}; \mathbf{a}) \geq \; u^m(\mathbf{q}^A, \mathbf{q}^B, \mathbf{Q}; \mathbf{a})$$

for $m = A, B$ (with at least one strict inequality), then

$$\mathbf{e}'(\tilde{\mathbf{q}}^A + \tilde{\mathbf{q}}^B + \tilde{\mathbf{Q}}) > \mathbf{e}'(\mathbf{q}^A + \mathbf{q}^B + \mathbf{Q})$$

[7] On the contrary, it can readily be checked that when price variations are available, the demand stemming from a model entailing income-dependent weights will not in general satisfy Slutsky symmetry.

This definition is quite general since no assumption whatsoever is made upon the form of individual preferences, the public or private nature of consumption goods or particular features of the intrahousehold decision process (beyond efficiency). Yet, strong restrictions on household demand functions obtain.

Our first important result is stated in the following proposition, which provides a *necessary and sufficient* characterization for collective demands in a cross-sectional context:

Proposition 2 *Consider a point* $P = (x, \mathbf{a}, \mathbf{z})$ *at which* $\frac{\partial \xi_i}{\partial z_1} \neq 0$ *for all* i. *Without a priori restrictions on individual preferences* $u^m(\mathbf{q}^A, \mathbf{q}^B, \mathbf{Q}; \mathbf{a})$, $m = A, B$, *a given system of demand functions is compatible with collective rationality in some open neighborhood of* P *if and only if either* $K = 1$ *or it satisfies any of the following, equivalent conditions :*

i) there exist real valued functions $\Xi_1, \ldots.., \Xi_n$ *and* μ *such that :*

$$\xi_i(x, \mathbf{a}, \mathbf{z}) = \Xi_i[x, \mathbf{a}, \mu(x, \mathbf{a}, \mathbf{z})] \qquad \forall i \tag{7}$$

ii) household demand functions satisfy:

$$\frac{\partial \xi_i / \partial z_k}{\partial \xi_i / \partial z_l} = \frac{\partial \xi_j / \partial z_k}{\partial \xi_j / \partial z_l} \qquad \forall i, j, k, l \tag{8}$$

iii) there exists at least one good j *such that:*

$$\frac{\partial \theta_i^j(x, \mathbf{a}, \mathbf{z}_{-1}, \mathbf{q}_j)}{\partial z_k} = 0 \qquad \forall i \neq j \text{ and } k = 2, .., K \tag{9}$$

When all consumptions are private, condition (8), which is usually known as the *proportionality* condition, has been known to be necessary for quite a long time (see Bourguignon *et al* (1993); Browning *et al* (1994) and Thomas *et al* (1997)). Proposition 2 extends existing results in three directions. First, it shows that the condition is necessary even in the most general case (with public consumption, externalities, etc.). Second, it provides equivalent versions of the conditions; in particular, the $z-$conditional form. Third and most importantly, it shows that these conditions are also sufficient, in the sense that any demand function satisfying them is compatible with collective rationality.

How should Proposition 2 be interpreted? The basic idea is that, by definition, distribution factors do not influence the Pareto set. They may affect consumption, but only through their effect upon the *location* of the final outcome on the Pareto frontier - or, equivalently, upon the *respective weighting* of each member's utility that is implicit in this location. The key point is that this effect is one-dimensional. This explains why restrictions appear only in the case where there is more than one distribution factor. Whatever the number of such factors, they can only influence consumption through a single, real-valued function $\mu(.)$. This is what is expressed by the condition (7).

This simple idea has two important consequences. First, let us compute q_i as a $z-$conditional function of $(x, \mathbf{a}, \mathbf{q}_j, z_{-1})$. Then collective rationality implies that it should not depend on $\mathbf{z}_{-1}$. The reason is that, for given values of x and $\mathbf{a}$, whenever distribution factors $(z_1, \mathbf{z}_{-1})$ contain some information that is relevant for intrahousehold allocation (hence for household behavior), this information, which is one-dimensional (as we have seen above), is *fully* summarized by the value of q_j. Once we condition on q_j, $\mathbf{z}_{-1}$ becomes irrelevant. This is the meaning of condition (9).

A second, very important consequence relates to the question of the number of distribution factors to be taken into account. At the level of generality considered here, Proposition 2 says that at least two distribution factors are needed to *test* the hypothesis of collective rationality. Thus, in full generality, collective rationality imposes *no* restriction on household demand functions in the case where there is only one distribution factor. Now, this does not mean that no other restrictions can possibly be found, but rather that such restrictions require some *additional assumptions* to be made upon the form of individual preferences $u^m(\mathbf{q}^A, \mathbf{q}^B, \mathbf{Q}; \mathbf{a})$, $m = A, B$. In particular, we shall see below that further restrictions appear in the case of a single distribution factor - and come in addition to those in Proposition 2 in the case of more than one distribution factor - when some goods are private and/or consumed exclusively by one member of the household.

Proposition 2 provides two distinct ways of testing for the collective conditions. The first, condition (8), relies on testing for cross-equation restrictions in a system of unconditional demand equations. The other method, based on (9), tests for exclusion restrictions in a $z-$conditional demand framework. Empirically, the latter is likely to be more powerful for at least two reasons. First we can employ single equation methods (including nonparametric methods). Second, single equation exclusion tests are more robust than tests of the equality of parameters across equations. As an illustration, assume that the household has three sources of *exogenous* income, $\left(y^A, y^B, y^H\right)$ with $x = y^A + y^B + y^H$. Then, while x does enter the budget constraint, the two individual income sources, y^A and y^B $\left(\text{or equivalently their relative sizes } y^A/x \text{ and } y^B/x\right)$ do not, and can be taken as distribution factors. Hence Proposition 2 applies. In the present case, the partial derivatives in (8) and (9) may be interpreted as the household 'marginal propensity to consume' a given good with respect to the various components of household income. The unitary model would require that these propensities be equal for all goods. Through condition (8), collective rationality requires that these marginal propensities to consume must be proportional *across all goods*, whereas condition (9) requires them to be zero, conditionally on the demand for another good.

Note that proposition 2 generalizes easily to the Beckerian framework in which domestic goods produced by the household are taken into account. Adding a domestic production function to go from the market inputs to the goods actually consumed by household members and taking into account the allocation of domestic labor gives the restrictions on household demands for market goods derived above.

3.3 Bargaining.

Many papers that have analyzed intrahousehold decision processes have assumed a bargaining framework (see, fore example, Manser and Brown (1980) and McElroy and Horney (1981)). If we take an axiomatic approach and include efficiency as one of our axioms then necessarily the bargained outcome will satisfy the conditions in Proposition 2. Of course, the bargaining framework should be expected to impose *additional* restrictions. Chiappori and Donni (2006) show however that such additional restrictions exist only insofar as specific assumptions are made on the bargaining process, and specifically on the nature of the status quo point. Indeed, any efficient outcome can be constructed as a bargaining solution for well-chosen status quo values. An easy example of a specific assumption of this kind is the following. Assume that some distribution factors are known to be positively correlated with member B's (resp. A's) threat point. Then, in program (P), μ should be increasing (resp. decreasing) in that distribution factor. taking derivatives through (7) with respect to z_1 and z_2 gives the following proposition.

Proposition 3 *Assume that μ is known to be increasing in z_1 and decreasing in z_2. Then the demand functions consistent with any bargaining model are such that:*

$$\frac{\partial \xi_i/\partial z_1}{\partial \xi_i/\partial z_2} = \frac{\partial \xi_j/\partial z_1}{\partial \xi_j/\partial z_2} \leq 0 \qquad \forall i = 1,..,n; j = 1,..,n$$

Thus if we assume a priori that two distribution factors have these properties then we have a further testable restriction. The obvious factors to take are the incomes of the two partners. Indeed, if we are willing to go further and assume that it is only the relative value of these incomes, z_1/z_2 that matters then we have in addition:

$$\frac{\partial \xi_i}{\partial \ln(z_1)} + \frac{\partial \xi_i}{\partial \ln(z_2)} = 0 \qquad \forall i = 1,..,n$$

This is simple to test and easy to interpret. As an illustration, Browning *et al* (1994) test the above restrictions on Canadian data, and find they are not rejected.

3.4 Examples.

To round off this section we present two parametric examples. To simplify the exposition we shall assume that there are no preference factors **a** and that there are exactly two distribution factors, z_1 and z_2. We first model the unrestricted household demands as a quadratic in (x, z_1, z_2):

$$\xi_i = a_i + b_i x + c_i x^2 + d_i z_1 + e_i z_2 + f_i z_1^2 + g_i z_2^2 + h_i x z_1 + k_i x z_2 + l_i z_1 z_2 \tag{10}$$

The restrictions implied by the unitary model are simply $d_i = e_i = ... = l_i = 0$. The restrictions implied by collective rationality (condition (8)) are a little more difficult to determine. We can show that the ξ_i must be of one of the following two forms:

$$\xi_i = a_i + b_i x + c_i x^2 + \lambda_i (d.z_1 + e.z_2 + f.z_1^2 + g.z_2^2 + h.x.z_1 + k.x.z_2 + l.z_1.z_2) \tag{11}$$

or:

$$\xi_i = a_i + b_i x + c_i x^2 + \lambda_i (z_1 + \alpha z_2) + \mu_i.(z_1 + \alpha z_2)^2 + \omega_i.x.(z_1 + \alpha z_2) \tag{12}$$

Thus, either all the terms involving the distribution factors z_1 and z_2 must be proportional across all demand functions, or all the demand functions must be quadratic in the same linear function $(z_1 + \alpha z_2)$ of these factors. It is also easily shown that the $z-$conditional demands consistent with (12) have the following expression under collective rationality:

$$\theta_i = \alpha_i + \beta_i x + \gamma_i x^2 + \delta_i q_1 + (\phi_i + \psi_i x).\sqrt{1 + \beta x + \gamma x^2 + \delta q_1} \tag{13}$$

(where conditioning is made on q_1). If (11) holds we have in addition that $\phi_i = \psi_i = 0$. Note that in the absence of theoretical restrictions, $z-$conditional demands derived from the quadratic demand functions (10) would also involve terms in $z_2, z_2^2, x.z_2$ and $q_1.z_2$ both in the first part of the RHS of (13) and under the square root sign.

As a second example, consider the case where the household demand function take the following extended Working-Leser form:

$$\xi_i = a_i + b_i x + c_i x \ln x + d_i \ln z_1 + e_i \ln z_2 \tag{14}$$

The associated $z-$conditional demand functions, conditioning on q_1, are given by:

$$\theta_i = \alpha_i + \beta_i x + \gamma_i x \ln x + \delta_i \ln z_2 + \eta_i q_1 \tag{15}$$

It is then easily shown that collective rationality implies that d_i/e_i be the same for all i=1,..,n, or, equivalently, that $\delta_i = 0$ for $i = 2, ..n$.

4 Private goods and caring agents.

4.1 The sharing rule.

In the previous section we did not impose any restrictions on preferences (beyond assuming them representable by a utility function) or on the public or private nature of the goods which are consumed. In this section we concentrate on the allocation of private goods across the members of the household. To do so, we impose the following restriction on individual preferences:

$$u^m(\mathbf{q}^A, \mathbf{q}^B, \mathbf{Q}; \mathbf{a}) = \psi^m[\phi^A(\mathbf{q}^A, \mathbf{Q}; \mathbf{a}), \phi^B(\mathbf{q}^B, \mathbf{Q}; \mathbf{a}), \mathbf{a}] \quad m = A, B \tag{16}$$

Here A and B have 'egotistic' preferences represented by the felicity functions ϕ^A and ϕ^B respectively, defined over their own consumptions of private and public goods. Both felicity functions enter person m's over-all utility function ψ^m. Following Becker (1991) we refer to 16 as *caring.* In comparison with the general formulation in the preceding sections, we see that this hypothesis is equivalent to a form of separability in the preferences of the two household members. Of course, caring utility functions include the special case of *egotistic preferences* for which $\psi^m\left(\phi^A, \phi^B\right) = \phi^m$.

The caring representation embodies two important sets of restrictions. First, there are no externalities for individual felicities. For many goods this could be questioned. For example, if one person smokes then the other is affected. As another example, if the good is clothing then it may well be that people care about whether their spouse dresses well. Second, the altruism that partners may feel for each other is restricted to work only through their felicity function. That is, one spouse cares only about the other's felicity and not how it is attained; that is, they defer to the other in their choice of goods. If there were an element of non-deference ('you should stop smoking and spend the money saved on exercising') then this would not be captured by this structure. In the following analysis we shall derive the implications of the caring assumption.

We concentrate here on private goods and we ignore the decision concerning public goods $\mathbf{Q}$. One way to proceed would be to condition everywhere on public goods. For the sake of simplicity, we prefer to assume the following separability property between private goods and public goods in individual preferences:

$$\phi^m(\mathbf{q}^m, \mathbf{Q}; \mathbf{a}) = f^m[v^m(\mathbf{q}^m; \mathbf{a}), \mathbf{Q}; \mathbf{a}) \quad m = A, B \tag{17}$$

Also, from now on, x denotes total expenditure on private goods: $x = \mathbf{e}'(\mathbf{q}^A + \mathbf{q}^B)$. It must be stressed that all the preceding assumptions are only useful in empirical work if it is possible to distinguish *a priori* public and private goods. In that case, the consumption vectors $\mathbf{q}^A$ and $\mathbf{q}^B$, on one hand, and the vector $\mathbf{Q}$, on the other are defined on disjoint sets of goods. Such a requirement was not necessary in the preceding section.

We restrict our attention in this section to the case of a *single* distribution factor z. There is no loss of generality in doing so, since we have seen in Proposition 2 that collective rationality implies that various distribution factors affect the intrahousehold allocation of goods through the one-dimensional factor μ. If demand functions satisfy conditions (7)-(9), the effects of all distribution factors may be summarized into those of a single one. In this case, Proposition 2 has shown that collective rationality was not imposing any restriction to demand functions. Our objective in this section is precisely to show that this is not the case when one restricts individual preferences through assumptions 16 and 17 to the case of private goods and caring agents. Before doing so we introduce the fundamental notion of a *sharing rule*:

Proposition 4 ***(Existence of a sharing rule)*** *Let* $(\mathbf{q}^A, \mathbf{q}^B)$ *be functions of* $(x, \mathbf{a}, z)$ *compatible with collective rationality. Assume, in addition, that the corresponding individual utilities satisfy assumptions 16 and 17 above. Then there exists a function* $\rho(x, \mathbf{a}, z)$ *such that* $\mathbf{q}^m$ *is a solution to:*

$$\max \upsilon^m(\mathbf{q}^m; \mathbf{a}) \text{ subject to } \mathbf{e}'\mathbf{q}^m = x^m$$

with $m = A, B,\ x^A = \rho(x, \mathbf{a}, z)$ *and* $x^B = x - \rho(x, \mathbf{a}, z)$.

This proposition is a particular case of the general equivalence between a Pareto optimum and a decentralized equilibrium if there are no externalities or public goods. It thus requires no formal proof. The function $\rho(x, \mathbf{a}, z)$, which denotes the part of total expenditure on private goods that person A receives is the 'sharing rule'. It describes the rule of budget sharing that the two agents implicitly apply among themselves when choosing a particular Pareto efficient allocation. Of course, we are not assuming that households of caring agents explicitly use such a sharing rule. Proposition 4 simply states that the outcome of the household allocation process can be characterized in this way.

4.2 Collective rationality, private goods and caring: a first characterization.

In section 2 we showed that all demand functions (for public or private goods) were consistent with collective rationality if there was only one distribution factor. In this section we are restrict attention to the case of caring and separable preferences. The natural question arises of whether there are additional restrictions stemming from these hypotheses which would permit us to test collective rationality, in the case where observed demands depend on only one distribution factor or which would come in addition of those included in Proposition 2 in the case of two or more distribution factors.

The answer is positive. There are additional restrictions that must be satisfied by joint demand functions in the case of private goods and collectively rational caring agents. These can be expressed at different levels of generality. At a basic level, the restriction is equivalent to taking explicitly into account the sharing rule either in direct, or in $z-$conditional demands. At a higher level of generality, we shall then see that it is in fact possible to recover the sharing rule between caring agents from the observation of their joint demand for private goods, provided these demands satisfy some restrictions. In turn these restrictions provide a general test of the joint hypothesis of collective rationality, private goods and caring agents.

The basic restrictions that must be satisfied by joint demand functions is expressed in the following Lemma (preference factors $\mathbf{a}$ are dropped for convenience).

Proposition 5 *Assume collective rationality, 16 and 17. Then :*

i) direct demands must satisfy the following: there exists a real-valued function ρ and $2n$ real-valued functions α_i and β_i such that:

$$q_i(z,x) = \alpha_i[\rho(z,x)] + \beta_i[x - \rho(z,x)] \;\; for \; i = 1, ...n \tag{18}$$

ii) $z-$conditional demands must satisfy the following : there exist two real-valued functions F and G such that:

$$\theta_i[s+t, F(t) + G(s)] = \theta_i \; [t, F(t) + G(0)] + \theta_i[s, F(0) + G(s)] - \theta_i[0, F(0) + G(0)] \tag{19}$$

for all t, s in $\mathbb{R}_+$ and for $i = 2, ...n$.

In (18), α_i and β_i are A and B's respective Engel curves for good i. Condition (18) is restrictive because it must be fulfilled across goods for the same function ρ. In (19), t and s represent the total expenditures of A and B respectively, that is ρ and $(x - \rho)$, and F and G are the demands for the conditioning good (here taken to be good 1) by A and B respectively.[8] Again, the testable restriction in (19) is that the functions F and G must be the same across all goods but the conditioning one. Note that this condition does not put any restriction on the individual demands for the conditioning good. An equivalent but more direct set of restrictions will be given in the next subsection.

Although the conditions given in (18) and (19) may appear somewhat involved, they are not too difficult to work with for particular functional forms for θ_i. As an illustration, we may consider again the case of Working-Leser demand equation (14) above:

$$\begin{aligned}
\xi_i &= a_i + b_i x + c_i x \ln x + d_i \ln z_1 + e_i \ln z_2 \\
\theta_i &= \alpha_i + \beta_i x + \gamma_i x \ln x + \delta_i \ln z_2 + \eta_i q_1
\end{aligned}$$

As we have seen, collective rationality imposes $\delta_i = 0$, which is equivalent to the d_i's and e_i's being proportional across goods. Now, let us consider (19). We have that

$$\begin{aligned}
\theta_i &= \alpha_i + \beta_i.(t+s) + \gamma_i.(t+s).\ln(t+s) + \eta_i.[F(t) + G(s)] \\
&= \alpha_i + \beta_i.t + \gamma_i.t.\ln t + \eta_i[F(t) + G(0)] + \alpha_i + \beta_i.s \\
&\quad + \gamma_i.s.\ln s + \eta_i.[F(0) + G(s)] - \alpha_i - \eta_i.[F(0) + G(0)]
\end{aligned}$$

which gives :

$$\gamma_i.(t+s).\ln(t+s) = \gamma_i.t.\ln(t) + \gamma_i.s.\ln(s)$$

This imposes that $\gamma_i = 0$ for $i > 1$, so that the three sets of coefficients c_i, d_i and e_i must now be proportional. Then direct demands become :

$$\xi_i = \mathbf{a}_i + b_i.x + r_i.\pi, \qquad \text{where} \;\; \pi = x.\ln x + d.\ln z_1 + e.\ln z_2$$

[8] With a single distribution factor and no preference factors there are only two arguments in the z-conditional demands.

5 Exclusive and assignable goods.

5.1 One exclusive good.

The analysis of the previous section assumes that we only observe household demands for private goods and not their allocation between partners. We now enrich the data environment by assuming that some assignability is observable. We start with two particular cases where some information is available about *individual* consumption of household members. This provides new tests and alternative ways of recovering the sharing rule. While in principle rather specific (and somewhat tedious to go through), these cases are empirically very important; most existing empirical analyses rely on assumptions of this type.

We may, in some cases, observe how much of a particular good each person consumes; this good is then said to be 'assignable'. For instance, we may observe independently male and female clothing expenditures or individual food consumptions. Alternatively, some goods may be consumed by one person only. This is the 'exclusive' case. One example would be information on the smoking or drinking patterns of one household member, provided that the same commodity is not consumed by the spouse - an idea reminiscent of Rothbarth's 'adult goods' assumption. Note that, in the present framework with no price variation, an assignable good can always be thought of as a pair of exclusive goods, one being consumed by A and the other by B.[9]

Before considering successively these two cases, we may stress their common feature. Whenever one good is known to be exclusively consumed by one member (member A, say) this provides some information on the sharing rule, as described in the following proposition.

Proposition 6 *(One exclusive good) Assume collective rationality and 16 and 17. If the consumption of exactly one exclusive good (consumed by member A) is observed, and if the demand function of member A for this good is strictly monotone, then we can recover the sharing rule $\rho(z,x)$ up to a strictly monotone transformation. That is, if $\rho(z,x)$ is one solution, then any solution is of the form $F[\rho(z,x)]$, where F is strictly monotone.*

Note that if the implicit individual demand function is not (globally) monotone, then the result holds on any subset of income and distribution factors over which the demand function is monotone. In particular, the result holds locally almost everywhere.

The next step, of course, is to identify the transformation $F(.)$. This is what is done in the remainder of this section . Notice, however, that, except in the case where all goods are assignable, and therefore the total (private) consumption of both members can be observed, the sharing rule can only be identified up to an additive constant. In all the other cases, we can only observe how the sharing rule changes with total expenditure, x, and the distribution factor, z, but not total individual expenditures (see Chiappori (1992) for a precise statement).

The preceding proposition suggests that it is more convenient to use $z-$conditional rather than direct demand functions wherever a good may be safely assumed to be exclusive. Indeed, conditioning on that good is equivalent to considering combinations of z and x such that the sharing rule is constant and should permit us to easily identify the individual Engel curves for non-exclusive or non-assignable goods. This explains why many of the following propositions are expressed in terms of $z-$conditional demands.

[9] If we have prices, then the pair of 'exclusive' goods asscoaited with an assignable good always have the same price.

5.2 One assignable good.

We begin with the simplest case in which we observe both members' respective consumptions of an assignable good (or, equivalently, of an exclusive good for member A *and* an exclusive good for member B). Then the following restrictions on the two observed demand functions must hold.

Proposition 7 *(One assignable good) Assume collective rationality, 16 and 17. Assume in addition that good 1 is an exclusive good consumed by member A, and that good 2 is an exclusive good consumed by member B. Consider an open set on which the demand for good 2, $z-$conditional on that for good 1 is such that: $\frac{\partial\theta_2}{\partial x} \neq 0$ and $\frac{\partial\theta_2}{\partial q_1} \neq 0$. Then the following, equivalent properties hold:*

i) there exists a function $F(t)$ satisfying:

$$\theta_2[t+s, F(t)] = \theta_2[s, F(0)] \tag{20}$$

for all non-negative s and t

ii) there exists two functions $\beta(.)$ and $g(.)$ such that:

$$\theta_2(x, q_1) = \beta[x - g(q_1)] \tag{21a}$$

iii) θ_2 satisfies:

$$\frac{\partial}{\partial x}\left[\frac{\partial\theta_2/\partial q_1}{\partial\theta_2/\partial x}\right] = 0 \tag{22}$$

This proposition provides a way of testing collective rationality, private goods and caring in the case where the consumption of an exclusive good is observed for each household member. The test is presented in terms of $z-$conditional demand and is most easily implemented for condition (22). For example, if good 1 is women's clothing and good 2 is men's clothing then we work with the demands for men's clothing conditional on total expenditures and women's clothing. If condition (22) is rejected this can be seen as a rejection of the caring assumption or the assumption that clothing is a private good (recall that we cannot reject the collective model with a single distribution factor).

The transposition to direct demand can be made by the change of variables $(x, q_1) \to (x, z)$ based on the observation of the direct demand function for good 1, $q_1(x, z)$. Likewise, the sharing rule is easily recovered through that same change of variable. The function $g(q_1)$ in (21a) is the amount of private expenditures going to member A. This function is obtained, up to an additive constant, by integrating the following differential equation, which is derived from differentiating (21a):

$$g'(q_1) = -\frac{\partial\theta_2/\partial q_1}{\partial\theta_2/\partial x} \tag{23}$$

Then replacing q_1 by its direct demand expression $q_1(x, z)$ yields the sharing rule:

$$\rho(x, z) = g[q_1(x, z)]$$

It is also possible to use direct demand functions throughout, as shown in the following.

Proposition 8 ***(Recovering the sharing rule with one assignable good).*** *Assume collective rationality, 16 and 17, and that q_1 and q_2 are consumed exclusively respectively by members A and B. Assume that the direct demand for both goods (as functions of x and z) are observed and that* *the corresponding conditional demand for good 2, $\theta_2(x, q_1)$ fulfills the conditions of proposition 7. Then, the sharing rule is given, up to an additive constant, by the following equivalent differential equations:*

i)

$$\begin{aligned} g'(q_1) &= -\frac{\partial\theta_2/\partial q_1}{\partial\theta_2/\partial x} \\ \rho(x,z) &= g[q_1(x,z)] \end{aligned} \tag{24}$$

ii)

$$\begin{aligned} \frac{\partial\rho}{\partial x} &= \frac{\frac{\partial q_1/\partial x}{\partial q_1/\partial z}}{\frac{\partial q_1/\partial x}{\partial q_1/\partial z} - \frac{\partial q_2/\partial x}{\partial q_2/\partial z}} \\ \frac{\partial\rho}{\partial z} &= \frac{1}{\frac{\partial q_1/\partial x}{\partial q_1/\partial z} - \frac{\partial q_2/\partial x}{\partial q_2/\partial z}} \end{aligned} \tag{25}$$

Several remarks are in order. First, it is possible in the present case to recover not only the sharing rule, but also the Engel curves for each individual, up to an additive constant. Note that this identification result still holds when, say, the preferences are identical for the two household members, or when they are linear. With an assignable good, it is therefore possible to identify the sharing rule, and the Engel curves, up to a constant with no restriction at all on preferences; as we will see later, this is not possible in the general case. Secondly, the identification of the sharing rule and individual Engel curves can be performed using only the observed marginal propensities to consume out of the total budget and out of the distribution factor. In other words, identification requires to use only the first derivatives of the observed demand functions and does not rely upon nonlinearities. This is important, since identification based upon nonlinearities is generally less robust. Finally, since condition (25) allows us to compute the two partials of ρ independently, the cross derivative conditions $\frac{\partial}{\partial x}\left(\frac{\partial\rho}{\partial z}\right) = \frac{\partial}{\partial z}\left(\frac{\partial\rho}{\partial x}\right)$ generate cross equation restrictions on the demands q_1 and q_2. One can readily check that these are equivalent to the conditions of proposition 7.

5.3 One exclusive good and one private good.

A less demanding assumption is that one good only is known to be exclusive. This may be particularly adequate whenever the private nature of some consumption is debatable. For instance, Browning *et al.* (1994) assume that female clothing is indeed an exclusive consumption, whereas they allow for a public good component in male clothing. We thus consider a situation in which the (individual) consumption of an exclusive good and the aggregate consumption of a private non-assignable good are observed.

As before, the restrictions implied by collective rationality turn out to be easier to express (and to test) in terms of $z-$conditional demands. Specifically, the demand for good 2 conditional on good 1 are summarized in the following.

Proposition 9 *(One private and one exclusive good) Assume collective rationality, 16 and 17. Assume in addition that good 1 is an exclusive good consumed by member A, and that good 2 is a private joint consumption good. Consider an open set on which the $z-$conditional demand $\theta_2(x, q_1)$ is such that $\partial^2\theta_2/\partial x^2 \neq 0$ and $\partial^2\theta_2/\partial x\partial q \neq 0$. Then the following, equivalent properties hold :*

i) there exists a function F(t) satisfying:

$$\theta_2[t+s, F(t)] = \theta_2[t, F(t)] + \theta_2[s, F(0)] - \theta_2[0, F(0)] \tag{26}$$

for all positive s and t.

ii) equivalently, there exist three functions α, β and g such that:

$$\theta_2(x, q_1) = \alpha[g(q_1)] + \beta[x - g(q_1)] \tag{27a}$$

iii) equivalently, θ_2 is such that

$$\frac{\partial}{\partial x}\left[\frac{\partial^2\theta_2/\partial x\partial q}{\partial^2\theta_2/\partial x^2}\right] = 0 \tag{28}$$

The basic difference between the present case and that of an assignable good is essentially that both the identification of the sharing rule and the test for collective rationality, private goods and caring agents now rely on second (rather than first) derivatives of the observed demand functions. They may thus be less robust. For the same reason identification now requires demand functions to be non-linear.

One could also consider other cases where more than a private good, or more than one or two exclusive goods would be observed. As in the general case, these additional observations do not give more information on the sharing rule, but they provide further tests of the joint hypothesis of collective rationality, private goods (and, possibly, exclusiveness of the goods assumed to be so).

5.4 Examples.

To illustrate the preceding properties consider the case where good 1 is exclusive and the observed demand for it is linear in x and z, and where the observed demand for good 2 is quadratic.

$$\begin{aligned} q_1 &= a_0 + a_1 x + a_2 z \\ q_2 &= \alpha_0 + \alpha_1 x + \alpha_2 x^2 + \beta_1 z + \beta_2 z^2 + \gamma x z \end{aligned}$$

If only the demand for good 1 is observed then the sharing rule is of the type:

$$\rho(x, z) = F(a_1 x + a_2 z)$$

and identification can only be obtained through an additional arbitrary restriction. If both goods 1 and 2 are observed, then it is possible to derive the $z-$conditional demand for good 2. It is also quadratic in x and q_1:

$$\theta_2 = A_0 + A_1 x + A_2 x^2 + B_1 q_1 + B_2 {q_1}^2 + C x q_1$$

If good 2 is exclusive to member B then condition (22) implies that $B_2 = C = 0$ and Proposition 8 yields:

$$g'(q_1) = -[B_1 + 2B_2 q_1]/A_1$$

and, after integration:

$$g(q_1) = k - [B_1 q_1 + B_2 {q_1}^2]/A_1$$

where k is some constant. The corresponding sharing rule thus is:

$$\rho(x,z) = k - [B_1(a_1 x + a_2 z) + B_2(a_1 x + a_2 z)^2]/A_1$$

If good 2 corresponds to the joint consumption of both members, then Proposition 9 applies. Condition (28) does not impose any restriction because the $z-$conditional demand is quadratic on x and q_1. The sharing rule is given by:

$$g'(q_1) = -C/(2A_2); \rho(x,z) = k - [C/(2A_2)](a_1 x + a_2 z)$$

It is thus linear in x and z. Indeed, this is a particular case of the example analyzed in section 3 of a linear sharing rule consistent with two private goods and quadratic demand functions.

6 Estimation and test from joint demands : the general case.

6.1 The general argument.

The previous results suggest that whenever information is available about individual consumptions, then it is in general possible to recover the sharing rule and individual Engel curves (up to additive constants). We now show a much more surprising result - namely that, generically on preferences, identification obtains *even without information on the assignment of private goods*. Let us start with a single consumption good. According to (18), collective rationality implies that aggregate demand by the two household members is of the form:

$$q_i(z,x) = \alpha_i[\rho(z,x)] + \beta_i[x - \rho(z,x)]$$

This leads to the following partial derivatives:

$$\begin{aligned} \frac{\partial q_i}{\partial z} &= (\alpha_i' - \beta_i')\frac{\partial \rho}{\partial z} \\ \frac{\partial q_i}{\partial x} &= (\alpha_i' - \beta_i')\frac{\partial \rho}{\partial x} + \beta_i' \end{aligned} \tag{29}$$

where it is assumed that q_i does indeed depend on z. Then from (29), we can compute α_i' and β_i':

$$\begin{aligned} \alpha_i' &= \frac{\rho_z . q_{i,x} + (1-\rho_x) . q_{i,z}}{\rho_z} \\ \beta_i' &= \frac{\rho_z . q_{i,x} - \rho_x . q_{i,z}}{\rho_z} \end{aligned} \tag{30}$$

where $q_{i,x}$ is the derivative of q_i with respect to x (and similarly for $q_{i,z}$).

But α_i (resp. β_i) must be a function of $\rho(z,x)$ (resp. $x - \rho(z,x)$). Along the locus holding $\rho(z,x)$ constant, the derivative of α_i' must be equal to zero. This leads to the following partial differential equation in $\rho(z,x)$:

$$\begin{aligned} & \frac{1}{q_{i,z}}.[q_{i,xx}\rho_z + q_{i,xz}(1-2\rho_x) - q_{i,zz}\frac{\rho_x(1-\rho_x)}{\rho_z}] \\ = \quad & \frac{1}{\rho_z}.[\rho_{xx}\rho_z + \rho_{xz}(1-2\rho_x) - \rho_{zz}\frac{\rho_x(1-\rho_x)}{\rho_z}] \end{aligned} \tag{31}$$

This is a first information on the sharing rule $\rho(z,x)$. If one observes the aggregate demand function of the household for a given good, $q_i(z,x)$, then the sharing rule must satisfy the partial differential equation (31). Equivalently a test of collective rationality for an observed aggregate demand function $q_i(z,x)$ is that there exists a function $\rho(z,x)$ such that (31) hold. However, this equation is rather complex and does not say much on the way the sharing rule depends on the observed demand behavior for good i.

More can be obtained when the aggregate demand for two goods, rather than a single one, is observed. Without loss of generality, assume these are goods 1 and 2. Then (31) must be satisfied for $i=1$ and 2. Equalizing the left hand-side of (31) for $i=1,2$ then yields:

$$\mathbf{Q}_{xx}^{12} + \mathbf{Q}_{xz}^{12}\frac{1-2\rho_x}{\rho_z} - \mathbf{Q}_{zz}^{12}\frac{\rho_x(1-\rho_x)}{\rho_z^2} = 0 \tag{32}$$

with:

$$\mathbf{Q}_{at}^{ij} = \frac{q_{iat}}{q_{iz}} - \frac{q_{jat}}{q_{jz}}$$

In other words, when two demands are observed, the sharing rule must satisfy two second order PDE's; moreover, these can be combined into a first order PDE, so that the sharing rule must equivalently satisfy one first-order and one second-order PDE.

A result by Chiappori and Ekeland (2005) guarantees that, generically, (31) and (32) identify ρ, in our case up to a constant and a permutation of members. Intuitively, an equation such as (32) defines ρ up to some boundary condition (say, to some function $f(z) = \rho(z,\bar{x})$ for some given $\bar{x}$). Again in general, the equation (31) will be sufficient to pin down the function f. We provide below an illustration in the case of quadratic demands. Note, however, that identification is only generic. There may exist specific forms for which identification does not obtain, but they are 'non robust'. A counter example is provided below. Finally, overidentifying restrictions are usually generated.

That identification should obtain up to a permutation of members is no surprise: from the observation of aggregate demand, it may be possible to say that one individual in the household is getting $\rho(z,x)$ and has associated Engel curves $\alpha_1, \alpha_2, ...$, but certainly not whether that individual is A or B. Formally, one can readily check that whenever some function $\rho(x,z)$ satisfies equations (31) and (32) above, then the function $\bar{\rho}(x,z) = x - \rho(x,z)$ is also a solution. In order to pin down who is who in the absence of assignable or exclusive commodities, a bargaining argument may be used. If the distribution factor is known to favor member A, then ρ represents member A's allocation (instead of member B's) if and only if ρ is increasing in z.

Also, it is clear that recovering the sharing rule, up to a constant, implies at the same time recovering the individual Engel curves. Indeed, equations (30) give the individual marginal propensity to consume each good i as a function of z and x. Integrating these equations yield the individual curves up to a constant, and of course up to a permutation of the two individuals.

6.2 The linear case as a non generic exception.

Identification is only 'generic', in the sense that it relies on the nonlinearities of the demand functions. Estimation and tests might then lack robustness. More precisely, the following proposition shows that the identification of the sharing rule and the test of collective rationality is not possible in the case of linear or 'quasi-linear' demand functions.

Proposition 10 ***(Linear and quasi-linear demand systems)*** *Assume collective rationality, 16 and 17. The following two properties are equivalent :*

i) Direct demands are of the form

$$q_i = a_i + b_i x + c_i A(z, x) \tag{33}$$

ii) Conditional demands are linear :

$$\theta_i \ = \alpha_i + \beta_i x + \eta_i q_1 \tag{34}$$

Moreover, if these conditions are fulfilled, any function of the form $f[A(y, m)]$, and any function of the form $f[m - A(y, m)]$, where f is an arbitrary monotonic transformation, is a possible sharing rule.

6.3 A quadratic example.

We now illustrate the previous results on a specific example.[10] We consider the case where the demand functions may be assumed to be quadratic:

$$q_i = a_i + b_i x + c_i x^2 + d_i z + e_i z^2 + f_i x.z \quad i = 1, 2$$

Then equation (31) can be written as:

$$\begin{aligned} & \frac{1}{d_i + ie_i z + f_i x}.[2c_i\rho_z + f_i(1 - 2\rho_x) - 2e_i \frac{\rho_x(1-\rho_x)}{\rho_z}] \\ = \ & \frac{1}{\rho_z}.[\rho_{xx}\rho_z + \rho_{xz}(1 - 2\rho_x) - \rho_{zz}\frac{\rho_x(1-\rho_x)}{\rho_z}] \end{aligned} \tag{35}$$

while (32) becomes

$$\begin{aligned} 0 \ = \ & \frac{2c_1}{d_1 + 2e_1 z + f_1 x} - \frac{2c_2}{d_2 + 2e_2 z + f_2 x} \\ & + \left(\frac{f_1}{d_1 + 2e_1 z + f_1 x} - \frac{f_2}{d_2 + 2e_2 z + f_2 x} \right) \frac{1 - 2\rho_x}{\rho_z} \\ & - \left(\frac{2e_1}{d_1 + 2e_1 z + f_1 x} - \frac{2e_2}{d_2 + 2e_2 z + f_2 x} \right) \frac{\rho_x(1-\rho_x)}{\rho_z^2} \end{aligned} \tag{36}$$

As it turns out, the solution to equations (35) and (36) is linear. Indeed, if ρ_x and ρ_z are constant, so are

$$U = \frac{1 - 2\rho_x}{\rho_z} \quad \text{and} \quad V = \frac{\rho_x(1-\rho_x)}{\rho_z^2} \tag{37}$$

[10] We are grateful to an anonymous referee for suggesting this example to us.

so that (32) can be written as:

$$
\begin{aligned}
0 \quad = \quad & \frac{(2c_1 f_2 - 2c_2 f_1 - 2Ve_1 f_2 + 2Ve_2 f_1)}{(d_1 + 2e_1 z + f_1 x)(d_2 + 2e_2 z + f_2 x)} x \\
& + \frac{(4c_1 e_2 - 4c_2 e_1 + 2Uf_1 e_2 - 2Uf_2 e_1)}{(d_1 + 2e_1 z + f_1 x)(d_2 + 2e_2 z + f_2 x)} z \\
& + \frac{2c_1 d_2 + 2Ve_2 d_1 - 2c_2 d_1 + Uf_1 d_2 - Uf_2 d_1 - 2Ve_1 d_2}{(d_1 + 2e_1 z + f_1 x)(d_2 + 2e_2 z + f_2 x)}
\end{aligned}
$$

which is satisfied as soon as:

$$
\begin{aligned}
2c_1 f_2 - 2c_2 f_1 - 2Ve_1 f_2 + 2Ve_2 f_1 &= 0 \\
4c_1 e_2 - 4c_2 e_1 + 2Uf_1 e_2 - 2Uf_2 e_1 &= 0 \\
2c_1 d_2 + 2Ve_2 d_1 - 2c_2 d_1 + Uf_1 d_2 - Uf_2 d_1 - 2Ve_1 d_2 &= 0
\end{aligned}
$$

The first two equations give

$$
U = 2\frac{c_1 e_2 - c_2 e_1}{e_1 f_2 - e_2 f_1} \quad \text{and} \quad V = \frac{-c_1 f_2 + c_2 f_1}{-e_1 f_2 + e_2 f_1}
$$

One can readily check that the third equation is always satisfied for these values, which shows that the solution is indeed linear.

Knowing U and V, (37) enable us to recover the partials ρ_x and ρ_z. If z is favorable to A ($\rho_z > 0$) then the solution is:

$$
\begin{aligned}
\rho_x &= \frac{1}{2} - \frac{1}{2}\frac{U}{\sqrt{4V + U^2}}, \rho_z = \frac{1}{\sqrt{4V + U^2}} \quad \Rightarrow \\
\rho &= \left(\frac{1}{2} - \frac{1}{2}\frac{U}{\sqrt{4V + U^2}}\right) x + \frac{1}{\sqrt{4V + U^2}} z + K
\end{aligned}
$$

where K is an arbitrary constant. Finally, one can readily check that (31) holds true as well.

Knowing the sharing rule up to a constant and a permutation, one readily recover individual Engel curves; in our case, they are quadratic. Specifically, choosing the first solution and setting $K = 0$, we have for $i = 1$:

$$
\begin{aligned}
\alpha_1(\rho) &= A_1 \rho^2 + A_1' \rho + K' \\
\beta_1(t) &= B_1 t^2 + B_1' t + a_1 - K'
\end{aligned}
$$

where

$$
\begin{aligned}
A_1 &= \frac{e_1}{2}\left(4V + U^2\right) - \frac{\sqrt{4V + U^2}}{U}\left(c_1 - \frac{e_1}{2}\left(2V + U^2\right)\right) \\
A_1' &= \frac{1}{2} d_1 \left(\sqrt{4V + U^2} + U\right) + b_1 \\
B_1 &= \frac{\sqrt{4V + U^2}}{U}\left[c_1 - \frac{e_1}{4}\left(\sqrt{4V + U^2} - U\right)^2\right] \\
B_1' &= -\frac{1}{2} d_1 \left(\sqrt{4V + U^2} - U\right) + b_1
\end{aligned}
$$

and K' is a constant. A similar expression obtains for other commodities.

6.4 More than two commodities.

Finally, if the demand functions of a household is observed for three goods or more, stronger results obtain. Intuitively, the sharing rule can be identified (up to a constant and a permutation) from the first two commodities; compatibility with the third generates overidentifying restrictions. In fact, the identification is easier than previously, because one can now derive two (or more) first order PDE's, namely (32) and

$$\mathbf{Q}^{13}_{xx} + \mathbf{Q}^{13}_{xz} \frac{1 - 2\rho_x}{\rho_z} - \mathbf{Q}^{13}_{zz} \frac{\rho_x(1 - \rho_x)}{\rho_z^2} = 0 \tag{38}$$

Therefore, we have a system of two quadratic equations in two *scalar* unknowns ρ_x and ρ_z. Solving the system gives two solutions for the two partials, which must moreover satisfy cross-derivative restrictions. The important and remarkable result here is that collective rationality implies enough restrictions on aggregate household demand functions so as to recover the sharing rule and individual Engel curves from the observation of aggregate marginal propensities to consume and the way they change as a function of both total expenditures and the distribution factor.

7 Conclusion.

In this paper, we have investigated the properties of the 'collective' approach to household behavior. This only relies upon one general assumption : that decisions taken within a household are 'cooperative' or 'collectively rational', that is, lead to Pareto efficient outcomes. What we have shown is that this very general setting has considerable empirical implications. It leads in particular to a sequence of tests which throw some light into the usual black box that is used to analyze household consumption decisions. Remarkably enough, our techniques only require a distinction between those factors which may be behind the allocation process within the household ('distributions factors') and those that are likely to affect personal preferences. It does not require in particular any knowledge of the actual intrahousehold allocation of goods. The most general test of cooperation does not even require any assumption on the nature of the goods that are consumed or produced within the household.

The mere interpretation of the distribution factors raises interesting issues. A difficult question, in particular, is the following. Suppose that distribution factors are random variables which will be realized *after* the couple's marriage contract has been negotiated. If spouses are risk averse, efficiency requires that these variations be insured away in the marriage contract - in which case the impact on behavior should be nil.[11] In this context, two types of arguments may justify the role of distribution factors. One is incomplete contracts and unforeseen contingencies; that is, the spouses failed to consider some possible future situations when writing the initial contract. For instance, all British couples who married in the 1960's may not have taken into account the possibility of the reform of child benefits that took place in the 1980's or the changes in divorce settlements that took place in England in the early 2000's, as studied by Kapan (2008). A related but different argument relies on imperfect commitment. Perfect risk sharing relies on the parties' ability to fully commit. In practice, such a complete commitment may be difficult to achieve, if only because people cannot legally commit not to divorce. Under unilateral divorce, therefore, any change in the environment (that is, in the rules governing settlements after divorce) that has a sufficiently large impact on

[11] We are grateful to an anonymous referee for raising this important point.

a spouse's reservation utility *must* lead to a renegotiation of the existing agreement, at least for existing marriages.[12]

A second interpretation of distribution factors relies more on a 'comparative statics' perspective. Assume, for simplicity, perfect commitment, and consider agents belonging to two different 'marriage markets' (or submarkets); one in which single women are rare, and another in which they are in excess (say, rural Wisconsin versus New York). The distribution of resources between spouses will probably reflect this asymmetry; typically, when many men are competing for a few women, the latter should be able to attract a larger share of household resources. Empirically, when studying a cross-section of such couples, we may then find a correlation between the state of the market for marriage (which will provide the distribution factors needed) and the structure of household consumption. This strategy has been used in a number of empirical papers (for example, Chiappori, Fortin and Lacroix (2002)).

Finally, the results presented here are not exhaustive. Additional tests are available when one wants to go further and infer from the joint spending behavior on private goods by the household some information on who gets what. A general test is available in the case where the analysis is restricted to private goods only. It has even been shown that it is possible, if that test is satisfied, to recover from the observation of joint consumption behavior, information on the intrahousehold allocation of these goods and on individual preferences (Engel curves). More information and more restrictive tests may be obtained in the case where at least one individual consumption is observed. Whether those tests are robust and will actually provide more information on intrahousehold decision processes will be taken up in forthcoming empirical work.

[12]For marriages taking place after the reform, the logic is quite different, because the new rules are taken into account in the design of the marriage contract. Even in that case, however, the reform tends to influence the intrahousehold allocation; for instance, a reform increasing the settlements obtained by the wife in case of divorce will be compensated by a reduction in the share of marital surplus she will receive while married. See Chiappori, Iyigun and Weiss 2008 for a precise investigation along these lines.

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A Appendix.

Proof. Proposition 2.

Consider first the case where there are at least two distribution factors. From the Pareto-efficiency assumption, demands should be solutions of the following program :

$$\max_{\mathbf{q}^A,\mathbf{q}^B,\mathbf{Q}} \quad u^A(\mathbf{q}^A,\mathbf{q}^B,\mathbf{Q};\mathbf{a})+\mu.\ u^B(\mathbf{q}^A,\mathbf{q}^B,\mathbf{Q};\mathbf{a})$$

$$\text{subject to } \mathbf{e}'(\mathbf{q}^A+\mathbf{q}^B+\mathbf{Q})=x$$

Here, the set of Pareto efficient allocations is fully described when μ varies within $\mathbb{R}^n_+$. The particular location of the solution on the Pareto frontier should of course be allowed to depend on all relevant parameters: i.e., μ will in general be a function of $(x,\mathbf{a},z)$. Household demand functions can thus be written:

$$\xi_i(x,\mathbf{a},z)=\Xi_i[x,\mathbf{a},\mu(x,\mathbf{a},z)] \qquad \forall i=1,.....,n$$

as stated in condition (7). Then (8) comes from the fact that:

$$\frac{\partial\xi_i/\partial z_k}{\partial\xi_i/\partial z_1}=\quad\frac{\partial\mu/\partial z_k}{\partial\mu/\partial z_1}\qquad\forall i,k \tag{39}$$

Finally, in the neighborhood of any point where a $z-$conditional demand can be defined, (7) allows us to (locally) express μ as a function of $\mathbf{q}_j$, x and $\mathbf{a}$. Replacing in the direct demand function for good i leads to (9). Hence (7), (8) and (9) are equivalent necessary conditions for observed demand functions to be consistent with collective rationality.

For sufficiency, note that according to (7) there exists some function $\nu(x,\mathbf{a},z)$ such that $\xi(x,\mathbf{a},z)$ can be expressed as a function $\Xi(x,\mathbf{a},\nu)$ of $x,\mathbf{a}$ and ν alone. Take some arbitrary function $G(\xi_1,\xi_2,...,\xi_n;\mathbf{a})$ that is positive, increasing and quasi-concave with respect to the variables ξ_1. Define then:

$$M(x,\mathbf{a},\nu)=G[\Xi_1(x,\mathbf{a},\nu),....,\Xi_n(x,\mathbf{a},\nu)]$$

We will now show that there exist two increasing and quasi-concave utility functions $v^A(\mathbf{Q},\mathbf{a})$ and $v^B(\mathbf{Q},\mathbf{a})$ such that the observed demand functions are solutions of (P) for $\mu(x,\mathbf{a},z)=M[x,\mathbf{a},\nu(x,\mathbf{a},z)]$. Clearly, these utility functions $v^i(X,z)$ are particular cases of the general utility functions $u^i()$ appearing in (P) because they depend only on public goods.

The necessary and sufficient first order conditions implied by (P) are:

$$\forall(x,\mathbf{a},z)\qquad\qquad D_\xi\nu^A(\xi,\mathbf{a})+\mu.D_\xi\nu^B(\xi,\mathbf{a})=\lambda.\mathbf{e}$$

where $D_\xi v^i$ is the gradient of v^i and λ is an arbitrary scalar function of $(x,\mathbf{a},z)$. Define then:

$$\begin{aligned}\nu^B(\xi,\mathbf{a}) &= A(\xi_1+\xi_2+...\xi_n)+B[(G(\xi,\mathbf{a})]\\ \nu^A(\xi,\mathbf{a}) &= \mathbf{C}[(G(\xi,\mathbf{a})]\end{aligned}$$

where A, $\mathbf{C}$ are arbitrary increasing scalar functions and B is a scalar function defined by:

$$B'(\mathbf{q})=-\mathbf{q}.\mathbf{C}'(\mathbf{q})$$

A is taken to be large enough with respect to B so that ν^B is increasing. These functions ν^A and ν^B are thus increasing and quasi-concave. Moreover, it can easily be checked that they satisfy (A1). It follows that the solution of (P) with these functions ν^i is the set of observed demand functions $\xi(x, \mathbf{a}, z)$ which satisfy the equivalent conditions (7)-(9) in Proposition 2.

It remains to show that the proposition remains valid in the case where there is only one distribution factor, $K = 1$. On one hand, condition (7) is trivially satisfied since it corresponds to a mere change of variable of z, whereas conditions (8) and (9) become irrelevant. On the other hand, the above sufficiency argument in the case $K > 1$ remains valid when $K = 1$ since it is solely based on condition (7). This shows that in the case of a single distribution factor and without a priori restrictions on individual preferences all observed demand functions are consistent with collective rationality. ■

Proof. Proposition 6

From (18), an exclusive good consumed by member A is such that:

$$\mathbf{q}(\mathbf{z}, x) = \alpha[\rho(\mathbf{z}, x)]$$

The function ρ is thus some transformation of the observed demand function $\mathbf{q}(\mathbf{z}, x)$. ■

Proof. Proposition 7

Equation (20) is directly obtained from (19) and the exclusivity condition on good 2. (21a) expresses the fact that the demand for good 2 is that of member B and thus depends only on the share of private expenditure going to him/her. The function $g(\mathbf{q}_1)$ in that expression is the share going to member A and thus the inverse of his/her own demand function (as in proposition 9), which is in fact the function F( ) appearing in (20). Finally, (22) is a translation of (21a) into a partial differential equation. Differentiating (21a) with respect to x and $\mathbf{q}_1$ yields:

$$\partial\theta_2/\partial x = \beta'\left[x - g(\mathbf{q}_1)\right]$$

and

$$\partial\theta_2/\partial\mathbf{q}_1 = -g'(\mathbf{q}_1).\beta'\left[x - g(\mathbf{q}_1)\right]$$

Assuming that Θ is non linear in x, we have that:

$$g'(\mathbf{q}_1) = -\frac{\partial\theta_2/\partial\mathbf{q}_1}{\partial\theta_2/\partial x} \tag{40}$$

This must be a function of $\mathbf{q}_1$ alone, which generates condition (22). Reciprocally, (23) implies that $\theta_2(\ )$ is a transformation of a function that is additively separable in x and $\mathbf{q}_1$. ■

Proof. Proposition 8

Only a proof of (ii) is needed at this stage. From (18) for exclusive goods we have:

$$\mathbf{q}_1(z, x) = \alpha[\rho(z, x)];\quad \mathbf{q}_2(z, x) = \beta[x - \rho(z, x)]$$

Differentiating the observed demand functions with respect to z and x yields:

$$\begin{aligned}\frac{\partial\mathbf{q}_1}{\partial z} &= \alpha'.\frac{\partial\rho}{\partial z}\\ \frac{\partial\mathbf{q}_1}{\partial x} &= \alpha'.\frac{\partial\rho}{\partial x}\end{aligned}$$

$$\begin{aligned}\frac{\partial \mathbf{q}_2}{\partial z} &= -\beta'.\frac{\partial \rho}{\partial z}\\ \frac{\partial \mathbf{q}_2}{\partial x} &= \beta'.(1-\frac{\partial \rho}{\partial x})\end{aligned}$$

Solving for ρ_x and ρ_z yields (25). It may be shown that the condition under which that resolution is possible -i.e. $\frac{\partial \mathbf{q}^A}{\partial z} \neq 0$ $\frac{\partial \mathbf{q}B}{\partial z} \neq 0$ $\frac{\partial \mathbf{q}^A/\partial x}{\partial \mathbf{q}^A/\partial z} \neq \frac{\partial \mathbf{q}^B/\partial x}{\partial \mathbf{q}^B/\partial z}$ - is equivalent to the $z-$conditional demand $\theta_2(x, \mathbf{q}_1)$ being well defined -i.e. $\partial\theta_2/\partial x \neq 0$; $\partial\theta_2/\partial \mathbf{q} \neq 0$ - as in Proposition 8. It may also be shown that the integrability condition of (25), that is the cross-derivative restriction:

$$\frac{\partial}{\partial z}(\left(\frac{\frac{\partial \mathbf{q}^A/\partial x}{\partial \mathbf{q}^A/\partial z}}{\frac{\partial \mathbf{q}^A/\partial x}{\partial \mathbf{q}^A/\partial z}-\frac{\partial \mathbf{q}^B/\partial x}{\partial \mathbf{q}^B/\partial z}}\right) = \frac{\partial}{\partial x}\left(\frac{1}{\frac{\partial \mathbf{q}^A/\partial x}{\partial \mathbf{q}^A/\partial z}-\frac{\partial \mathbf{q}^B/\partial x}{\partial \mathbf{q}^B/\partial z}}\right)$$

is equivalent to condition (22) above after a change of variables. ■

Proof. Proposition 9

(i) is simply (19). ii) is a restatement of (18) where $g(\mathbf{q}_1)$ is the share of total expenditures going to member A, given that $\mathbf{q}_1$ is exclusively consumed by him/her. Finally (28) is the partial differential equation expression of (27a). The equivalent of relationship (23) above is obtained now by differentiating (27a) twice:

$$g'(\mathbf{q}_1) = -\frac{\partial^2\theta_2/\partial \mathbf{q}_1\partial x}{\partial^2\theta_2/\partial x^2} \tag{41}$$

which leads to (28). Reciprocally (28) implies that θ_{2x} is the transformation of a function that is additively separable in x and $\mathbf{q}_1$. Hence (27a).

As in the preceding case, the sharing rule may be easily recovered from the preceding differential equation in x_1 and the direct demand function $\mathbf{q}_1(x,z)$ through $\rho(x,z) = g[\mathbf{q}_1(x,z)]$. As before, it is thus defined up to an additive constant. Things are a little more complex in the present case when one uses direct demand functions, although, as in the preceding case, all properties on $z-$conditional demands have their counterpart on direct demand functions. We leave these derivations to the interested reader. ■

Proof. Proposition 10

That (33) and (34) are equivalent is obvious. Also, for any f, define α_i and β_i by :

$$\begin{aligned}\alpha_i(u) &= c_i f^{-1}(u) + b_i u\\ &\text{and}\\ \beta_i(v) &= a_i + b_i v\end{aligned}$$

Then (29) is obviously fulfilled. Note that, in this case, the conditions of Proposition 9 do not apply. Also, it is interesting to note that all equations (30) are proportional, so that considering several consumption goods does not bring additional information. The only way to identify the sharing rule in that case is to observe an assignable good. ■