Notes on the Farm-Household Model

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Contents

Part I
Household Models

Agenda Household Models

Idea is to outline basic model, a la Singh et al. [1986]. Show that separation is implied by basic model. Discuss how to estimate demand system, labor supply, all in an environment with no risk.

Contents

1 Outline of Basic Model

Elements of Basic Model Household-level Decision-making

Of course, the "household" can't really make decisions; really what's being assumed here are three things:

- That household resources are pooled;
- That allocations within the household are efficient; and
- That the organization of production is efficient.

1.1 Household Preferences

1.1.1 Commodity Space

Elements of Basic Model Commmodity Space

- Bardhan-Udry (following Singh et al. [1986]) assume that the household derives utility from the consumption and leisure of its members. In addition, land and labor are used in production of some *numeraire* good.
- In a more general formulation, we can think of some x as a "netput" vector of commodities.
- In either case, generally assume that the set of feasible allocations (both for consumption and production) is convex and compact.

Example Problem for the Farm-Household

$$\max_{c_i,l_i,A,L} U(c_1, c_2, l_1, l_2)$$

such that

$$p(c_1 + c_2) + w(l_1 + l_2) \leq [F(A, L, \epsilon) - rA - wL] + rE^a + w(E_1^L + E_2^L),$$

and subject to a collection of non-negativity constraints on consumptions, leisures, and farm inputs.

1.1.2 Objective Function

Objective Function

- For Bardhan-Udry, objective is $U(c_1, c_2, l_1, l_2)$, where there are two household members $i = 1, 2$, and where (c_i, l_i) is the consumption-leisure pair for person i.
- There's little of importance lost by assuming that the *household* utility can be additively decomposed,

$$U(c_1, c_2, l_1, l_2) = \sum_{i=1}^{n} U_i(c_i, l_i).$$

- Basically, will typically want to assume that objective function is increasing and concave.

1.2 Feasible Set

Feasible Set

- In the Bardhan-Udry formulation, the feasible set depends on:
  1. Total household land endowment E^A (note pooling of resources among household members).
  2. Total time available to each household member: E_i^L, $i = 1, 2$.
  3. Prices for consumption, labor, and land: (p, w, r).
- The household takes endowments (E^A, E_1^L, E_2^L) and prices (p, w, r) as given.
- Let $\Gamma(E^A, E_1^L, E_2^L, p, w, r)$ denote the feasible set for the household.

2 Solution to the Household-Farm's Problem

2.1 Solving the Problem

Solution Solving

- With concave, increasing objective function and a non-empty, compact, convex feasible set, theory of the maximum implies a unique solution.
- Since the objective function *doesn't* depend on any of the variables that determine the feasible set, the "separation property" is satisfied. Households can solve their problem in two separate steps:
  1. Maximize farm profits; and
  2. Given "total income" (including, but not limited to, farm profits), choose a consumption-leisure allocation to maximize utility.

2.2 General Properties of the Solution

Solution General Properties

- Demands for leisure and consumption should depend only on total income and prices; **not** on (E^A, E^L_1, E^L_2) (except to the exent these determine total income) and **not** on production decisions such as the choice of the allocation of land and labor to production (A, L).
- Operation of farm should **not** depend on household characteristics which influence only objective function.
- Since **other** farm-households presumably face the same prices, marginal products of labor and land should be equated across farms.

3 Estimation & Inference

Estimation & Inference

- Making additional progress on understanding behavior of farm households will require additional assumptions on either the feasible set (perhaps particularly the farm production function) or on the household utility function.
- Ideally, we'd like to use data to allow us to recover both U and Γ; then we'd have a complete model of the farm-household.
- In practice, this model may be too simple to capture important elements of the problem facing the farm-household. We can get at this by testing.

Example: Estimating utility functions

- *If* separation property is satisfied, then we can ignore production side, and just look at demands for goods and leisure.
- Parameterize utility function; e.g.,

$$U(c_1, c_2, l_1, l_2; X_1, X_2) = \theta_1 \exp(\delta' X_1) \left[\alpha_1 \frac{(c_1 - \phi_1)^{1-\gamma} - 1}{1-\gamma} + (1-\alpha_1)\frac{(l_1)^{1-\gamma} - 1}{1-\gamma}\right] + \theta_2 \exp(\delta' X_2) \left[\alpha_2 \frac{(c_2 - \phi_2)^{1-\gamma} - 1}{1-\gamma} + (1-\alpha_2)\frac{(l_2)^{1-\gamma} - 1}{1-\gamma}\right]$$

Features of Utility

These preferences feature:

- Linear Engel Curves (necessary for aggregation)
- "Subsistence" parameters ϕ_i
- Demands can depend on individual characteristics X_i.
- Demands depend on "disposable" total income $\bar{x} = y - p\sum_{i=1}^{i}\phi_i$.

Demands for consumption and leisure take the form

$$c_i(\bar{x}, p) = \left(\frac{\theta_i \alpha_i e^{\delta' X_i}}{p}\right)^{1/\gamma} \bar{x} + \phi_i$$

and

$$l_i(\bar{x}, w) = \left(\frac{\theta_i (1-\alpha_i) e^{\delta' X_i}}{w}\right)^{1/\gamma} \bar{x}$$

Estimating Demands

If we have data on c_i, l_i, prices (p, w), and disposable total income $\bar{x}$, then we can imagine using these to try and estimate these demand relationships.

Taking logs of the expression for $c_i(\bar{x}, p)$ and re-arranging,

$$\log(c_i - \phi_i) = \frac{1}{\gamma}[\log(\theta_i) + \log(\alpha_i) + \delta' X_i - \log(p)] + \log(\bar{x}),$$

which is *almost* something we could use OLS to estimate. Or if prices aren't observed, rearrange again to get

$$\log \frac{p(c_i - \phi_i)}{\bar{x}} = (1 - \frac{1}{\gamma}) \log p + \frac{1}{\gamma}[\log(\theta_i) + \log(\alpha_i) + \delta' X_i],$$

which is an expression which would allow one to relate budget shares to household characteristics.

Testing

If we can estimate demand system or Engel curves using previous, we can recover the utility function!

Question: How will we know if our estimates of preference parameters are adequate?

Answer: We won't. We can only know if they're *not* adequate.

Check the following, in this order:

1. Are residuals from estimating equations independent of functions of prices and net total income? If not, suggests a problem with specification of utility function.
2. Are residuals independent of production side characteristics? If not, suggests a problem with separation hypothesis.

4 When Separation Fails

Possible reasons for failure of separation

"Shallow" reasons for failure of separation:

- Mis-specified utility function.
- Utility depends directly on production arrangements (e.g., prefer to work on own land).
- Production depends directly on consumption side (e.g., marginal product of labor depends on how well-fed workers are).
- "Transaction" costs.

"Deep" reasons for possible failure of separation involve so-called *missing markets*, though this begs the question of why markets may be missing.

Farm-Households and Missing Contingent-Claims Markets Feasible Set

Suppose that there's some randomness to production, so that output is given by $F(A, L, \epsilon)$, where ϵ is a random variable, instead of simply $F(A, L)$.

Let's suppose that $\epsilon \in \Omega$, and that Ω has a finite number of elements.

Now, the farm household's constraints still have to be satisfied, as before, but *now* they have to be satisfied for every value of ϵ which may be realized. This gives us additional constraints; we write the new set as

$$\Gamma(E^A, E_1^L, E_2^L, \epsilon, p, w, r).$$

Farm-Households and Missing Contingent-Claims Markets Feasible Set

Decisions about how land and time are allocated have to be made before ϵ is observed. So what's the household's new problem? To maximize *expected* utility subject to this new constraint set:

$$\max_{c_i(\epsilon), l_i, A, L} \sum_{\epsilon} \Pr(\epsilon) U(c_1(\epsilon), c_2(\epsilon), l_1, l_2)$$

such that

$$p(c_1(\epsilon) + c_2(\epsilon)) + w(l_1 + l_2) \leq [F(A, L, \epsilon) - rA - wL] + rE^a + w(E_1^L + E_2^L)$$

for all ϵ. Associate a multiplier $\Pr(\epsilon)\lambda(\epsilon)$ with the budget constraints. Solution must also satisfy a collection of non-negativity constraints.

Notice that consumption depends on ϵ*!*

First-order conditions Consumption side

$$c_i(\epsilon): \quad \Pr(\epsilon)\frac{\partial U}{c_i} = p\Pr(\epsilon)\lambda(\epsilon)$$

$$l_i: \quad \mathrm{E}\frac{\partial U}{l_i} = w\mathrm{E}\lambda(\epsilon)$$

Summing the FOC w.r.t. $c_i(\epsilon)$ over ϵ, we obtain the optimality condition

$$\frac{\mathrm{E}\partial U/\partial c_i}{\mathrm{E}\partial U/\partial l_i} = \frac{p}{w}.$$

Contrast with case of no risk; effect on consumption and leisure depends on curvature of utility function.

First-order conditions Production side

$$A: \quad \mathrm{E}\left[\lambda(\epsilon)\frac{\partial F}{\partial A}\right] = r\mathrm{E}\lambda(\epsilon)$$

$$L: \quad \mathrm{E}\left[\lambda(\epsilon)\frac{\partial F}{\partial L}\right] = w\mathrm{E}\lambda(\epsilon)$$

Thing to note is that the first order conditions for production can't in general be disentangled from the multiplier $\lambda(\epsilon)$. As a consequence, the choice of productive inputs will depend on the probability distribution of the marginal utility of income for the household.

Example

Here's a particular example. Use the preferences described above, with $\gamma = 1$ and $\phi_i = 0$ (i.e., logarithmic utility) and a Cobb-Douglas production function

$$y = A^\beta L^{1-\beta} e^\epsilon,$$

where $\mathrm{E}e^\epsilon = 1$.

Now, optimality on the consumption side implies

$$\frac{\alpha_i}{1-\alpha_i}\mathrm{E}\frac{l_i}{c_i(\epsilon)} = \frac{p}{w}.$$

Since the left-hand side of this is a convex function of a random variable, the expected marginal rate of substitution consumption must be greater than in the case of no risk (Jensen's inequality). Accordingly, in this case the farm household will work more than it would if separation held [see Kochar, 1999, for evidence on this point].

Key points from example

- Failure of separation
- "Precautionary labor" (but direction of distortion depends on preferences and technology)
- Farm inputs and consumption demands now become a complicated function of just about everything in the environment. The approach to estimation taken above won't work (in the sense that the demands are mis-specified, so we can't expect to use this as the basis for a (consistent) estimator).

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