

## ARE 251/ECON 270A ASSIGNMENT 3

ETHAN LIGON

- (1) Consider a household consisting of only two people, each of whom lives for two periods. Each derives utility from a von Neumann-Morgenstern utility function  $U : \mathbb{R} \rightarrow \mathbb{R}$ , and discounts future utility by a factor  $\beta = 0.9$ .

There are two possible states in each period  $t$ , denoted by  $\omega_t \in \{1, 2\}$ . Person  $i$ 's endowment in period  $t$  and state  $\omega_t$  is equal to  $y_{it}(\omega_t)$ . The probability that  $\omega_t = 1$  is 0.5. Assume that  $U(c) = \log(c)$ . There is no storage, and neither person has any interaction with others outside the household.

Each person's endowments in each of the two periods is given by:

		Person	
		1	2
$\omega$	$t$	0	3
1	1	5	7
2	1	2	4
1	2	3	3
2	2		

- a) Write down an event tree for this economy. Write down the commodity space.
  - b) Calculate the expected discounted utility for agent 2 if he simply consumes his endowment in each period.
  - c) Suppose that in addition to utility from own consumption, person one also enjoys a share  $\theta_2^1$  of person two's utility, and conversely person two enjoys a share  $\theta_1^2$  of person one's utility. Show that a set of state-contingent altruistic transfers between the two people can yield Pareto optimal outcomes, even in the absence of any other kinds of transfers.
  - d) Show that an income-pooling scheme can yield Pareto optimal outcomes. What is the minimum share that agent two would accept as an inducement to participate in this scheme?
  - e) Using the fact that  $\frac{U'(c_{it})}{\beta E U'(c_{it+1})} = 1 + r_t$ , compute implicit interest rates (in each of the two first-period states) given that the couple implements the income pooling scheme.
  - f) Do interest rates depend on shares?
  - g) Would we expect to observe any borrowing or lending between this couple?
- (2) Download the files `vlss_extract.txt` and `vlss_readme.txt`. These contain a randomly selected sub-sample of households from the *Vietnam Living Standards Survey*, including data on household expenditures, income, and demographics.
- a) Produce plots of three types of Lorenz curves: for food expenditures, for total expenditures, and for income in both 1993 and 1998. Comment on the relationship between these curves, both across time and across types.

- b) Compute Atkinson's inequality measure for both years, taking the "inequality aversion" parameter to be equal to 2. Comment on changes you see across years.
- (3) Keep the environment described initially above, but now suppose that instead of a single isolated household there are  $n$  households similar to the household described above. Index these households by  $i = 1, \dots, n$ , and let the probability of state 1 occurring for household  $i$  be equal to  $p_i$  (instead of  $1/2$ ). The parameter  $p_i$  is, in turn, drawn from the uniform distribution on the open interval  $(0, 1)$ .

- a) Show that if all households are permitted to engage in exchange with each other that in a competitive equilibrium an individual in household  $i$  will have consumption which will depend on  $p_i$ , but not on the households' endowment realization. In particular, show that each individual  $j \in \{1, 2\}$  in household  $i$  in period  $t$  will have consumption realizations which take the form

$$\log c_{it}^j = \alpha_i + \nu_j + \eta_t,$$

where  $\alpha_i$  can be thought of as a "household effect,"  $\nu_j$  reflects the distribution of resources in the household, and  $\eta_t$  reflects aggregate endowment shocks.

- b) If all households are now permitted to engage in exchange with one another and  $n$  is large, what will prevailing interest rates in the first period be if there's full insurance?
- c) Produce two Lorenz curves for this model economy in each of the two periods: The first for household level consumption inequality, the second for household level income inequality. Discuss the relationship between inequality over time, as well as on the differences between income and consumption inequality.
- d) Produce two Lorenz curves for this model economy in each of the two periods: The first for individual level consumption inequality, the second for individual level income inequality. Discuss the relationship of these curves with the household level Lorenz curves.
- e) Using the data extract from the VLSS, test the predictions of the full risk-sharing model you derived above. What do these results suggest about the right way to measure poverty or inequality?
- f) Compare and contrast the Lorenz curves from the data with the Lorenz curves from the model. What are the principal successes and failures of the model when it comes to explaining the patterns you've observed in the data?
- g) Following Ravallion's 1994 advice, compute the 'usual' Foster-Greer-Thorbecke poverty measures ("headcount", "poverty-gap" and "squared poverty gap") for a range of possibly poverty lines using the 1993 data. Produce a plot of these measures where the poverty line appears on the horizontal axis, and the poverty measures on the vertical axis. How does the relationship between these measures and Atkinson's inequality measure vary as the poverty line increases?