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# Household Models

**I. The Basic Separable Household Model** (Singh, I., Squire, L., and Strauss, J. (eds.) *Agricultural Household Models*. Chapters 1 and 2. Baltimore, MD: The Johns Hopkins University Press, 1986)

Two producer goods: food ($a$) and cash crops ($c$)
Two factors of production: labor ($l$) and other variable inputs ($x$)
Three consumer goods: food ($a$), manufactured goods ($m$), and leisure ($l$)

Definitions:
$q_a$ production of food crop with price $p_a$
$q_c$ production of cash crop with price $p_c$
$q_l$ labor used in farm production with wage $p_l$
$q_x$ other variable inputs with price $p_x$
$z^q$ fixed factors in production and producer characteristics

$c_a$ consumption of food product with price $p_a$
$c_m$ consumption of manufactured good with price $p_m$
$c_l$ consumption of leisure with price $p_l$
$z^h$ household characteristics in consumption
$l^s$ time worked
$E$ total time endowment
$p_l$ wage on labor market
$y$ income
$S$ exogenous cash transfers

## 1.1. The structural model

Assume: perfect markets for all products and factors, including food and family labor.
Household optimization problem:

$$\underset{q_a, q_c, q_l, q_x, c_a, c_m, c_l}{\text{Max}} \quad U\left(c_a, c_m, c_l; z^h\right)$$

s.t.

(1) $g\left(q_a, q_c, q_l, q_x; z^q\right) = 0$, production function

(2) $p_x q_x + p_m c_m = p_a\left(q_a - c_a\right) + p_c q_c + p_l\left(l^s - q_l\right) + S$, liquidity constraint

(3) $l^s + c_l = E$, time constraint

Substituting $l^s$ in (2) for its value in (3) gives the full income constraint:

$$p_a c_a + p_m c_m + p_l c_l = \left(p_a q_a + p_c q_c - p_l q_l - p_x q_x\right) + p_l E + S$$
$$= \Pi + p_l E + S$$

where $\Pi = p_a q_a + p_c q_c - p_x q_x - p_l q_l$, restricted profit in agriculture.

The household optimization problem can be rewritten as:

$$\underset{q_a, q_c, q_l, q_x, c_a, c_m, c_l}{\text{Max}} \ W = U + \phi g + \lambda\left[\Pi - p'c + p_l E + S\right]$$

Assume interior solution with $q$ and $c > 0$. First order conditions:

(4) $\frac{\partial W}{\partial q_i}$: $\phi g_i' = -\lambda p_i,\ \ i = a,c$ (producer goods)

(5) $\frac{\partial W}{\partial q_j}$: $\phi g_j' = \lambda p_j,\ \ j = l,x$ (factors)

(6) $\frac{\partial W}{\partial \phi}$: $g = 0$ (technology constraint)

(7) $\frac{\partial W}{\partial c_k}$: $U_k' = \lambda p_k,\ k = a,m,l$ (consumption goods)

(8) $\frac{\partial W}{\partial \lambda}$: $p'c - (\Pi + p_l E + S) = 0$ (full income constraint)

This indicates recursivity, called separability or separation, i.e.:

Equations (4)–(6) $\Rightarrow$ optimum levels of outputs, inputs, and maximum profit $\Pi^*$.

Equations (7) and (8) identical to a pure consumer problem.

Production decisions influence consumption only through profit $\Pi^*$.

**1.2. Recursive solution: the reduced form**

First step: Solve the producer problem for maximum agricultural profit:

$$\max_{q_a, q_c, q_l, q_x} \quad \Pi = p_a q_a + p_c q_c - p_x q_x - p_l q_l, \qquad s.t.\ g(q_a, q_c, q_l, q_x; z^q) = 0\ .$$

This gives the reduced form:

Supply functions $q_i = q_i(p_a, p_c, p_l, p_x; z^q),\ \ i = a,c$

Factor demands $q_j = q_j(p_a, p_c, p_l, p_x; z^q),\ \ j = l, x$

Maximum restricted profit $\Pi^* = \Pi^*(p_a, p_c, p_l, p_x; z^q)$

Second step: Solve the consumer problem for maximum utility given the level of profit $\Pi^*$ achieved in production

$$\max_{c_a, c_m, c_l} \quad U(c_a, c_m, c_l; z^h)$$

s.t. $p_a c_a + p_m c_m + p_l c_l = \Pi^* + p_l E + S$, full income constraint

This gives the reduced form:

Final demand functions: $c_k = c_k(p_a, p_m, p_l, y^*; z^h),\ \ k = a,m,l$

where $y^* = \Pi^*(p_a, p_c, p_l, p_x; z^q) + p_l E + S.$

Hence: $c_k = c_k(p_a, p_c, p_l, p_x, p_m; z^q, z^h, E, S)$

Note: under separability, the prices of consumption goods not produced at home ($p_m$) and the $z^h, E,$ and $S$ variables do not influence production decisions. This will provide a test of separability.

**II. Household model with missing markets for food and labor**
(de Janvry, A., Fafchamps, M., and Sadoulet, E. "Peasant Household Behavior with Missing Markets: Some Paradoxes Explained." *Economic Journal*, Vol. 101, No. 409 (November, 1991), pp. 1400-1417.)

**2.1. The structural model**

Market failures for food (*a*) and labor (*l*): non-tradables
Perfect markets for cash crops (*c*), other inputs (*x*), and manufactured goods (*m*): tradables with exogenous idiosyncratic prices:

$p_c$ farm gate sale price of cash crop
$p_x, p_m$ farm gate purchase prices of other inputs and manufactured goods

$$\underset{q_a, q_c, q_l, q_x, c_a, c_m, c_l}{\text{Max}} U(c_a, c_m, c_l; z^h)$$

s.t.

$p_x q_x + p_m c_m = p_c q_c + S$ cash income constraint,

$g\left(q_a, q_c, q_l, q_x; z^q\right) = 0$ production technology.

$p_i = \overline{p}_i$ for $i = c, x, m$ exogenous effective prices for tradables

$\begin{cases} c_a = q_a \\ c_l = E - q_l \end{cases}$ equilibrium conditions for non-tradables

**2.2. The first order conditions**

$$\underset{q_a, q_c, q_l, q_x, c_a, c_m, c_l}{\text{Max}} W = \left[U + \lambda\left(p_c q_c + S - p_x q_x - p_m c_m\right) + \phi g + \mu_a\left(q_a - c_a\right) + \mu_l\left(E - q_l - c_l\right)\right]$$

First-order conditions:

$\frac{\partial W}{\partial q_c}:\ \phi g_c' = -\lambda p_c;\ \ \frac{\partial W}{\partial q_x}:\ \phi g_x' = \lambda p_x$ (tradables)

$\frac{\partial W}{\partial q_a}:\ \phi g_a' = -\mu_a;\ \ \frac{\partial W}{\partial q_l}:\ \phi g_l' = \mu_l$ (non-tradables)

$\frac{\partial W}{\partial c_m}:\ u_m' = \lambda p_m$ (tradables)

$\frac{\partial W}{\partial c_k}:\ u_k' = \mu_k, \quad k = a, l$ (non-tradables)

$\frac{\partial W}{\partial \phi}:\ g = 0$ (technology constraint)

$\frac{\partial W}{\partial \lambda}:\ p_x q_x + p_m c_m = p_c q_c + S$ (cash income constraint)

$\frac{\partial W}{\partial \mu_a}:\ c_a = q_a$ (equilibrium condition for food)

$\frac{\partial W}{\partial \mu_l}:\ c_l = E - q_l$ (equilibrium condition for labor).

Define decision prices $p^*$ as follows:

$p_a^* = \mu_a / \lambda, p_l^* = \mu_l / \lambda$ shadow prices for the nontradables *a* and *l*
$p_i^* = \overline{p}_i$ effective market prices for the tradables *c, x*, and *m*.

Combining the last three conditions gives the full income constraint:

$$p_x q_x + p_m c_m + p_a^* c_a + p_l^* c_l = p_c q_c + p_a^* q_a + p_l^* \left(E - q_l\right) + S \qquad .$$

By analogy with the first-order conditions for the separable model in 1.1, the first order conditions for the non-separable model can be rewritten using decision prices $p^*$ as:

| | |
|---|---|
| $\phi g_i' = -\lambda p_i^*,\ i = c, a$ | products |
| $\phi g_j' = \lambda p_j^*,\ j = l, x$ | factors |
| $g = 0$ | technology |

| | |
|---|---|
| $u_k' = \lambda p_k^*,\ \ k = m, a, l$ | consumer goods |
| $\sum_{k=a,m,l} p_k^* c_k = \sum_{i=a,c} p_i^* q_i - \sum_{j=l,x} p_j^* q_j + p_l^* E + S$ | full income constraint |

$$\begin{cases} c_a = q_a \\ c_l = E - q_l \end{cases} \qquad \text{equilibrium conditions for non-tradables}$$

### 2.3. The household's decision structure (semi-structural form)

Production decisions from profit maximization: supply and derived demand:

$$q_i = q_i\left(p_a^*, p_c^*, p_l^*, p_x^*; z^q\right),\ \ i = a, c, l, x \qquad .$$

Profit and full income:

$$\Pi^* = \sum_{i=a,c} p_i^* q_i - \sum_{j=l,x} p_j^* q_j$$
$$y^* = \Pi^* + p_l^* E + S.$$

Consumption from utility maximization (with prices $p^*$ and income $y^*$)

$$c_k = c_k\left(p_a^*, p_m, p_l^*, y^*; z^h\right),\ \ k = a, m, l$$

Equilibrium conditions

$$\left.\begin{aligned} c_a\left(p^*, y^*; z^h\right) &= q_a\left(p_a^*, p_c^*, p_l^*, p_x^*; z^q\right) \\ c_l\left(p^*, y^*; z^h\right) &= E - q_l\left(p_a^*, p_c^*, p_l^*, p_x^*; z^q\right) \end{aligned}\right\} \quad \text{for non-tradables}$$

Solving these equilibrium conditions for the shadow prices of non-tradables:

$$p_j^* = p_j^*\left(p_c, p_x, p_m; z^q, z^h, E, S\right),\ \ j = a, l \qquad .$$

The $p^*$ for nontradables are function of the prices of tradable consumption goods and of $z^q, z^h, E,$ and $S$.

The semi-structural solution of the model is thus:

$$q_i = q_i\left(p_a^*, p_c, p_l^*, p_x; z^q\right),\ \ i = a, c, l, x$$
$$c_k = c_k\left(p_a^*, p_m, p_l^*, y^*; z^h\right),\ \ k = a, m, l$$

and $\quad p_j^* = p_j^*\left(p_c, p_x, p_m; z^q, z^h, E, S\right),\ j = a, l$

Hence, household characteristics in consumption, $z^h$, *E*, and *S* and consumption prices, $p_m$, affect production decisions, as opposed to the separable model. The system would be recursive if there were only tradables.

### 2.4. The reduced form

Substituting the expression just derived for the shadow price $p_j^*$ into the production and consumption decisions give:

$$q_i = q_i\left(p_c, p_x, p_m; z^q, z^h, E, S\right),\ i = a, c, l, x$$
$$c_k = c_k\left(p_c, p_x, p_m; z^q, z^h, E, S\right),\ k = a, m, l$$

### 2.5. Price elasticities (*E*)

Supply response

$$E^G\left(q_i / p_j\right) = E\left(q_i / p_j\right) + E\left(q_i / p_a^*\right)E\left(p_a^* / p_j\right) + E\left(q_i / p_l^*\right)E\left(p_l^* / p_j\right),\ i = a, c; j = c.$$

where $E^G$ is the global elasticity.

Consumption

$$E^G\left(c_k / p_j\right) = E^H\left(c_k / p_j\right) + E^H\left(c_k / p_a^*\right)E\left(p_a^* / p_j\right) + E^H\left(c_k / p_l^*\right)E\left(p_l^* / p_j\right),\ k = m, l; j = m$$

where $E^H$ is the elasticity in the separable household model with endogenous income effects:

$$E^H\left(c_k / p_k\right) = E\left(c_k / p_k\right) + E\left(c_k / y^*\right)E\left(y^* / p_k\right),\ k = a, m.$$

## III. Examples of empirical analyses:


Benjamin, D. "Household Composition, Labor Markets, and Labor Demand: Testing for Separation in Agricultural Household Models." *Econometrica*, Vol. 60, No. 2 (1992), pp. 287-322.

Jacoby, Hanan. "Shadow Wages and Peasant Family Labour Supply: An Econometric Application to the Peruvian Sierra." *Review of Economic Studies*, Vol. 60 (1993), pp. 903-921.

Bowlus, Audra J. and Terry Sicular, "Moving toward markets? Labor allocation in rural China," *Journal of Development Economics*, Vol. 71, No. 2 (2003), pp. 561-583

Strauss, J. 1986. "Does better nutrition raise farm productivity." *Journal of Political Economy* 94(2), pp. 297-320.

Key, Nigel, Elisabeth Sadoulet, and Alain de Janvry. "Transactions Costs and Agricultural Household Supply Response", *American Journal of Agricultural Economics*, Vol. 82, No. 2, (May 2000), pp 245-59.

**Simulation Results**

| | 2.1. Impact of a 10 percent increase in the price of cash crops | | | | 2.2. Impact of a 10 percent increase in the price of manufactured goods | | | | 2.3. Impact of a monetary head tax | | | | 2.4. Impact of a 10 percent increase in productivity of food production | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Market failures | | | | Market failures | | | | Market failures | | | | Market failures | | | |
| | Food and labor | Labor | Food | None | Food and labor | Labor | Food | None | Food and labor | Labor | Food | None | Food and labor | Labor | Food | None |
| | Percentage changes over base | | | | Percentage changes over base | | | | Percentage changes over base | | | | Percentage changes over base | | | |
| Consumption | | | | | | | | | | | | | | | | |
| Food | -0.5 | 3.0 | -0.8 | 2.1 | 1.1 | 1.8 | 1.0 | 1.7 | -4.9 | -9.1 | -4.3 | -7.0 | 8.8 | 4.5 | 8.8 | 3.0 |
| Leisure | 0.4 | 0.6 | 4.0 | 2.7 | 0.2 | 0.3 | 0.9 | 0.6 | -3.9 | -4.2 | -10.2 | -9.0 | 0.8 | 0.5 | 1.3 | 3.9 |
| Manufactured good | 15.8 | 7.7 | 9.5 | 5.6 | -12.8 | -14.5 | -14.0 | -14.8 | -33.6 | -23.6 | -22.3 | -18.7 | 1.0 | 11.4 | 0.2 | 8.0 |
| Production | | | | | | | | | | | | | | | | |
| Food crop | -0.5 | -6.4 | -0.8 | -5.4 | 1.1 | -0.2 | -1.0 | | -4.9 | 2.3 | -4.3 | | 8.8 | 16.4 | 8.8 | 18.0 |
| Cash crop | 1.8 | 9.3 | 5.5 | 9.9 | -1.7 | -0.1 | -1.0 | | 10.7 | 1.7 | 4.1 | | 0.7 | -8.8 | 1.2 | -7.7 |
| Fertilizer | 4.7 | 2.8 | 3.1 | 2.2 | 0.5 | 0.1 | 0.2 | | -3.7 | -1.4 | -0.8 | | 0.0 | 2.4 | -0.2 | 1.5 |
| Labor | -0.6 | -1.0 | 3.9 | 1.7 | -0.4 | -0.4 | 0.5 | | 5.8 | 6.3 | -2.0 | | -1.2 | -0.7 | -0.6 | 3.7 |
| Prices | | | | | | | | | | | | | | | | |
| Food crop | 8.8 | --[a] | 5.8 | | 1.9 | -- | 1.3 | | -10.7 | -- | -5.4 | | -11.0 | -- | -11.4 | |
| Cash crop | 10.0 | 10.0 | 10.0 | 10.0 | | | | | | | | | | | | |
| Fertilizer | | | | | | | | | | | | | | | | |
| Labor | 9.3 | 4.5 | | | 1.7 | 0.7 | | | -16.4 | -10.5 | | | 1.3 | 7.4 | | |
| Manufactured good | | | | | 10.0 | 10.0 | 10.0 | 10.0 | | | | | | | | |
| Net labor supply[b] | | | -10.6 | -6.1 | | | -1.7 | -0.8 | | | 15.2 | 11.7 | | | -1.7 | -9.7 |
| Marketed surplus of food[b] | | -10.1 | | -7.9 | | -2.0 | | -1.5 | | 11.2 | | 7.0 | | 10.3 | | 12.7 |

[a]Blanks indicate no change relative to base value.
[b]Net labor supply in percent of household labor effort, and marketed surplus in percent of food production.
Source: A. de Janvry, M. Fafchamps, and E. Sadoulet, "Peasant Household Behavior with Missing Markets: Some Paradox Explained", *Economic Journal,* Vol. 101, No. 409 (November 1991), pp. 1400-17.