Fall 2008 **ARE211**

PROBLEM SET #10

FIRST NPP PROBLEM SET

DUE DATE: NOV 20

(1) Consider the following maximization problem (solve it graphically):

$\max_{x_1,x_2} f(x_1,x_2)$ with $f(x_1,x_2) = -x_1$

subject to $g_1 : -x_1^3 + x_2 \leq 0$ and $g_2 : -x_1^3 - x_2 \leq 0$.

a) What is the solution to the maximization problem?

b) Are the KKT conditions satisfied for the solution to part a). If yes, write the gradient of the objective function as a postive linear combination of the gradients of the contstraints that are satisfied with equality. If not, explain why?

c) Now, slightly change the problem and let the second constraint be $g_2 : -x_1^3 - \epsilon x_1 - x_2 \leq 0$ for $\epsilon > 0$ Again, what is the solution to your problem?

d) For the revised problem in part c), is the Mantra satisfied. If yes, write the gradient of the objective function as a postive linear combination of the gradients of the contstraints that are satisfied with equality. If not, explain why?

(2) Consider the following minimization problem

$$\min_{x_1,x_2} 2x_1^2 + 2x_2^2 - 2x_1x_2 - 9x_2$$

subject to the following constraint set:

$$
\begin{aligned}
g_1 &: \quad x_1 && \geq 0 \\
g_2 &: \quad x_2 && \geq 0 \\
g_3 &: \quad 4x_1 + 3x_2 && \leq 10 \\
g_4 &: \quad -4x_1^2 + x_2 && \geq -2
\end{aligned}
$$

Write down the Lagrangian and solve the first order necessary condition.

Hint: (1) show that the cases $x_1 = 0, x_2 > 0$ and $x_1 > 0, x_2 = 0$ and $x_1 = x_2 = 0$ lead to a contradiction. Infer that a possible solution must satisfy: $x_1 > 0, x_2 > 0$. **Note well:** You can only make this inference once you have checked in each of the three cases that the constraint qualification is satisfied. Otherwise you haven't excluded the possibility that you've reached a contradiction even though a solution does exist.

(2) Look at the 4 positivity cases for λ_3, λ_4 (i.e., where each of them is strictly greater than zero or zero).

(3) Find the maximum and minimum distance from the origin to the ellipse $x_1^2 + x_1 x_2 + x_2^2 = 3$. (Hint: instead of using the distance $\sqrt{x_1^2 + x_2^2}$, maximize or minimize the square of the distance which is much easier)

a) Set up the Lagrangian and derive the first order necessary conditions.

b) Solve the first order necessary conditions.

c) Check the bordered Hessian for the second-order sufficiency conditions.