

PROBLEM SET #04

FIRST LIN ALGEBRA PROBLEM SET

DUE DATE: OCT 02

- (1) Simon & Blume question 10.5 (page 208).
- (2) Simon & Blume question 10.10 (page 220)
- (3) Suppose you know that the angle between two vectors is as given below. What do you know about the sign of the inner product of the two vectors?
 - a) 180
 - b) 53
 - c) 320
 - d) 90
- (4) Using the definition of linear independency, i.e., a set of vectors $\{v^1, \dots, v^k, \dots, v^m\}$ is a *linear independent set* if for all $\mathbf{t} \in \mathbb{R}^m$, $\sum_{k=1}^m t_k v^k = 0$ implies $\mathbf{t} = 0$, prove the following properties: (Note: once you have shown a property, you can use it to show the following ones)
 - a) A singleton vector is a linear independent set *if and only if* it is not the zero vector.
 - b) Two nonzero vectors are linearly independent *if and only if* they are not colinear (or proportional, i.e. for two vectors (u, v) , there exists $\lambda \in \mathbb{R}$ such that $u = \lambda \cdot v$)
 - c) for $n > 1$, (v^1, \dots, v^n) is a linear dependent set *if and only if* one of the vector in the set v^i is a linear combination of the other $n - 1$ vectors.
 - d) If (v^1, \dots, v^n) is a linear independent set, and y is a different vector, (v^1, \dots, v^n, y) is linearly dependent *if and only if* y is a linear combination of v^1, \dots, v^n
 - e) If (v^1, \dots, v^n) is a linear independent set, then any subset of this set (such as v^1, \dots, v^i , with $i < n$) is also linear independent.

(5) Show that if \mathbf{v}^1 and \mathbf{v}^2 are linearly independent vectors in \mathbb{R}^2 , then any vector $\mathbf{w} \in \mathbb{R}^2$ can be written as a linear combination of \mathbf{v}^1 and \mathbf{v}^2 . To show this, use *only* the fact that \mathbf{v}^1 and \mathbf{v}^2 are linear independent iff they are not colinear. (i.e., do not use properties of matrices or determinants.)

Hint #1: Try to write \mathbf{w} as $\alpha\mathbf{v}^1 + \beta\mathbf{v}^2$. Under what conditions can you solve for α and β ?

Hint #2: one way of utilizing hint #1 is to use the following rather clumsy characterization of colinearity:

$$\mathbf{v}^1 \text{ and } \mathbf{v}^2 \text{ are colinear iff } \left\{ \begin{array}{l} \text{neither } v_1^1 \text{ nor } v_1^2 \text{ is zero} \implies \frac{v_2^1}{v_1^1} = \frac{v_2^2}{v_1^2} \quad \text{or} \\ \text{neither } v_2^1 \text{ nor } v_2^2 \text{ is zero} \implies \frac{v_1^1}{v_2^1} = \frac{v_1^2}{v_2^2} \quad \text{or} \\ \text{at least one of the vectors is zero} \end{array} \right. \quad (1)$$

The second and third branches of this characterization say, respectively, that either “rise over run” or “run over rise” must be the same for the two vectors.