

## PROBLEM SET #04

## FIRST LIN ALGEBRA PROBLEM SET

DUE DATE: OCT 02

- (1) Simon & Blume question 10.5 (page 208).
- (2) Simon & Blume question 10.10 (page 220)
- (3) Suppose you know that the angle between two vectors is as given below. What do you know about the sign of the inner product of the two vectors?
  - a) 180
  - b) 53
  - c) 320
  - d) 90
- (4) Using the definition of linear independency, i.e., a set of vectors  $\{v^1, \dots, v^k, \dots, v^m\}$  is a *linear independent set* if for all  $\mathbf{t} \in \mathbb{R}^m$ ,  $\sum_{k=1}^m t_k v^k = 0$  implies  $\mathbf{t} = 0$ , prove the following properties: (Note: once you have shown a property, you can use it to show the following ones)
  - a) A singleton vector is a linear independent set *if and only if* it is not the zero vector.
  - b) Two nonzero vectors are linearly independent *if and only if* they are not colinear (or proportional, i.e. for two vectors  $(u, v)$ , there exists  $\lambda \in \mathbb{R}$  such that  $u = \lambda \cdot v$ )
  - c) for  $n > 1$ ,  $(v^1, \dots, v^n)$  is a linear dependent set *if and only if* one of the vector in the set  $v^i$  is a linear combination of the other  $n - 1$  vectors.
  - d) If  $(v^1, \dots, v^n)$  is a linear independent set, and  $y$  is a different vector,  $(v^1, \dots, v^n, y)$  is linearly dependent *if and only if*  $y$  is a linear combination of  $v^1, \dots, v^n$
  - e) If  $(v^1, \dots, v^n)$  is a linear independent set, then any subset of this set (such as  $v^1, \dots, v^i$ , with  $i < n$ ) is also linear independent.

- (5) Show that if  $\mathbf{v}^1$  and  $\mathbf{v}^2$  are linearly independent vectors in  $\mathbb{R}^2$ , then any vector  $w \in \mathbb{R}^2$  can be written as a linear combination of  $\mathbf{v}^1$  and  $\mathbf{v}^2$ . (Hint #1: you will need to use the fact that  $\mathbf{v}^1$  and  $\mathbf{v}^2$  are linear independent iff they are not colinear. For the purposes of this particular computation, the most useful representation of colinear is the following extremely clumsy one:

$$\mathbf{v}^1 \text{ and } \mathbf{v}^2 \text{ are colinear iff } \left\{ \begin{array}{ll} \text{either vector is zero} & \text{or} \\ v_1^1 = v_1^2 \neq 0 \text{ and } \frac{v_2^1}{v_1^1} = \frac{v_2^2}{v_1^2} & \text{or} \\ v_2^1 = v_2^2 \neq 0 \text{ and } \frac{v_1^1}{v_2^1} = \frac{v_1^2}{v_2^2} \end{array} \right.$$

Hint #2: Try to write  $\mathbf{w} \in \mathbb{R}^2$  as  $\alpha\mathbf{v}^1 + \beta\mathbf{v}^2$ ; find out under what conditions you can solve for  $\alpha$  and  $\beta$ .)