

PROBLEM SET #11

FIRST COMPSTAT PROBLEM SET

DUE DATE: DEC 11

- (1) In problem 3 on your last problem set, you found the maximum and minimum distance from the origin to the ellipse $x_1^2 + x_1x_2 + x_2^2 = 3$. Generalize this problem to “minimize/maximize the distance from the origin to the ellipse $x_1^2 + x_1x_2 + \alpha x_2^2 = 3$ ” and use the Envelope theorem (starting from $\alpha = 1$) to estimate the maximum and minimum distance from the origin to the following ellipse,

$$x_1^2 + x_1x_2 + 0.9x_2^2 = 3$$

- (2) a) Prove that the expression $\alpha^2 - \alpha x^3 + x^5 = 17$ defines x implicitly as a function of α .
in a neighborhood of $(\bar{\alpha}, \bar{x}) = (5, 2)$
- b) Estimate the x -value which corresponds to $\alpha = 4.8$ using a first order approximation.
- (3) Consider the equation $\alpha_1^3 + 3\alpha_2^2 + 4\alpha_1x^2 - 3x^2\alpha_2 = 1$. Does this equation define x as an implicit function of α_1, α_2
- a) in a neighborhood of $(\bar{\alpha}_1, \bar{\alpha}_2) = (1, 1)$
- b) in a neighborhood of $(\bar{\alpha}_1, \bar{\alpha}_2) = (1, 0)$
- c) in a neighborhood of $(\bar{\alpha}_1, \bar{\alpha}_2) = (0.5, 0)$
- If so, compute $\frac{\delta x}{\delta \alpha_1}$ and $\frac{\delta x}{\delta \alpha_2}$ at this point.

(4) The economy of Iceland can be expressed by the following three variables:

x : hard-core liquer to survive dark cold winters

y : imported oil for heating in the cold winter

z : beaver pelts for people crazy enough to leave the house

The equilibrium is given by the two equations:

$$2xz + xy + z - 2\sqrt{z} = 11 \quad (1)$$

$$xyz = 6 \quad (2)$$

Assume the economy finds itself at the initial equilibrium where $x = 3, y = 2, z = 1$. The government fixes the number of allowances to hunt beaver exogenously.

- a) If the prime minister raises z to 1.1, calculate the change in x and y .
- b) The anti-drug alliance argues that too much alcohol is consumed in the country. They therefore argue to fix the amount of alcohol consumed exogenously by law at 2.95 and instead abolish the beaver hunting constraint. Can they use the implicit function theorem to estimate the changes in y and z ?

(5) Assume you are given a competitive industry of identical firms with open entry. (i.e., two equations characterize the system: (1) producers are profit maximizers and the derivative of the price w.r.t a firm's own quantity is zero $\frac{\delta p(Q)}{\delta q_i} = 0$ and (2) new entry into the market occurs until profits are zero).

The government is thinking about imposing a lump-sum tax on firms. What would be the effects on a firm's individual output q , its profit π , the total market output Q , the price $p(Q)$ and the number of firms in the industry n ?

This problem involves one of the many internal contradictions that economists make all the time. First you assume that $\frac{\delta p(Q)}{\delta q_i} = 0$, i.e., that individual firms face perfectly elastic demand curves. Then you assume that the industry faces a downward sloping demand curve, i.e., that $\frac{dp(Q)}{dQ} < 0$. Mathematically this is of course absurd, since $Q = nq_i$, but economists have been doing this shamelessly for centuries. There's a way of "fixing" this contradiction, but we're not going to get into it. So just do it and shudder quietly to yourselves.