

PROBLEM SET #07

FIRST CALCULUS PROBLEM SET

DUE DATE: OCT 30

Please note: There are parts of one question in this assignment to which you may want to reply: “I beg your pardon?” or something possibly ruder than this. That kind of response is entirely appropriate, but you should add a sentence explaining why you are led to make such a remark.

- (1) Consider the function $f(x) = 20x^3 - 120x^2 - 5x + 36$
 - a) What is the derivative of this function?
 - b) What is the derivative of this function evaluated at $x_0 = 4$?
 - c) What is the differential of this function at $x_0 = 4$?
 - d) Approximate the change in the function when moving from $x_0 = 4$ to $x_1 = 5$.
 - e) What is the actual change in the function when moving from $x_0 = 4$ to $x_1 = 5$?
- (2) Multivariate calculus drill: Simon and Blume, question 14.1, parts (a), (d) and (f);
- (3) Gradients: Simon and Blume, question 14.19
- (4) More differential approximations: Simon and Blume, question 14.4.
- (5) Still more differential approximations: Simon and Blume, question 14.8b.
- (6) And one more: Simon and Blume, question 14.10
- (7) Consider the function $f(\mathbf{x}) = x_1x_2$. Let $\mathbf{x}_0 = (3, 2)$ and $\mathbf{h} = (3, 4)$.
 - (a) Directly compute the directional derivative of f at \mathbf{x}_0 in the direction \mathbf{h} by the method used in class and in the lecture notes.
 - (b) Indirectly compute the directional derivative of f at \mathbf{x}_0 in the direction \mathbf{h} using the differential
 - (c) Do both computations give the same answer. If not, why not?

(8) Consider the function $f(x, y) = \begin{cases} \frac{xy}{\sqrt{x^2+y^2}} & \text{if } (x, y) \neq (0, 0) \\ 0 & \text{if } (x, y) = (0, 0) \end{cases}$

(a) Using the chain rule, compute $f_x(x, y)$, for $(x, y) \neq (0, 0)$.

(b) Compute $f_x(0, 0)$, using the following definition: $f_x(x, y) = \lim_{t \rightarrow 0} \frac{f((x, y) + (t, 0)) - f(x, y)}{t}$.

(c) Compute the directional derivative of f at $(0, 0)$ in the direction $(3, 3)$, using the method presented in class (and the lecture notes).

(d) Write down $f_y(0, 0)$ and evaluate the expression $\nabla f(0, 0) \cdot (3, 3)$.

(e) For $x_0 = y_0 = 0$ and $\mathbf{h} = (3, 3)$, what can be said about the expression

$$\lim_{|k| \rightarrow \infty} \frac{(f((x_0, y_0) + \mathbf{h}/k) - f(x_0, y_0)) - \nabla f(x_0, y_0) \cdot \mathbf{h}/k}{\|\mathbf{h}\|/k}?$$

(f) What can you conclude about the differentiability or otherwise of f ?

(g) With x on the horizontal axis, plot $f_x(x, x)$, for $x \in [-1, 1]$.

(i) From your graph, what can you say about the continuity or otherwise of $f_x(\cdot, \cdot)$ at $(0, 0)$.

(ii) From your graph, what can you say about the continuity or otherwise of $f_x(0, 0)$.

(iii) Explain why your answers to 8(g)i and 8(g)ii are different.

(iv) What is the relationship between your answers to 8(g)i and 8(g)ii and your answer to 8f?