Problem Set #04

Fourth Analysis Problem Set

Due date: Oct 06

Thanks to Bob Anderson for allowing me to crib the following old exam questions...

Problem 1

Let Ψ be a correspondence from X to Y which is compact-valued and upper hemicontinuous, and let C be a compact subset of X. Let $\Psi(C) = \cup_{x\in C}\Psi(x)$.

a) Using one of the three definitions of upper-hemi-continuity given in class, prove that $\Psi(C)$ is compact.

b) Using a second, different one of the three definitions of upper-hemi-continuity given in class prove that $\Psi(C)$ is compact.

Problem 2

Let $F : C \times \mathbb{R}^m \to \mathbb{R}^n$ be a continuous function, where $C \subset \mathbb{R}^n$.

a) Given $\epsilon > 0$, define

$$\Psi_\epsilon(\omega) = \{x \in \mathbb{R}^n : |F(x,\omega)| < \epsilon\}$$

Show directly from the definition of Ψ that for each $\epsilon > 0$, Ψ_ϵ is a lower hemicontinuous correspondence.

b) Let

$$\Psi(\omega) \quad = \quad \{x \in \mathbb{R}^n : F(x, \omega) = 0\}$$

Show directly from the definition of Ψ that if C is compact, then Ψ is an upper hemicontinuous correspondence.

c) Construct a counter-example proving that if C were not compact, then Ψ would not necessarily be upper hemicontinuous. (Hint: look at the lecture notes.)

Problem 3

Suppose $F : X \times \Omega \to \mathbb{R}$ is continuous, where X is a compact subset of $\mathbb{R}^m$ and $\Omega \subset \mathbb{R}^n$. Define

$$\Psi(\omega) \quad = \quad \{x \in X : F(x, \omega) = \sup\{F(z, \omega) : z \in X\}$$

a) Show that for all $\omega \in \Omega$, $\Psi(\omega) \neq \emptyset$.

b) Show that Ψ is upper hemicontinuous.

c) Construct a counter-example proving that if X were not compact, then Ψ would not necessarily be upper hemicontinuous.