

1 Ricardian model

The Ricardian model provides the simplest setting to illustrate comparative advantage and the gains from trade in a general equilibrium setting. This chapter has the following objectives:

- Make sure that everyone knows what comparative advantage means. *Definition of comparative advantage*: A country (e.g. Canada) has a comparative advantage in the production of a commodity (e.g. corn) if the ratio between its pre-trade marginal costs of that commodity (corn), and its pre-trade marginal cost of producing “the other” commodity (umbrellas) is lower than that of its trading partner.
- Show how you use equilibrium conditions (here, zero profit conditions) to determine endogenous variables (here, relative prices).
- Provide a simple setting in which to understand the relation between commodity prices and factor prices.
- Understand the meaning of "real returns to a factor" and show how trade affects this "real return".
- Put all of this together in order to be able to do comparative statics experiments. In particular, show how a change in technology affects equilibrium welfare.

I will use a simple example to discuss the Ricardian model. Two countries, Canada and the US produce two commodities, corn and umbrellas. (The obvious alliteration is a mnemonic device.)

1.1 Technology and markets

The Ricardian model assumes that production uses only 1 input (labor), with constant returns to scale. This assumption means that the technology in each country and each sector is entirely determined by the labor requirement per unit of output. The other assumptions are that (a) labor moves freely between sectors within a country, but (b) labor cannot move between countries. Assumption (a) implies that in a particular country, the wage must be the same in both sectors;

assumption (b) means that the wage need not be the same (and typically is not the same) in the two countries. In addition, all agents are price takers, i.e. there is perfect competition.

In my example, the unit labor requirements are

	unit labor requirement	
	Corn (good 1)	Umbrellas (good 2)
US	$a_1^u = 1$	$a_2^u = 1$
Canada	$a_1^c = 3$	$a_2^c = 6$

Table 1, Labor requirements

(Corn is good 1, umbrellas are good 2. Subscripts indicate commodity, superscripts indicate country.) I assume that both goods require one unit of labor to produce one unit of output in the US. This assumption is without loss of generality; it merely amounts to a choice of units.

Production of both goods require less labor in the US than in Canada. In this sense, the US has an absolute advantage in the production of both goods. I make this assumption not because of any anti-Canadian bias, but in order to emphasize that comparative and absolute advantage are distinct concepts. The gains from trade are *not related* to absolute advantage. In this and in more general models, a “country” benefits from trade if and only if the relative prices at which it can trade differ from the equilibrium autarkic relative prices. In this particular example of the Ricardian model, Canada is less productive than the US, and in that sense is at a disadvantage. However, at least one country, and possibly both countries gain from trade.

Note that I speak of a “country” as benefiting from trade. In this model there is a single homogenous factor of production means; this fact means that there is no distinction between the benefit to individuals and the “benefit to the country”. We return to this issue later, where it is important to recognize that trade liberalization, as with many other policy changes, has different effects for different types of agents.

In the numerical example above, the relative production costs in the two countries are different. This fact means that the efficient pattern of production requires at least one country to specialize. To convince yourself of this conclusion, suppose to the contrary that both countries produce both goods. How should a social planner reallocate labor to increase output of both goods?

The *opportunity cost* of producing corn, for example, is the number of umbrellas that need to be sacrificed to obtain an additional unit of corn.

$$\begin{aligned} \text{US corn opportunity cost} &= \frac{a_1^u}{a_2^u} = \frac{1}{1} > \\ \text{Canadian corn opportunity cost} &= \frac{a_1^c}{a_2^c} = \frac{3}{6} \end{aligned}$$

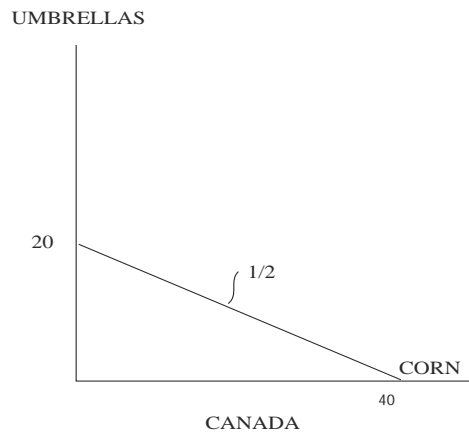


Figure 1

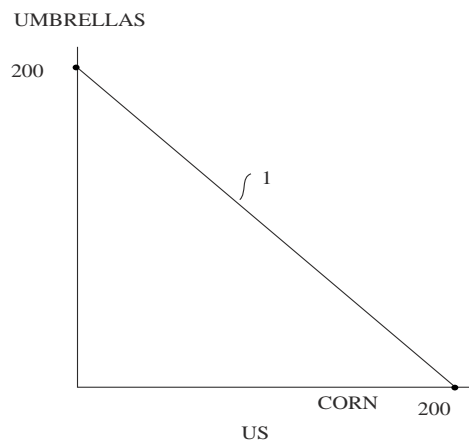


Figure 2

In Canada, the opportunity cost of producing one unit of corn is the loss of 1/2 umbrella. The extra unit of corn requires three units of labor, leading to the loss of 1/2 unit of umbrella. In the United States, the opportunity cost of producing 1 unit of corn is the loss of 1 umbrella. Canada has a lower opportunity cost of producing corn.

The statement that Canada has a lower opportunity cost of corn (something that is determined only by the relation between relative – as distinct from absolute – input requirements) is equivalent to the statement that Canada has a comparative advantage in corn. Recall the definition of comparative advantage above. The marginal cost of producing corn in Canada is $w^c a_1^c$ and the marginal cost of producing umbrellas is $w^c a_2^c$. Obviously the ratio of these marginal costs depends only on the technological parameters, the labor input requirements – not on the wage. Therefore, the technology in the two countries determines the pattern of comparative advantage. In a more general model, the technology is one of several factors that might determine the pattern of comparative advantage.

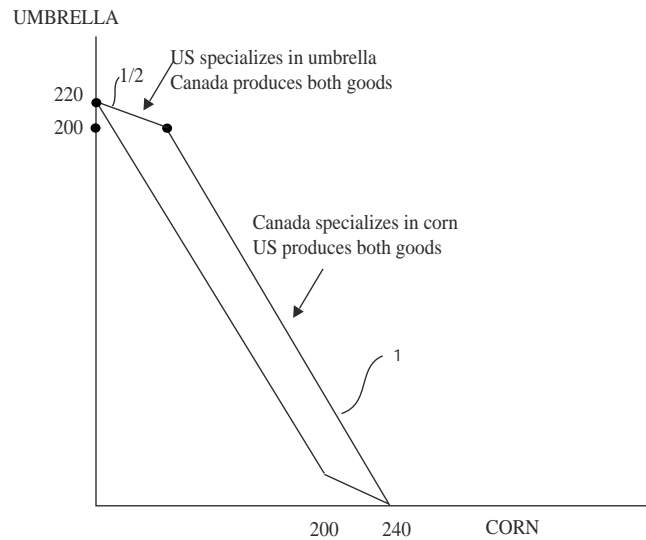


Figure 3

In the Ricardian model, where the only factor of production is labor, knowledge of the labor input coefficients and of the total stock of labor, enable us to obtain the production possibility frontier. For this example, suppose that the amount of labor in Canada is $L^c = 120$ and the amount of labor in the US is $L^u = 200$. Denoting the amount of corn and umbrella production as c and u , the full employment condition in Canada is $3c + 6u = 120$, and the full employment condition in the US is $c + u = 200$. Figures (1) and (2) graph these equations, the production possibility frontiers (PPF) for the two countries. The simplicity of this example makes it useful as an illustration of how various equilibrium conditions – here, the full employment condition – are used to construct an equilibrium. (The figures are not to scale. I use a wavy curve to indicate the slope of a line.)

The slope of the PPF depends on the ratio of labor input requirements, not the absolute amounts of the requirements. The marginal rate of transformation of umbrellas for corn is $\frac{a_1}{a_2}$. The units of this ratio are

$$\text{units} \left(\frac{a_1}{a_2} \right) = \frac{\frac{\text{labor}}{\text{corn}}}{\frac{\text{labor}}{\text{umbrella}}} = \frac{\text{umbrella}}{\text{corn}}. \quad (1)$$

The trapezoid in Figure (3) shows the combinations of world output that result in full employment. At any point on the trapezoid, labor is fully employed in both countries. The "outer part" gives the world production possibility frontier (PPF), the set where production is efficient. On the PPF Canada always produces corn and US always produces umbrellas, the commodities for which they have a comparative advantage. On the "inner part" of the trapezoid production is inefficient. There Canada always produces umbrellas and US always produces corn.

Note that the full employment conditions lead to the trapezoid shape because of our as-

sumption that the ratio of input requirements $\frac{a_1}{a_2}$ is different in the two countries. If this ratio was the same in the two countries, the trapezoid would collapse to a straight line.

1.1.1 Commodity Prices

Economic modeling uses equilibrium conditions to determine endogenous variables. I noted above that the full employment condition is used to construct the production possibility frontier. Now we consider the determination of prices, where the relevant equilibrium conditions are the zero-profit conditions. Again, it is simple in this context to understand the use of these conditions to determine equilibrium relative prices. In some cases, neither the relevant equilibrium condition nor the way it which it is used is so obvious. Treat the current exercise as an example of how to proceed. You need to understand what you are looking for (here, the equilibrium relative prices), what the equilibrium conditions are (here, the zero profit conditions) and how to use them (here, simple algebra).

In the Ricardian model, autarkic relative prices depend on technology, but not on preferences, provided that both goods are produced in equilibrium. (“Autarkic” means “no trade”.) Hereafter we will assume that in equilibrium both commodities are consumed. Since national production equals national consumption under autarky, both countries are incompletely specialized in autarky. Trade breaks the equality between national production and national consumption.

Let p_1 be the price of corn and p_2 the price of umbrellas. The wage is w^c, w^u in Canada and the US. (Remember, labor is only input in the Ricardian model.) What are the autarkic prices in Canada? Suppose that both goods are produced in autarky. With perfect competition and constant returns to scale, profits (rents) must be 0. We use the 0-profit conditions to find the equilibrium prices in Canada.

$$p_1^c - w^c a_1^c = 0 \tag{2}$$

$$p_2^c - w^c a_2^c = 0.$$

Denote p^c as the relative price of corn (the price of corn relative to the price of umbrellas) in autarky in Canada. Taking the ratio of these 0-profit conditions gives

$$p^c = \frac{p_1^c}{p_2^c} = \frac{a_1^c}{a_2^c} = \frac{1}{2} \tag{3}$$

In this model, as in most economic models, only relative prices matter. For example, if we double all commodity prices and also double the wage, all relative prices remain the same; nothing of substance has changed. We therefore have one degree of freedom. We can set one price equal to 1 without loss of generality. It does not matter which commodity or factor we chose as the numeraire. (In autarky there is only one factor, but with trade there are two factors,

labor in the US and labor in Canada. These are distinct factors because of the assumption that there is no international labor mobility.)

The units of the relative price are (using equation (1))

$$\text{units} \left(\frac{p_1^c}{p_2^c} \right) = \frac{\frac{\text{dollars}}{\text{corn}}}{\frac{\text{dollars}}{\text{umbrella}}} = \frac{\text{umbrellas}}{\text{corn}} = \text{units} \left(\frac{a_1^c}{a_2^c} \right). \quad (4)$$

I have made a point of identifying the units of a ratio for a couple of reasons. First, it is easy to get confused about whether we are talking about a ratio like $\frac{p_1}{p_2}$ or its inverse. If you check the units you can usually figure it out. (Checking the units also provides a way of doing a “consistency check” for computations.) Second, equation (4) shows that the units of relative prices are the same as the units of the marginal rate of transformation (MRT, the slope of the production possibility frontier). We know that this relation has to hold, because in equilibrium the relative price has to equal the MRT.

Given incomplete specialization, *the autarkic relative price is determined by technology in the Ricardian model*; this is a consequence of the very special production technology – it is not true in more general models.

The autarkic relative price in the US is

$$p^u = \frac{p_1^u}{p_2^u} = \frac{a_1^u}{a_2^u} = \frac{1}{1}.$$

Canada has a comparative advantage in corn if and only if the Canadian autarkic relative price of corn is lower than the U.S. autarkic relative price. This inequality holds in my example here.

1.2 Relative prices under free trade

The equilibrium relative price under trade is the price that actually arises when the two countries trade. With trade, the equilibrium relative price depends not only on technology (as is the case under autarchy), but also on demand. Demand depends on taste, i.e. on preferences, as well as on prices and income. In Chapter 2 we will discuss how the equilibrium price is determined. For the time being, we take this price as given. Depending on the equilibrium price, world production could occur on regions A or C or at point B, on the production possibility frontier shown in figure (4). I want to describe the pattern of production and trade, and the levels of welfare (relative to the autarkic levels) in these three regions.

Assume that there are 0 transportation costs and no government interventions in the market, e.g. no commodity taxes or trade restrictions. These assumptions imply that under trade, the relative price *is the same in both countries*. The (assumed) difference between the autarkic relative prices is the reason for trade. Note that the inequality in relative autarkic prices is source of the gains from trade (the “reason” for trade). The equality of relative prices under

trade is a *consequence* of trade. If, under free trade, the relative prices within the two countries were different then there would be further opportunities for arbitrage. People could make money by buying a commodity where it is cheap and selling it where it is expensive. Free trade eliminates these opportunities for arbitrage.

This distinction between the “reason for trade” and the “consequence of trade” is quite obvious, but similar distinctions crop up elsewhere in equilibrium problems where it is less obvious, e.g. in a model in Chapter 4 of these notes. Once you recognize the distinction, you would not ask “Why is there trade, given that the (relative) prices in the two countries are the same?” You know that the (relative) prices in the two countries are same precisely because there is trade – and given the assumptions that there are no transport costs or government intervention.

The *levels* of the two commodity prices (as distinct from their ratios) are related by the exchange rate, e , defined as the number of units of Foreign currency that exchanges for a unit of Home currency (e.g. the number of pesos per dollar). The Home and Foreign price of commodity i are related by $p_i = ep_i^*$. (It is a convention to use the "*" to denote Foreign variables.) Equality of relative prices implies equality of price *levels* if and only if the exchange rate is $e = 1$. There is no loss in generality in assuming that the exchange rate is 1. For example, suppose that initially $e = 0.1$, so one dollar equals 10 pesos; invent a new unit of currency, say the peseta, where one peseta equals 10 pesos. With this invention, one dollar equals one peseta, and the exchange rate with this new unit of currency is equal to 1. Therefore, there is no loss in generality in speaking as if the *levels* of commodity prices are also equal when the relative prices are equal; it is nothing more than a “normalization”, i.e. a choice of units. Hereafter I will use this normalization.

Eventually we will be interested in determining the actual levels of production in the two countries. For the time being, we put that question aside and merely ask what must be true under the assumption that in equilibrium production occurs on regions A or C or at point B. We consider these possibilities in turn.

First consider Region A, where Canada is incompletely specialized and the US produces only umbrellas. In constant, in Region C Canada produces only corn and the US is incompletely specialized. At Point B both countries are completely specialized. (Point B is also consistent with complete specialization by one, but not by both countries. This generalization is not important, and I will hereafter assume that both countries are completely specialized at point B.)

In the subsection above, I stated the 0-profit condition using the equation “price = average cost”. There is another way in which profits can be zero in a sector: the sector can shut down. Thus, the complete description of the 0-profit condition is “price = marginal cost if production is positive, and price is less than or equal to marginal cost when production is 0”.

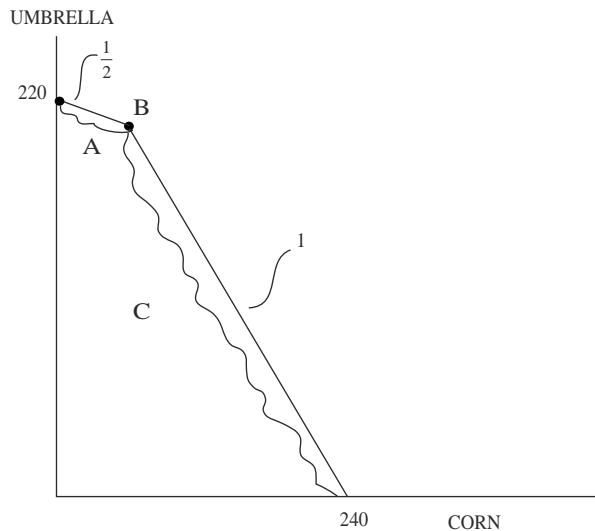


Figure 4

(This kind of relation is sometimes described as a “complementary slackness” condition.) The only possibility excluded by the 0-profit condition is that price is strictly greater than average costs.

Now we want to confirm (just as an exercise!) that the full 0-profit conditions hold at each point on the world production possibility frontier. Consider a free trade equilibrium in region A. In this region, Canada is incompletely specialized. As we saw above, the equilibrium conditions (the 0 profit conditions) requires that if Canada is incompletely specialized, then the relative price of corn in Canada must be $\frac{1}{2}$. (See equation (3)). Under free trade (given the assumptions above about 0 transport costs and the absence of government tax intervention) relative prices are equal in the two countries. Therefore, if at the equilibrium production occurs in region A, the world relative price of corn must be $\frac{1}{2}$ in both countries.

To confirm that this relative price "supports" production in region A, we need to make sure that at this price it is an equilibrium for the US to be completely specialized. Recall that I can choose any good or factor as the numeraire. I will choose the U.S. wage as numeraire, i.e. set $w^u = 1$. Then in Region A, where the US produces umbrellas, 0 profits in the umbrella sector requires

$$p_2 = w^u a_2^u = 1. \quad (5)$$

Using equation (5) and the fact that in Region A the relative price of corn is $1/2$ implies $p_1 = \frac{1}{2}$. In Region A the US does not produce corn, because profits in the corn sector in the US would be negative (therefore the sector shuts down in the US). Our calculations show that profits in

the corn sector in the US are indeed negative:

$$p_1 = \frac{1}{2} < w^u a_1^u = 1.$$

Using the same logic you can show that in Region C, where the US is incompletely specialized, the equilibrium world relative price of corn is 1. This is the only price at which, 0 profits in the US are consistent with production of both commodities. At this price, 0 profits in Canada require that the umbrella sector shuts down.

At point B the relative world price of corn is between 1 and $\frac{1}{2}$ so both countries are specialized. (I am ignoring the knife-edge case where production occurs at point B and one-country is incompletely specialized.)

We see that there is a simple relation between the equilibrium world price and the type of production equilibrium (i.e., whether production occurs on regions A or C or at point B).

1.3 Wages in the two countries

We want to know the relation between US and Canadian wage under trade. This relation depends on the equilibrium world price, which we saw above is related to the equilibrium levels of production under trade. Again, I am taking as given the equilibrium world price for this exercise, and asking how the relation between the wages in the two countries varies with the price. As above, use the normalization $w^u = 1$. In Region A we saw that this normalization implies $p_2 = 1$ (equation (5)). The zero profit condition for Canada, equation (2), implies $w^c = \frac{1}{6}$. Thus, the Canadian wage is 1/6th the US wage if production occurs in Region A. Both countries produce umbrellas (in Region A), which under free trade (with zero transport costs) sells for the same price. Since the US is six times as productive as Canada in the umbrella sector, the US equilibrium wage must be six times as large as the Canadian wage. We summarize the equilibrium prices in Region A, using the normalization $w^u = 1$:

$$p_1 = \frac{1}{2}, \quad p_2 = 1, \quad w^c = \frac{1}{6}, \quad w^u = 1. \quad (6)$$

The last equality is due to our choice of numeraire; it is a normalization.

Similarly, if production occurs in Region C, the US wage is three times as large as the Canadian wage. If production occurs at point B, the US wage is between three and six times as large as the Canadian wage, depending on the exact equilibrium price (which depends on tastes).

The fact that the US wage is always higher than the Canadian wage is due to the assumption that the US has an absolute advantage in both commodities – it can produce either commodity using less labor, compared to Canada. By changing the technological coefficients in my example, you should convince yourself that matters are different if each country has an absolute

advantage in one commodity. In that case, as we allow the world price to range between the two extremes (these being the autarkic prices in the two countries) either country might have the higher wage. You should be able to figure out the world price at which each country has the higher wage.

1.4 Trade

Recall that we assumed that in equilibrium both countries consume a positive amount of each good. This assumption implies (for this example) that in equilibrium Canada exports corn and the US exports umbrellas. More generally, countries export the commodity for which they have the comparative advantage. This conclusion does not depend on the world equilibrium price, i.e. it does not depend on whether production is on A,B or C. For example, in Region A, the US is specialized in umbrellas, so it must import Corn, which Canada must export. To pay for its imports, the US must export umbrellas.

The Ricardian model demonstrates a very simple but profound point: Providing that two countries have different *relative* labor input requirements (i.e., providing that the ratio $\frac{a_1}{a_2}$ differs in the two countries), each country has a comparative advantage in one good. This fact does not depend on whether a country has an *absolute* advantage in either good. In equilibrium the wage adjusts so that the *production cost* (i.e., the cost in dollars) of a commodity is lower (more precisely: “not higher”) in the country that has the lower *opportunity cost* for that commodity.

For example, consider again Region A, and use the normalization $w^u = 1$ and the resulting equilibrium prices summarized in equation (6). In Canada it costs $\$ \frac{3}{6} = \0.5 to produce a unit of corn. In equilibrium (in Region A) the US does not produce corn, but if it were to do so at the prevailing US wage, it would cost \$1. Corn is cheaper to produce in Canada. Since (in Region A) both countries produce umbrellas, the dollar cost of production of umbrellas must be the same in the two countries. Similarly, in Region C the dollar cost of producing umbrellas is lower in the US than in Canada (despite the fact that the US wage is three times higher); the dollar cost of producing corn is the same in the two countries. At point B, where both countries are completely specialized, the dollar cost of producing corn is lower in Canada, and the dollar cost of producing umbrellas is lower in the US.

1.5 National income

In a competitive equilibrium, profits are 0. Because of the assumption of constant returns to scale, all revenue from the sale of products is returned to the factors of production. In this model the only factor of production is labor, so national income equals labor income. Therefore, national income in a country is wL , where L is the stock of labor.

"Real" income is a measure of the amount that the country is able to consume. Thus, real income depends both on nominal income (the number of dollars that the country earns) and the commodity prices. In the Ricardian model, since labor income is equal to national income, real income is equal to the real wage times the amount of labor. I am holding the aggregate amount of labor in each country fixed in all of the comparative static experiments considered here. Therefore, in order to know whether the country's real income has increased as a consequence of some exogenous change, we need to know how this change alters the real wage.

1.6 The real wage

Suppose that each worker sells one unit of labor. The *nominal income* of this worker is therefore equal to the *nominal wage*, e.g. the salary per month (the thing that I denote as w). The *real income* (or equivalently the *real wage*)¹ of the worker depends on the quantity of commodities that she is able to buy, and this depends on the prices that she faces.

This subsection makes two points. The first point is that the real wage is a function of the ratios of the wage and the commodity prices, $\frac{w}{p_i}$. The second point is that a movement from autarchy to trade increases the real wage if and only if the relative commodity price under trade is *different* than the autarchic relative commodity price.

There are two general approaches to defining the real wage. We will be interested only in determining whether an exogenous change increases or decreases welfare. For this purpose, the two ways of defining the real wage are equivalent.

One method defines the real wage as the number of "consumption bundles" that an agent can purchase by selling one unit of labor. For example, a consumption bundle in the two-commodity setting can be written as (a, b) where a, b are non-negative numbers; one bundle consists of a units of commodity 1 and b units of commodity 2. n consumption bundles consist of an units of commodity 1 and bn units of commodity 2.

The second method defines the real wage as the amount of "utility" that an individual can obtain by selling one unit of labor.

To use the first definition, I need to specify what constitutes a "consumption bundle", i.e. the values of a and b ; to use the second, I need to specify the utility function. Using the first definition, we have

Remark 1 *A necessary and sufficient condition for the real wage to increase, for any consump-*

¹Why are the real wage and the real income "equivalent"? Because I chose units so that each worker has one unit of labor to sell. Of course, if some workers have more labor to sell than others, their real wage is the same but their real income differs. If a worker sells ten units of labor then her nominal income is 10 times the nominal wage.

tion bundle (i.e. for any nonnegative values of a and b) is that $\frac{w}{p_i}$ strictly increases for at least one i , and does not fall for either i .

Using the second definition, we have

Remark 2 A necessary and sufficient condition for the real wage to increase, for any well behaved utility function is that $\frac{w}{p_i}$ strictly increases for at least one i , and does not fall for either i .

These Remarks mean that regardless of the approach that I take to construct the real wage, determining unambiguously the change in the real wage requires knowing that both $\frac{w}{p_i}$ (weakly) decrease or both (weakly) increase. (“Unambiguously” in the previous sentence means that the conclusion does not depend on the specific utility function or consumption bundle.) That is, the real wage is a nondecreasing function of $\frac{w}{p_i}$, $i = 1, 2$:

Problem Set 1 asks you to confirm these two Remarks. These exercises give you practice in totally differentiating equilibrium conditions to obtain comparative statics results. Proving Remark 2 also provides a useful review of the definition of and the properties of the indirect utility function (Section , and a reminder of the Envelope Theorem.

To repeat the main point: Whichever approach I take to defining the real wage, I can conclude that the real wage increases following an exogenous change if and only if $\frac{w}{p_i}$ increases for one i and does not decrease for the other i . I can conclude that the real wage decreases following an exogenous change if and only if $\frac{w}{p_i}$ decreases for one i and does not increase for the other i . If the two ratios change in different directions (one strictly decreases and the other strictly increases) the change in the real wage is ambiguous. In that case, I would need to make additional assumptions in order to determine the effect of the change on the real wage.

Recall that this model assumes perfect competition and constant returns to scale. Therefore profits (or rent) are zero, and the firm’s revenue equals its payments to factors. Since there is only one factor in this model (labor), workers receive all revenue. Therefore (in this model) *social welfare increases if and only if the real wage increases*. (Spend a moment to consider how this conclusion would change if there were pure rents, or more than one factor of production, or another source of revenue such as a revenue from tariffs.)

In the Ricardian model a nation gains from trade if and only if the relative commodity price at which it trades is *different* from its autarkic relative price. If the relative price at which it trades is the same as its autarkic relative price, welfare is the same under autarky and free trade. We illustrate this claim by considering Region A, where we have already determined equilibrium prices and wages under trade, in equation (6). For Canada, the zero profit conditions in equation (2) must still hold, since Canada produces both commodities.

Therefore the ratio

$$\frac{w^c}{p_i} = \frac{1}{a_i^c}$$

is independent of the nominal price and wage. A movement from autarky to free trade does not alter the technological parameters a_i^c . Consequently, the Canadian real wage (and thus, social welfare) is the same under autarky and under free trade when production occurs on A.

For the US, the autarkic ratios are

$$\frac{w^{u,autarky}}{p_i} = \frac{1}{a_i^u}.$$

Under trade, when production occurs in Region A, the ratios are

$$\frac{w^{u,trade}}{p_1} = \frac{1}{1/2} = 2 > 1 = \frac{w^{u,autarky}}{p_1} \tag{7}$$

$$\frac{w^{u,trade}}{p_2} = \frac{1}{a_2^u} = \frac{w^{u,autarky}}{p_2}.$$

The first line of equation (7) states that the movement from autarky to trade has increased $\frac{w^u}{p_1}$; the second line states that the movement has left $\frac{w^u}{p_2}$ unchanged. Therefore, the US real wage is higher under trade (when production is at A) than under autarky. Note that here I am comparing the real wage in a particular country under autarky and under trade. In the section “the relative wage” I compared the US and the Canadian nominal wages.

1.6.1 Review of Indirect Utility Function

The indirect utility function $V(p_1, p_2, y)$ is defined as

$$\begin{aligned} V(p_1, p_2, y) &= \max U(x_1, x_2) \\ \text{subject to } p_1x_1 + p_2x_2 &\leq y, \end{aligned}$$

where y is the individual’s income and U is the utility function. The Lagrangian is

$$\mathcal{L} = U(x_1, x_2) + \lambda(y - p_1x_1 - p_2x_2)$$

where the constraint multiplier λ is the shadow value of income. The indirect utility function is

$$V(p_1, p_2, y) = \mathcal{L}^*(p_1, p_2, y) = \max_{x_i} \min_{\lambda} \mathcal{L},$$

where a superscript * denotes the value of a function evaluated at the optimum.

The indirect utility function is homogenous of degree 0 in prices and income, i.e. doubling all prices and income leaves the individual no better or worse off. This fact enables us to write the indirect utility function in a number of ways. For example, if an individual’s entire income

derives from labor, and they sell one unit of labor, then $y = w$, the wage per unit time. In this case, we can write the indirect utility function as

$$V(p_1, p_2, y) = V(p_1, p_2, w) = V\left(\frac{p_1}{w}, \frac{p_2}{w}, \frac{w}{w}\right) \equiv v\left(\frac{p_1}{w}, \frac{p_2}{w}\right).$$

That is, the indirect utility depends on the ratio between output price and the nominal wage. Taking the derivative of the last expression with respect to these relative prices, provides an almost immediate proof of Remark 2.

It is often convenient to choose one of the output prices as numeraire. For example, if commodity 1 is the numeraire then the relative price $p = \frac{p_2}{p_1}$ gives the number of units of commodity 1 that can be exchanged for one unit of commodity 2 in the market; this ratio therefore gives the price of commodity 2 in units of commodity 1. The ratio $Y = \frac{y}{p_1}$ gives the number of units of commodity 1 that can be bought with nominal income y . We can write the indirect utility function as

$$V(p_1, p_2, y) = V\left(\frac{p_1}{p_1}, \frac{p_2}{p_1}, \frac{y}{p_1}\right) = V(1, p, Y) \equiv v(p, Y).$$

We can write the total derivative of the last function as

$$dv = v_p dp + v_Y dY.$$

Dividing by v_Y we obtain

$$\frac{dv}{v_Y} = \frac{v_p}{v_Y} dp + dY = -C_2 dp + dY, \quad (8)$$

where the last equality uses consumption of good 2 = C_2 , by virtue of Roy's equality (also known as Roy's identity). The units on both sides of the equation are money, e.g. Euros. Since Y is in units of money, dY is obviously in units of money (e.g. Euros).

The ratio $\frac{dV}{V_y}$ measures the change in real income in units of money. To see this, identify the units of this ratio:

- units V : utils
- units dV : change in utils
- units V_y : change in utils divided by change in income.

Therefore,

- units $\frac{dV}{V_y}$: change in utils over change in income equals change in income.

The part to remember is that if you divide a change in utility (dV) by the marginal utility of money (V_y) you have a change in utility expressed as a change in income, i.e. in units of money.

1.7 Gains from trade: another view

Above I showed that the welfare effect of moving from autarky to trade can be determined by considering the effect of trade on the real wage. In a more general setting (with more factors and/or government transfers), it is useful to examine the effect of trade on real national income. Real national income is defined analogously to the real wage. It is the number of “consumption bundles” (defined above) that a country can purchase with total income. Alternatively, it is the amount of utility that the representative citizen can obtain.

In the Ricardian model (with free trade) all income accrues to labor, so there is no difference between the wage bill and the national income; there is consequently no difference between the real wage and the real national income. (Remember that the modifier “real” means that I am deflating a nominal wage or income, either by using a consumption bundle or by using the marginal utility of income, based on a utility function.)

If there were two or more factors (and free trade) payments to factors other than wages would also be part of national income. If there were trade restrictions such as tariffs, the tariff revenues would also be part of national income.

In general, the gains from trade arise because of the possibility of separating consumption from production. The production point has to lie on the PPF, and the consumption point has to lie on a budget constraint. Under autarky, the market clearing constraint requires that consumption equals production for each good. Under trade, the market clearing constraint requires that world production equals world consumption for each good. The market clearing constraint under trade is therefore weaker than the market clearing constraint under autarky (in the sense that the latter implies the former, but the former does not imply the latter). Trade relaxes a binding constraint, and therefore leads to the possibility of welfare gains.

An individual country (just like an individual consumer) does not concern itself with the requirement that markets clear. However, the country, just like the consumer, has to live within its means. If we exclude the possibility of borrowing or saving (because we are working with a static model), then the value of domestic consumption must equal the value of national income. But national income equals the value of domestic production. (In the Ricardian model, but not in more general models, the value of national income equals the wage bill.)

Let S_i and D_i equal domestic supply and demand of commodity i . The value of national income is

$$y = p_1 S_1 + p_2 S_2$$

and the value of consumption is

$$p_1 D_1 + p_2 D_2$$

when the country faces prices p_i . Here I assume that domestic consumer and producer prices

are the same; they both equal the world prices. There are no taxes or trade restrictions. When we relax this assumption it will be important to distinguish among world prices, domestic consumer prices, and domestic producer prices.

The national income accounting constraint, i.e. the requirement that the value of consumption equals the value of production is

$$p_1 S_1 + p_2 S_2 = p_1 D_1 + p_2 D_2. \quad (9)$$

We can rearrange this equation to obtain

$$p_1 (S_1 - D_1) = p_2 (D_2 - S_2) \quad (10)$$

or

$$p_1 X_1 = p_2 M_2$$

with $X_1 \equiv S_1 - D_1$ exports of commodity 1
and $M_2 \equiv D_2 - S_2$ imports of commodity 2

Equation (9) states that the value of consumption equals the value of national income; equation (10) states that the value of imports equals the value of exports, i.e., trade balances. The important point is that these two conditions are equivalent. In future lectures I use the constraint (10), which I refer to as the “Balance of payments constraint”.

Now we see how a change in the budget constraint changes a country’s welfare level. Suppose that Canada is able to trade at a relative price of corn $p > 1/2$, i.e. at a price greater than its autarkic price. In this case, we have seen that Canada specializes in corn. Given the assumptions above, Canada produces 40 units of corn. ($S_1 = 40$ and $S_2 = 0$ using the notation above.) I choose umbrellas as the numeraire, i.e., set the price of umbrellas equal to 1, so that I can interpret p as both the relative price of corn (the price of corn relative to the price of umbrellas) and as the nominal price of corn. Canada’s national income is $40p$. The balance of payments (BOP) constraint (equivalently, the national income accounting identity) in this case is

$$p(40 - D_1) = p_2 D_2. \quad (11)$$

Figure (5) graphs Canada’s PPF from figure (1), and adds the graph of the balance of payments constraint, the line through points b and c. This BOP constraint is analogous to the income constraint in a consumer’s utility maximization problem. The ability to trade at prices different than the autarkic price means that the BOP constraint lies strictly above the PPF (except at the corners). An increase in p causes the BOP to rotate upwards, making it possible to achieve higher levels of consumption. Absent other distortions (as we assume), an

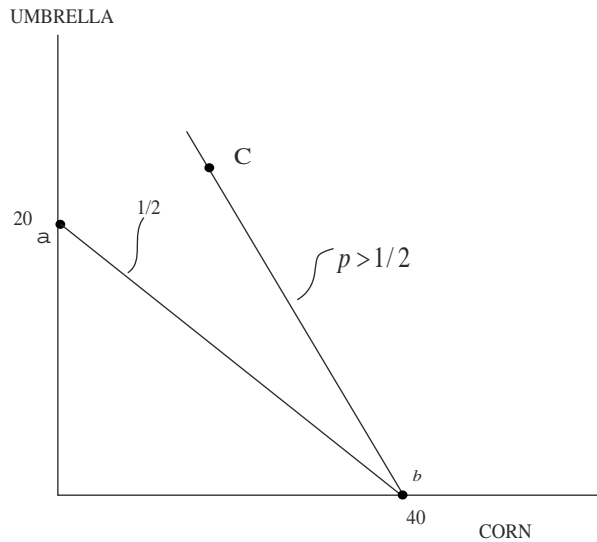


Figure 5

increase in p raises national welfare. Here an increase in p means that the price of exports rises, i.e. Canada's terms of trade improve.

Of course Canada also benefits from trade if it faces world prices $p < 1/2$. In that case, Canada imports corn. Lowering p means that the price of imports falls, again leading to an improvement in its terms of trade, and an increase in its welfare. The graph of Canada's gains from trade, as a function of the world price, reaches a minimum of 0 at the autarchic price. The graph is decreasing for world price lower than the autarchic price and increasing for world price higher than the autarchic price. Provided that the utility function is continuous in the consumption goods, so that the indirect utility function is continuous in prices and income, the gains from trade is a smooth function of the world price. Since it reaches a minimum at the autarchic price, it must be convex in the neighborhood of that price. (Thus, if the world price equals the autarchic price, a small change in the world price causes only a second order increase in the gains from trade. This fact will be useful to recall when thinking about a problem set involving an environment externality.)

1.8 Some comparative statics experiments

When I ask these comparative statics questions, assume that the exogenous change is small enough that it does not change the "nature" of the equilibrium. For example, if the initial equilibrium is on Region A, then it remains on that region after the change.

(i) What is the effect on trade and welfare (in both countries) of a small improvement in US corn technology, i.e. a small reduction in a_1^u ? Using figure (4), this change causes the line

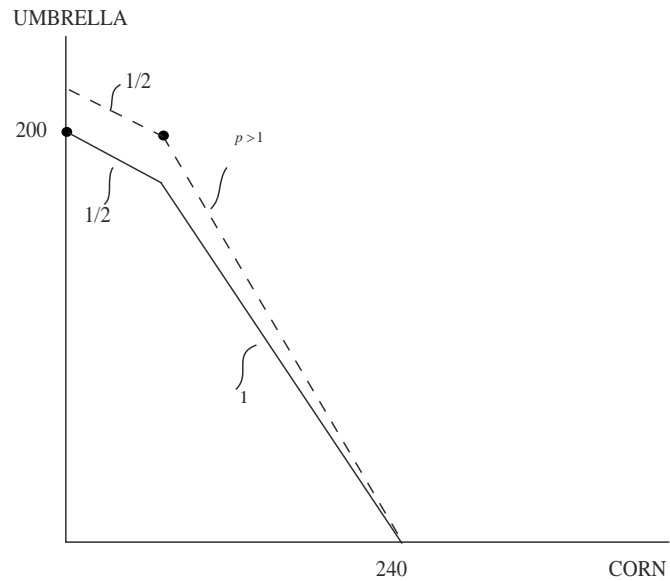


Figure 6

segment labelled "C" to rotate counter-clockwise (become flatter) at point B. If the equilibrium is in Region A or at point B nothing happens in either country under trade, because the US corn sector does not operate. In the absence of trade, the technological improvement increases the real wage in the US, and of course has no effect on the Canadian economy. If the equilibrium is initially on the segment C, then the improvement in US technology reduces the world equilibrium price of corn. Since Canada exports corn, the change lowers the price of Canadian exports and therefore lowers Canadian welfare. The price of US imports falls, so US welfare increases.

(ii) What is the effect on trade and welfare in both countries of a technical improvement in U.S. umbrella production, i.e. a small reduction in a_2^u ? This change causes the world PPF in figure (4) to shift to the dotted lines shown in figure (6). This question is a bit more interesting; I ask it in problem set #1.