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Open economy microeconomics
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Solution Problem Set 1

1) Given prices p_1, p_2 one consumption bundle (a, b) costs $p_1a + p_2b$. Given wage w, a person who sells one unit of labor can afford to buy n consumption bundles, where n solves

$$\begin{aligned} w &= n\left(p_1a + p_2b\right) \Rightarrow \\ n &= \frac{w}{(p_1a + p_2b)} = \frac{1}{\frac{a}{\left(\frac{w}{p_1}\right)} + \frac{b}{\left(\frac{w}{p_2}\right)}}. \Longrightarrow \\ dn &= \frac{1}{\left(\frac{a}{\left(\frac{w}{p_1}\right)} + \frac{b}{\left(\frac{w}{p_2}\right)}\right)^2}\left(\frac{a}{\left(\frac{w}{p_1}\right)^2}\left(d\left(\frac{w}{p_1}\right)\right) + \frac{b}{\left(\frac{w}{p_2}\right)^2}\left(d\left(\frac{w}{p_2}\right)\right)\right). \end{aligned}$$

I used the quotient rule twice to obtain the last equality. If $d\left(\frac{w}{p_1}\right)$ and $d\left(\frac{w}{p_2}\right)$ are of the same sign, then dn (the change in the number of consumption bundles) is also of the same sign. However, if $d\left(\frac{w}{p_1}\right)$ and $d\left(\frac{w}{p_2}\right)$ are of opposite signs, then dn can be either positive or negative, depending on the magnitude of a and b.

2) Using the envelope theorem,

$$\frac{\partial V}{\partial p_i} = -\lambda x_i^*. \tag{1}$$

Assuming non-satiation (all of income is consumed), $\lambda > 0$; assuming that the commodity is consumed ($x_i^* > 0$) equation (1) implies that an increase in the price of a commodity decreases the level of utility – an obvious result.

Suppose that each worker owns one unit of labor. The only source of income in this model is labor income, as noted above, so $y = w$; we can write $V(p_1, p_2, y) = V(p_1, p_2, w)$. The indirect utility function is homogenous of degree 0 in prices and income, i.e. doubling all prices and income leaves the individual no better or worse off. Therefore we can write

$$V(p_1, p_2, y) = V(p_1, p_2, w) = V\left(\frac{p_1}{w}, \frac{p_2}{w}, \frac{w}{w}\right) \equiv v\left(\frac{p_1}{w}, \frac{p_2}{w}\right).$$

Equation (1) implies (under the assumptions of non-satiation and positive consumption) that

$$v_i \equiv \frac{\partial v}{\partial\left(\frac{p_i}{w}\right)} < 0.$$

(Here, and in (some) other contexts a subscript on a function denotes a partial derivative.) The change in v following an exogenous change in $\frac{p_i}{w}$ is

$$dv = v_1 d\left(\frac{p_1}{w}\right) + v_2 d\left(\frac{p_2}{w}\right).$$

If $d\left(\frac{p_i}{w}\right) < 0$ and $d\left(\frac{p_j}{w}\right) \leq 0$ (so that $d\left(\frac{w}{p_i}\right) > 0$ and $d\left(\frac{w}{p_i}\right) \geq 0$), then $dv > 0$, i.e. utility increases. If we reverse the inequalities, utility falls. However, if $d\left(\frac{w}{p_i}\right) > 0$ and $d\left(\frac{w}{p_i}\right) < 0$ we have one positive and one negative effect. We cannot determine the sign of the change in utility, dv, without putting more structure on the problem, i.e. without putting additional restrictions on the utility function.