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Open economy microeconmics
August 7, 2008

Problem set 1

(I repeat part of the lecture notes for convenience.)

We are interested in determining whether an exogenous change increases or decreases welfare. For this purpose, the two ways of defining the real wage are equivalent. One method defines the real wage as the number of "consumption bundles" that an agent can purchase by selling one unit of labor. For example, a consumption bundle in the two-commodity setting can be written as (a, b) where a, b are non-negative numbers; one bundle consists of a units of commodity 1 and b units of commodity 2. n consumption bundles consist of an units of commodity 1 and bn units of commodity 2.

The second method defines the real wage as the amount of "utility" that an individual can obtain by selling one unit of labor.

To use the first definition, I need to specify what constitutes a "consumption bundle", i.e. the values of a and b; to use the second, I need to specify the utility function. Using the first definition, we have

Remark 1 *A necessary and sufficient condition for the real wage to increase, for any consumption bundle (i.e. for any nonnegative values of a and b) is that $\frac{w}{p_i}$ strictly increases for at least one i, and does not fall for either i.*

Using the second definition, we have

Remark 2 *A necessary and sufficient condition for the real wage to increase, for any well behaved utility function is that $\frac{w}{p_i}$ strictly increases for at least one i, and does not fall for either i.*

5

1) Prove Remark 1. Hint: The budget constraint is $w = n\,(p_1 a + p_2 b)$, where w is the wage (equal to the income of a person who sells one unit of labor, and n is the number of consumption bundles, (a, b). Solve the budget

constraint for n, the equilibrium number of consumption bundles that the person buys, and rewrite the expression as a function of the relative prices $\frac{w}{p_i}$. Totally differentiate the resulting equation to obtain the differential for dn. Use this equation to prove the claim.

2) Prove Remark 2 using the indirect utility function, defined as

$$\begin{aligned} V(p_1, p_2, y) &= \max U(x_1, x_2) \\ \text{subject to } \; p_1 x_1 + p_2 x_2 &\leq y, \end{aligned}$$

where y is the individual's income and U is the utility function. The Lagrangian is

$$\mathcal{L} = U(x_1, x_2) + \lambda (y - p_1 x_1 - p_2 x_2)$$

where the constraint multiplier λ is the shadow value of income. The indirect utility function is

$$V(p_1, p_2, y) = \mathcal{L}^*(p_1, p_2, y) = \max_{x_i} \min_{\lambda} \mathcal{L},$$

where a superscript $*$ denotes the value of a function evaluated at the optimum.