

Exam ARE 201 (International Trade) 2007.
Answer all parts of all three questions (see reverse)
Answer Key

1) Consider the problem

$$y(\alpha) = \max_x \frac{g(\alpha x)}{\alpha}$$

where α is a parameter. Denote $x^*(\alpha)$ as the solution to this problem.

(a) (10 points) Write the expression for the elasticity of x^* with respect to α .

(b) (10 points) Write the expression for $\frac{dy}{d\alpha}$.

Answer¹

a) Denote $x^*(\alpha)$ as the solution to this problem. The first order condition is

$$\frac{\alpha g'(\alpha x)}{\alpha} = g'(\alpha x) = 0.$$

Taking the differential and setting it equal to 0 gives

$$g''(\alpha x) x d\alpha + g''(\alpha x) \alpha dx = 0.$$

This equation implies

$$\frac{dx^*}{d\alpha} = -\frac{x^*}{\alpha}.$$

Convert to elasticity to obtain

$$\frac{dx^*}{d\alpha} \frac{\alpha}{x^*} = -1$$

b) Using the envelope theorem and the quotient rule we have

$$\frac{dy}{d\alpha} = \frac{\alpha g'(\alpha x) x - g(\alpha x)}{\alpha^2} = -\frac{g(\alpha x)}{\alpha^2} = -\frac{y}{\alpha}$$

¹Some people gave as the answer for part "a" $\frac{dx}{d\alpha} \frac{x}{\alpha}$. I wasn't asking whether you knew the formula for the elasticity; I wanted to see whether you could apply it. If I had asked for an expression for the derivative of x^* with respect to α , would you have written $\frac{dx^*}{d\alpha}$?

2) In a two commodity world (food and cloth) Home imposes an ad valorem production tax τ on cloth, returning tax revenues to consumers in a lump sum. Home moves from autarchy to free trade, keeping the tax constant. Food is the numeraire good, and p is the consumers' relative price of cloth.²

a) (5 points) Denote the equilibrium levels of production of food and cloth in autarchy as F and C . Denote as y the level of national income. What is the value of y ? (Write the equation for y in terms of the notation given above.)

b) (15 points) State and briefly explain a sufficient condition under which the movement to free trade increases Home's welfare.

c) State a circumstance under which the movement to free trade reduces Home's welfare.

i) (10 points) First give an informal explanation for your answer. (One way to provide this informal answer is to explain (**briefly**) the relation between this problem and one or more model we worked through during the course.)

ii) (10 points) Provide a graphical illustration – using a general equilibrium model – of the situation where trade reduces welfare.

ii) (10 points) Using the indirect utility function $V(y, p)$, show mathematically that the circumstance you identified does indeed reduce welfare. Hint: (i) take the total differential of $V(y, p)$ evaluated at the autarchic price, (ii) use Roy's identity, and the condition for equilibrium production, and your assumption of how p changes with trade.

Answer

a)

$$y = (1 - \tau)pC + F + \tau pC = pC + F$$

b) In autarchy the tax causes the country to produce too little cloth and too much food, relative to the first best (0 tax) level. If the world price of cloth is higher than the autarchic price, the movement to free trade increases Home welfare.

²There is no trade in factors in this problem. Under trade home remains incompletely specialized. The tax is exogenous to the model and there are no other distortions.

c) A necessary condition for trade to lower welfare is that the world price of cloth is lower than the autarchic price, i.e. $dp < 0$. A sufficiently small decrease in price lowers welfare.

i) To answer this question informally, you should recognize the parallels between this problem and the poaching model and the pollution externality problem set. In those problems, a distortion causes one sector to produce too much (and the other sector to produce too little) relative to the first best level. If the price change following trade liberalization is small, the standard gains from trade are small. If the price change exacerbates the externality, by increasing production in the sector that is already "overproducing" (relative to the social optimum), trade can reduce welfare.

ii) See figure on final page. (The line labelled p^w is slightly flatter than the line labelled p , to indicate that the world relative price of cloth is slightly lower than the autarchic relative price of cloth. The important features of the graph are: 1) Production = consumption under autarchy, point A; 2) at autarchy the indifference curve is tangent to the domestic autarchic consumer price; 3) the domestic consumer relative price is steeper than the tangent to the PPF at A, since the tangent of the PPF equals the producer relative price of cloth, which is less than the consumer relative price; 4) opening up to trade causes the production point to move to the food axis, i.e. toward point B; 5 the BOP constraint through B has slope equal to the world price, which is less than the domestic consumer price; 6 consumption occurs on the BOP curve below the autarchic indifference curve.)

iii) To proceed formally, let $V(y, p)$ be the indirect utility function. Income is

$$\begin{aligned} y &= (1 - \tau) pC + F + \tau pC = pC + F \implies \\ dy &= pdC + Cdp + dF \end{aligned}$$

where C, F are the production levels. Rearranging the differential of indirect utility gives

$$\begin{aligned} \frac{dV}{V_y} &= dy + \frac{V_p}{V_y} dp \\ &= pdC + Cdp + dF - C^{con} dp \\ &= pdC + dF - (C - C^{con}) dp \\ &= \left(p + \frac{dF}{dC} \right) \frac{dC}{dp} dp - (C - C^{con}) dp \end{aligned}$$

The second equality uses Roy's identity and the differential for price; the next two equalities are just rearrangements. If the change in price is small, the first order approximation of the change in welfare is given by the last line evaluated at $p = \text{autarchic price}$, where $(C - C^{\text{con}}) = 0$. In this case we have

$$\frac{dV}{V_y} = \left(p + \frac{dF}{dC} \right) \frac{dC}{dp} dp$$

Use

$$\frac{dC}{dp} dp < 0$$

because the necessary condition for trade to reduce welfare is $dp < 0$; in addition

$$\begin{aligned} \frac{dF}{dC} &= -p(1 - \tau) > -p \implies \\ \left(p + \frac{dF}{dC} \right) &> 0 \end{aligned}$$

so conclude

$$\frac{dV}{V_y} = \left(p + \frac{dF}{dC} \right) \frac{dC}{dp} dp < 0$$

3) (30 points total; each part worth 10 points) Consider a small open two-commodity (food and cloth) economy. Initially (before any tax) the economy produces both commodities. Now society imposes a small tax on food production. What effect does this tax have on real returns to factors in the (a) Ricardo, (b) Ricardo-Viner, and (c) HOS models?.

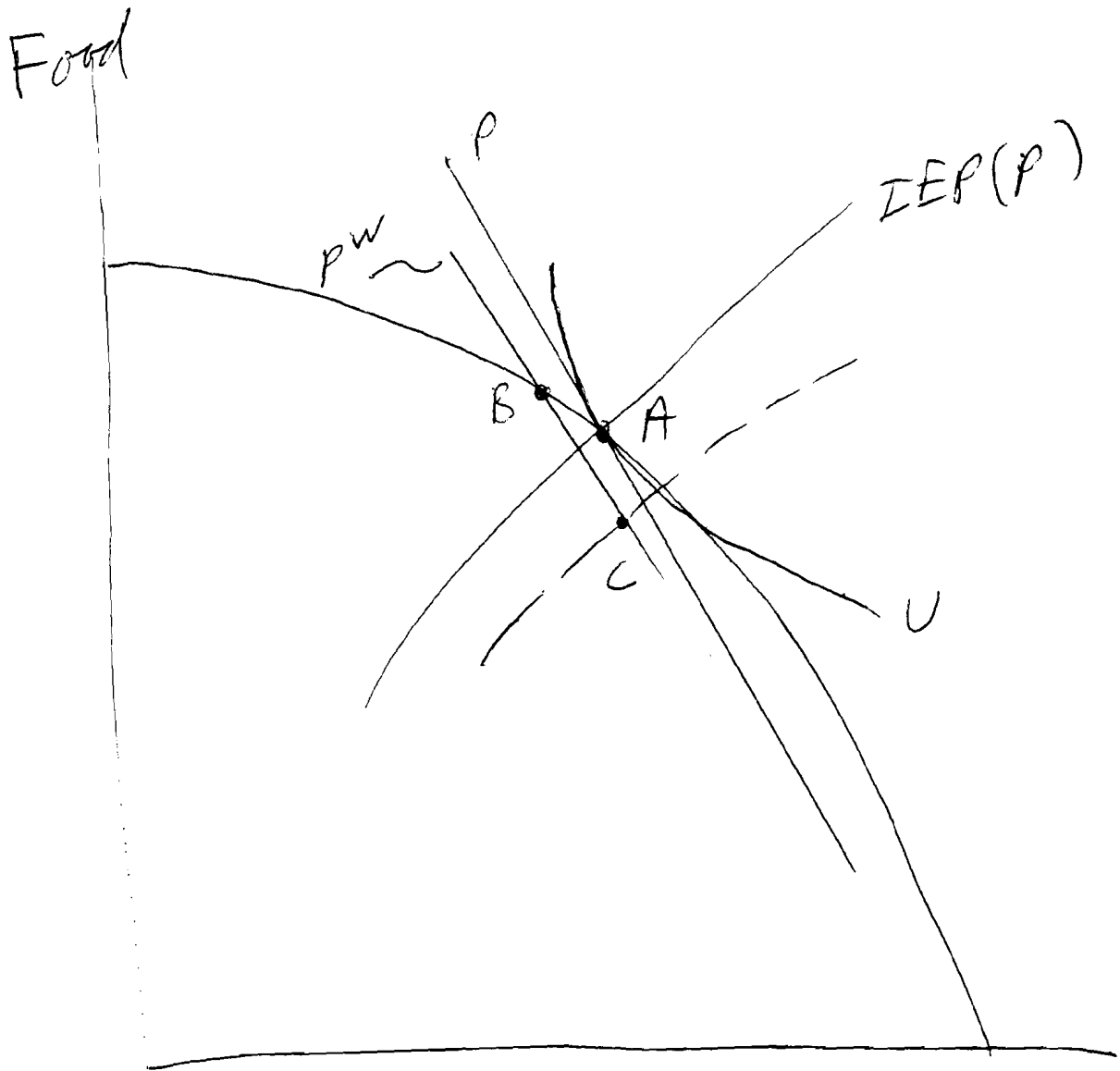
Answer

The tax has no effect on consumer prices in any of these models, because the country is small. Therefore, to determine the effect of the tax on the real return to factors, you need to know only the effect of the tax on the nominal return to the factors.

(a) In the Ricardian model, the tax induces the country to specialize in the cloth sector, but has no effect on the wage (since the economy was initially incompletely specialized), and thus no effect on the real return to labor.

(b) In the Ricardo-Viner model the tax reduces the producer price of food, decreasing the nominal wage. Since prices have not changed, the real return to the labor falls. The amount of labor in the food sector falls, so the marginal productivity of capital in the food sector falls. Since the producer price of food also falls, the value of marginal product of capital in the food sector (= the nominal return to capital in that sector) falls. The value of marginal product of capital in the cloth sector rises with the increase in labor in that sector, so the price of capital in that sector (and thus its real return) rises. (You could have shown all of this using graphs, but that was not necessary.)

(c) The answer depends on which good is relatively capital intensive. If cloth is relatively capital intensive, the increase in the relative producer price of cloth increases the rental rate and decreases the wage. In this case, the real return to capital rises and the real return to labor falls. If, instead, food is capital intensive, the direction of changes is reversed. (I took off one point if you invoked the Stolper-Samuelson theorem. That theorem assumes that consumer and producer prices are equal – which is not the case here.)



A: autarky production & consumption. ^{Cloth}

$P^w < P$ (the autarky price.)

Under trade, production moves to B and consumption moves to ~~A~~ C

Free trade utility = U

Point C below indifference curve U.