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Solution to Problem Set 4

Some of the figures that are needed to provide "graphical proofs" appear rather complicated, making the arguments appear difficult. This apparent complication is a disadvantage of graphical arguments, relative to mathematical arguments. For the latter, the sequential nature of the proof is apparent: one thing comes before another. In contrast, with a graph, "everything appears at the same time". One way to overcome this difficulty is to use lots of graphs, so that the reader can see the development of the argument. I have tried to do that, but there is no substitute for "thinking with a pencil". As you read the text of the argument, you should draw the figure, and compare your drawings with the ones in the answer key.

Case I (Constraint on food consumption)

a) Under free trade the equilibrium consumption of food is $F_T > F^*$ so the constraint is violated. (Figure 1)

b) The optimal policy is a consumption tax/subsidy. The domestic consumer price equals slope of the tangent to indifference curve at point A, figure 2, denoted p^d . The ad valorem consumption tax on food, t , solves

$$p^d = \frac{p^w}{1+t}.$$

You can write the tax in terms of the slopes of lines p^d and p^w . The first best consumption tax/subsidy leads to consumption at A and utility U_1 . Imposition of the constraint leads to a lower level of utility compared to free trade: there are points on the balance of payments constraint, above point A, that result in higher utility. (The indifference curve U_1 is not tangent to the BOP line.) Note that the consumption tax does not alter the production point nor the BOP constraint.

c and d) First consider the production policy. I've drawn the constraint as the dashed line at F^* in figure 3. If a production tax/subsidy is used, the consumer price equals the world price, so consumption must be on $IEP(p^w)$. This fact, and the fact that the constraint is binding, means that consumption must occur at point B. Utility at this point involves the same amount of food consumption and strictly less cloth consumption than at A (the consumption point under the first best policy), so utility is lower than in the

first best policy. Production must be at point C , in order to satisfy the BOP equilibrium. Note that growth cannot alter the consumption point (which is uniquely determined by the IEP curve and the constraint) so growth neither increases nor decreases welfare. Growth can change the production point (e.g. growth may result in production shifting away from point C); growth cannot shift the consumption point along the line at F^* when a production policy is used. Therefore, growth cannot be immiserizing if a production policy is used - but neither can it lead to an increase in welfare.

Now consider the trade policy. Note that point A gives the highest feasible level of consumption of cloth, subject to the constraint that consumption of food does not exceed F^* . Any point to the right of A on the line F^* and on a BOP line (i.e. a line with slope p^w) lies strictly outside the production possibility frontier, and therefore is not feasible. Thus, a tariff that achieves the constraint must result in a lower level of welfare than at point A . A tariff changes both consumer and producer prices. The IEP under a tariff lies strictly below $IEP(p^w)$ shown in figure 3. The consumption point under a tariff must therefore lie between points A and B in figure 3, and the production point must lie on the PPF below point C . (These are points D and E in figure 3. Note that the BOP line - not shown in order to reduce clutter - between these two points has the same slope as the two BOP lines that are shown.)

Now I want to show that when the constraint is achieved by means of a tariff, growth can be immiserizing. First, consider the economic intuition. We know that the optimal policy is to change only consumer prices. The tariff changes both producer and consumer prices. It lowers food consumption (to achieve the constraint) by means of a substitution effect and an income effect. The income effect occurs because the tariff moves the production point away from the free trade level, thereby decreasing the value of national income.

We know that the optimal way to achieve the constraint is by changing consumer prices "a lot" and changing producer prices "not at all". When we achieve the constraint by means of a tariff we change the consumer prices by less, (compared to the use of the optimal consumption tax/subsidy). The tariff forces the income effect to do "too much work" in reducing food consumption, and forces the substitution effect to do "too little work" in reducing consumption. Growth might be immiserizing if it increases the income effect, while reducing the substitution effect.

Now consider the graphical analysis. Figure 4 reproduces important parts

of figure 3, leaving other parts out to reduce clutter. In order for growth to reduce welfare, it must be the case that the equilibrium consumption point moves left from D (the pre-growth level of consumption under the tariff), say to point G . You should be able to convince yourself that the IEP through point G lies above the IEP through point D . Therefore, the domestic relative price of food under the post-growth tariff is lower than the domestic relative price of food under the pre-growth tariff. That is, *in this example* growth leads to a *reduction* in the tariff.

I've drawn a portion of the pre-growth PPF through point E , the pre-growth production point under the tariff. Note that the tangent of the PPF at E is equal to the tangent of the indifference curve at D , since these two tangents equal, respectively, the producer relative price and the consumer relative price – and those two prices are equal under a tariff.

To complete the analysis, you should draw an indifference curve through point G . Note that the tangent at this point – call it p' – is lower than the tangent at point D (which equals p^t). The tangent of the PPF at point H must equal p' . You can easily draw a PPF through H with this characteristic. That PPF lies outside the original PPF, so growth has indeed occurred.¹

My answer to the question began with a discussion of the intuition for the answer, and then provided the (graphical) analysis. When you actually solve this problem, most people would proceed in the reverse order. You would first demonstrate the possibility and then use the demonstration to interpret the economics. If you proceed systematically, this is not hard to do. Just begin with the observation that in order for growth to lower welfare under a tariff, given the constraint, consumption must move to the left from D in figure 4. Then recognize that in order for this to occur, the IEP must shift up, and that means that the tariff must decrease.

e) We have used a general equilibrium setting to examine the effects of growth under the different types of policies (given the same constraint). A partial equilibrium (PE) model ignores the fact that income changes (possibly it decreases) when the production possibility frontier shifts out. It is interesting to see how far you can get with a PE model. Draw a supply and demand curve with a consumption constraint that is violated under free trade. Show:

¹Students who are on the ball will object that my construction does not show that points G and H are the optimal consumption and production points under a tariff that achieves the constraint. In order to demonstrate this, you can show that there is unique tariff that satisfies the constraint.

i) Welfare (producer + consumer surplus plus tax or tariff revenue) is higher when the constraint

is achieved using a consumer tax rather than a tariff.

ii) A production tax/subsidy is completely ineffective in reaching the target.

iii) Growth can lead to lower welfare, but growth does not change the equilibrium tariff. Figure 5 illustrates the last possibility. The initial supply and demand curves are D and S , and the tariff needed to achieve the constraint is shown as t . Suppose that growth causes the supply curve to shift to S' , so that imports are eliminated in equilibrium. There is no change in consumer surplus since neither the level of consumption (F^*) nor the demand curve have changed. Producer surplus has increased by the area aeb but tariff receipts have fallen by $abdc$, so social welfare (the sum of producer and consumer surplus and tax revenue) has fallen. Growth causes greater substitution of low priced imports for high cost domestic production.

Case II: Constraint of Imports

a) M is the free trade level of imports, with $M > M^*$ (figure 6).

b) The optimal policy is a tariff that leads to production at A , consumption at B , and utility U_t , with imports M^* (figure 7). I showed the free trade and tariff-ridden levels of imports in two distinct graphs, in order to reduce clutter. You should draw them in the same graph so that you can compare the levels of imports in the two cases. I reminded you above how to express the level of the tariff in terms of the slopes of the price lines.

c) Show that either a production or a consumption policy which satisfies the constraint $M \leq M^*$ leads to a lower level of utility (relative to the level under the tariff). I will provide the argument for the case of a production policy. An analogous argument holds for the case of a consumption policy. I use a proof by contradiction and figure 8. Remember that the height of the trade triangle associated with a production and consumption point equals the level of imports (because the imported good is on the vertical axis). In figure 8 the height of the trade triangle is M^* by construction.

We want to show that it is not feasible to satisfy the constraint $M \leq M^*$ using a production tax/subsidy and achieve the level of utility obtained under at tariff, U_t . Suppose, to the contrary, that it is possible to satisfy this constraint, obtaining utility U_t . (This is the hypothesis that we want to falsify.) The income expansion path at world price, $IEP(p^w)$ lies strictly above the income expansion path under the tariff (not shown in order to reduce

clutter) which goes through point B . (See figure 8.) Under a production policy, consumer prices are unchanged, so consumption occurs on IEP (p^w). If society enjoys the same level of utility as under a tariff, consumption must be at point C . If consumption occurs at point C , then BOP equilibrium requires that production occurs at point D . (I have not drawn the BOP constraint in order to avoid clutter, but note that a line through point D with slope p^w intersects the indifference curve U_t at point C .) It remains only to show that the trade triangle (not drawn) corresponding to points C and D violates the constraint $M \leq M^*$.

Figure 9 reproduces important features of figure 8, and removes other features to achieve clarity. The dashed lines through points A and B with slope p^t are the tangents to the PPF and the indifference curve, respectively. Because of the curvature of the PPF and the indifference curve, the points C and D lie above/below the dashed lines, as shown in figure 9. (Take another look at figure 8 to convince yourself of the truth of the last sentence.) The two trade triangles (not shown), associated with the points A and B and the points C and D are "similar" (because of the "angle-angle-angle rule" of geometry). (You should draw these two trade triangles. I have not drawn them, so that the figure is not cluttered.) The hypotenuse of the trade triangle associated with C and D is longer than the hypotenuse of the triangle associated with B and A ; because the triangles are similar, the height of the former is also larger than the height of the latter. By construction, the height of the trade triangle associated with B and C is equal to the constrained level of imports, M^* . Therefore, contrary to our hypothesis, if we achieve the same level of utility (as under a tariff) using a production policy, we violate the constraint on imports.

d) If the constraint $M \leq M^*$ is achieved using a production policy, growth cannot be immiserizing. In figure 10, the constraint is achieved using relative producer prices p^d leading to production at A and consumption at B . Growth may induce a change in producer prices, but not in consumer prices. (Remember that when a production tax/subsidy is used, the producer prices become endogenous. They are chosen in order to satisfy a constraint. Anything that changes the optimization problem - here, growth - can change the equilibrium producer prices.)

In order for growth to reduce welfare, given that consumer prices remain constant, the consumption point would have to move down the IEP, in the direction of C . If this occurs, and imports remain at M^* , BOP equilibrium requires that production occur beneath the original PPF. However, with com-

petitive producers, production must occur on the new (post-growth) PPF, which lies above the original PPF. Therefore, growth cannot reduce welfare with a production tax/subsidy is used to achieve the import constraint.

Now show that growth can be immiserizing if the non-economic objective is achieved using a consumption tax/subsidy. (See figure 11.) Before growth, under the consumption tax/subsidy, production is at point A and consumption at point B , with utility at U^d . By construction, the height of the trade triangle (not shown) is equal to M^* . After growth (with constant relative producer price equal to the world price), production moves to point D . In order to satisfy the constraint, at this new level of production, consumption would have to move to the point marked x . (The distance between x and D equals the distance between B and A , so the height of the trade triangles associated with those two points is the same.) Since point x lies below U^d , growth leads to a loss in welfare.

Intuition: The "target" (or constraint) in this problem involves trade. Given that world price is fixed, and given the requirement that trade balance, constraining imports is equivalent to constraining exports. The "Principle of Targeting" says that you should use a trade policy to achieve a trade target. Imports involve both production and consumption. It is possible to alter imports by altering only production or consumption (i.e. using a consumption or a production policy) but it is optimal to meet the target by altering both production and consumption. It is optimal to achieve the constraint by "distorting" both consumption and production, rather than requiring one or the other to bear the entire brunt.

Why can growth be immiserizing with one type of policy but not the other? When the target is reached using a production policy, the reduction of imports is achieved by reducing income (measured at world prices). In order for growth to reduce welfare still further, it would be necessary for growth to reduce income (measured at world prices). However, such a reduction in income is not consistent with maintaining the same level of imports and also producing on the production possibility frontier. In contrast, when a consumption policy is used to achieve the target, national income (measured at world prices) is not changed, but a consumption distortion is introduced. Growth may require an increase in the consumption distortion in order to maintain the constraint. This increase in the distortion can lead to a fall in welfare, even though income (measure in world prices) rises.