

September 18, 2006

Extra problem for Chapter 2 of Notes.

Consider a small country that produces both corn and umbrellas; this country imports umbrellas. The world relative price of corn is p^w . (Umbrella is the numeraire good: its price is set to 1.) The country imposes a small tax on corn production. Using a graph, identify the production and consumption points under two scenarios: (i) the government returns the tax revenue to consumers in a lump sum, and (ii) the government throws the tax revenue into the sea.

Identify the production point under free trade as A; the consumption point under free trade as B; the consumption point when the tax revenue is returned to consumers as G and the consumption point when the tax revenue is thrown into the sea as D. Identify other points as needed.

Answer: First recognize that the production point depends only on relative producer prices, not on what is done with the tax revenue. Therefore, the production point in both scenarios (with the tax) is on the PPF above A, where the tangent equals the negative of the relative producer price for corn. Identify this point as E. The tangent to the PPF at point E is $-(p^w - t)$, where t is the *unit tax* on corn producers.¹

i) Draw a line with slope $-p^w$ tangent to the PPF at point A; this line is the BOP constraint in the absence of the tax. Point B is the intersection of the BOP constraint and the IEP(p^w). Draw a line with slope $-p^w$ through E. This line is the BOP constraint when production is at point E, i.e. in the presence of the tax. The intersection of this BOP constraint and the IEP (which is unaffected by the producer tax) is point G. (Note that the slope of the BOP line is not changed by the tax, because of the small country assumption

¹You should know the difference between a unit and an ad valorem tax. If the market price is fixed at p and producers pay a unit tax of t , they receive $p - t$ for each unit sold. If they pay an ad valorem tax of τ , they receive $p(1 - \tau)$. The unit and ad valorem tax result in the same producer price if $p(1 - \tau) = p - t$, i.e. if $p\tau = t$.

The production point E corresponds to U_E units of umbrellas and C_E units of corn; see the attached figure. The attached figure shows the horizontal intercept of this line as point F. The value of every point on the line CEF is the same, valued at world prices. Thus

$$\begin{aligned} p^w F &= p^w C_E + U_e \implies \\ F &= C_E + \frac{1}{p^w} U_E. \end{aligned}$$

i)) If the tax revenues are thrown into the sea, consumer's only source of income is the payment they receive as factors of production, which equals the revenue (net of taxes) that producers receive from the sale of output. This revenue (= income) is

$$y = (p^w - t) C_E + U_e$$

Given this revenue, consumers can afford to purchase the corn, umbrella bundle C, U that satisfies

$$\begin{aligned} p^w C + U &= y \implies \\ U &= (p^w - t) C_E + U_e - p^w C. \end{aligned}$$

The graph of the last equation is a line with slope $-p^w$ and horizontal intercept (obtained by setting $U = 0$ and solving the last equation for C)

$$H \equiv C^{\text{intercept}} = \frac{(p^w - t) C_E + U_e}{p^w}.$$

The attached figure shows the horizontal intercept, H . Note that $H < F$. The difference, $F - H$ equals the value of the tax revenue (expressed in units of corn) that was thrown away. The tax revenue, expressed in dollars, is simply tC_E . This revenue, expressed in units of corn (i.e., the amount of corn that can be bought with this revenue) is simply $\frac{tC_E}{p^w}$. To check calculations, note that:

$$\begin{aligned} \text{tax revenue (expressed in units of corn)} &= \\ F - H &= C_E + \frac{1}{p^w} U_E - \left(\frac{(p^w - t) C_E + U_e}{p^w} \right) = \\ \frac{p^w - (p^w - t) C_E}{p^w} &= \frac{tC_E}{p^w}. \end{aligned}$$

Think of the units of the last ratio. The units of t is $\$/\text{units corn}$ and C is units of corn, so the product of the two (the numerator of the ratio) is in units of dollars. The numerator measures the tax revenue in dollars. The denominator (the relative price of corn), has units: $\text{dollars}/\text{units of corn}$, so the ratio has units of corn.

