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3 The Ricardo-Viner Model

The Ricardo-Viner or "Sector-Specific Factors"model assumes that both commodities require a sector specific input (e.g. land for corn and machines for umbrellas) as well as a mobile factor. In contrast, the Ricardo model assumes that there is a single factor of production, which is mobile across sectors. The Heckscher-Ohlin-Samuelson (HOS), studied in the next section, assumes that there are two or more factors of production, all of which are mobile across sectors. We can interpret HOS as a long run model and the Ricardo-Viner model as a medium run model.

I first outline the model and discuss its comparative statics properties. Then I use the model for two second-best applications: (i) trade in the presence of imperfect property rights and (ii) trade in the presence of a minimum wage. I close this chapter by studying an "intermediate" model in which the sector-specific factors can adjust slowly across sectors. This extension serves as a bridge between the Ricardo-Viner and the HOS models.

3.1 Equilibrium in the basic model

The basic building block in this model is the optimality condition that the value of the marginal product of labor (the mobile factor) is equal to the wage, i.e. the price of labor. Since labor can move freely from one sector to the other, the wage in the two sectors must be equal (as in the Ricardo model). Since producers maximize profits by setting the value of marginal product of labor equal to the wage, so these values of marginal product must be equal in the two sectors.

Let K_c and K_u be the amount of sector-specific capital in the corn and umbrella sector; L_c and L_u are the stocks of labor in the two sectors. Let $C(K_c, L_c)$ and $U(K_u, L_u)$ be the sectoral production functions, and denote partial derivatives using subscripts, so C_L and U_L are the marginal products of labor in corn and umbrellas. (To conserve notation, I do not use subscripts on the subscripts, but the meaning should be clear from the context.) These marginal products are functions of the labor and the amount of capital in the sector. I assume that the production function in each sector is concave in labor and that the inputs are technical substitutes, i.e. that an increase in either factor in a sector increases the marginal product of the other factor in that sector. For the corn sector, for example, these assumptions mean:

$$C_{LL} < 0 \text{ and } C_{LK} = C_{KL} > 0. \tag{40}$$

Let W be the nominal wage. The optimality conditions are

$$\begin{aligned} p_c \cdot C_L &= W \\ p_u \cdot U_L &= W. \end{aligned}$$

It is convenient to work with relative prices, so define $p = p_c/p_u$ and $w = W/p_u$. I choose umbrellas as the numeraire. Notice that the units of w are units of umbrellas/labor, so w tells how many umbrellas exchange for a unit of labor.

Divide the optimality conditions by p_u to rewrite them as

$$\begin{gathered} p \cdot C_L = w \\ U_L = w. \end{gathered} \tag{41}$$

The total supply of labor is $\bar{L}$, so full employment requires $L_c + L_u = \bar{L}$. Figure 1 graphs $p \cdot C_L$ and U_L as functions of labor allocation. The width of the horizontal axis is $\bar{L}$. Moving left to right increases the stock of labor in umbrellas and decreases the stock of labor in corn. The relative price p is taken as given. The intersection of the two graphs determines the equilibrium allocation of labor and the equilibrium wage w. The equilibrium umbrella wage is shown in figure 1 as c.[11]

3.2 Comparative statics

I will consider two types of comparative statics questions. What is the effect of the commodity price on labor allocation and real returns to factors? What is the effect of a change in factor supplies on real returns to factors, holding fixed the commodity price?

3.2.1 The effect of the commodity price on labor allocation and the wage

I will answer the first comparative statics question using both graphical and mathematical analysis. Note that full employment of labor requires $L_u + L_c = \bar{L}$. Holding the total supply of

[11] This kind of a diagram – where the length of one axis represents the total availability of something (here, labor), and a movement along the axis represents reallocations of that variable – is useful in a variety of settings (not only to describe the Ricardo-Viner model).

labor fixed, the differential of this equation implies

$$dL_c = -dL_u, \tag{42}$$

which says that labor entering one sector must come from the other.

First consider the effect of an increase in p (the relative price of corn) on labor allocation. The increase in p causes the curve DE to shift up to the dotted line $D'E'$, causing the allocation of labor to shift from F to F' in figure 1. Labor leaves the umbrella sector and enters the corn sector.

The graphical analysis may be the easiest way to understand how the model works, but it is worth being able to show the results mathematically. Totally differentiating the equilibrium conditions, equation (41), using equation (42) , gives

$$\begin{aligned} dw &= U_{LL} \cdot dL_u \\ dp &= \tfrac{(pC_{LL}+U_{LL})}{C_L} \cdot dL_u. \end{aligned} \tag{43}$$

The second line of equation (43) implies that

$$\frac{dL_u}{dp} = \frac{C_L}{(pC_{LL}+U_{LL})} < 0, \tag{44}$$

where the inequality follows from the assumption that the production functions are concave. Thus, an increase in p decreases the amount of labor in the umbrella sector.

Now consider the effect of the price on the real return to labor. An increase in the price shifts up the curve pC_L in figure 1 to the dashed curve. The relative wage w increases, so W/p_u increases from c to b. Recall that in order to determine the effect of the price increase on the real wage, it is necessary to know how the price increase changes both of the the ratios W/p_u and W/p_c. We still need to determine how the increase in the relative price changes

$$\frac{W}{p_c} = \frac{w}{p}.$$

Remember, this ratio equals the number of units of corn that a worker can purchase by selling one unit of labor.

I will use a graphical and a mathematical demonstration that an increase in p lowers the ratio $\frac{w}{p}$. In view of the discussion of the real wage in Chapter 1, this result implies that the effect of the change in the relative commodity price on the real return to labor is ambiguous.

First consider the graphical approach, using figure 1. For this demonstration I evaluate all of the derivatives at the initial equilibrium allocation, point F. The height of the point a is $p'C_L = (p + \Delta p)\, C_L$ and the height of the point c is pC_L. Therefore,

$$\frac{a-c}{c} = \frac{(p+\Delta p)\, C_L - pC_L}{pC_L} = \frac{\Delta p}{p}.$$

Using the same logic,

$$\frac{b-c}{c} = \frac{\Delta w}{w}.$$

By inspection of figure 1,

$$\frac{\Delta p}{p} > \frac{\Delta w}{w}.$$

This inequality implies that

$$\frac{\Delta w}{\Delta p}\frac{p}{w} < 1 \tag{45}$$

This inequality states that (for example) a 10% increase in p leads to a less than 10% increase in w. This result implies that an increase in p leads to a fall in $\frac{w}{p}$.

To obtain this result mathematically, divide the first line in equation (43) by w and the second line by p, and then divide one equation by the other. We obtain the following expression for the elasticity of the wage with respect to the price:

$$\frac{dw}{dp}\frac{p}{w} = \frac{\frac{dw}{w}}{\frac{dp}{p}} = \frac{\frac{U_{LL}}{pC_L}}{\frac{pC_{LL}+U_{LL}}{pC_L}} = \frac{U_{LL}}{pC_{LL}+U_{LL}} < 1. \tag{46}$$

Again, this inequality implies that a 10% increase in the price causes a less than 10% increase in the wage. Equations (45) and (46) contain the same information. I present both derivations to give you practice with both mathematical and graphical approaches to deriving a result.

The return to the specific factors Thus far I have assumed only that the production function is concave in labor and that labor and capital are technical substitutes, so that the cross partials of the production functions are positive (i.e., more labor increases the marginal product of capital). The fixed factor is the “residual claimant” in this model. This means that the value of production, minus the wage bill, accrues to the fixed factor. The fixed factor receives rent.

The fact that the fixed factors do not move from one sector to the other means that the rents these factors receive are (typically) different in the two sectors. Thus, it does not make sense to speak of the rental rate of "capital". We have to distinguish whether we mean capital in the umbrella sector, or capital in the corn sector. Therefore I use subscripts to distinguish these two rental rates. We want to know how an exogenous change (such as a change in the relative commodity price) changes both the nominal and the real return to these two factors (capital in the two sectors).

For concreteness, consider the umbrella sector, where the value of production is $U(K_u, L_u)$; recall that by my choice of numeraire, the price of umbrellas is 1, so U is both the quantity and the value of umbrella output. I added subscripts to the arguments of the production function in order to emphasize that umbrella production depends on the amount of capital and labor in that sector. The value of umbrella output, less the wage bill is (using equation (41))

$$U(K_u, L_u) - wL_u = U(K_u, L_u) - U_L L_u.$$

This residual value accrues to umbrella capital, so the rent per unit of capital is simply

$$r_u = \frac{U(K_u, L_u) - wL_u}{K_u}. \tag{47}$$

There is a similar relation for the corn sector.

In the interest of simplicity (only) I will now assume that in each sector there are constant returns to scale. Euler's Theorem implies that

$$\begin{aligned} U_K(K,L)\,K + U_L(K,L)\,L &= U(K,L) \Longrightarrow \\ U_K(K,L)\,K + wL &= U(K,L) \Longrightarrow \\ \frac{U(K,L) - wL}{K} &= r_u = U_K. \end{aligned} \tag{48}$$

(I used the second line of equation (41) to obtain the second line of equation (48).) Thus, with constant returns to scale, the rent to a factor (equal to its price) is equal to its value of marginal productivity. If I did not assume constant returns to scale, I would use equation (47) to determine the price of the sector-specific factor. Alternatively, I could assume that each sector specific factor earns its value of marginal product; but in that case, I would have to introduce some other factor that is the residual claimant.

With the assumption of constant returns to scale, the equilibrium factor prices in the two sectors are given by the value of their marginal product

$$\begin{matrix} r_c = p \cdot C_K \\ r_u = U_K \end{matrix} . \tag{49}$$

(Remember, the normalization that $p_u = 1$ means that the value of the marginal product of capital in the umbrella sector is equal to the marginal product of capital in that sector.) Since capital is immobile across sectors, there is no reason for the return to capital to be equal in the two sectors.

An increase in p increases the real return to capital in the corn sector and decreases the real return to capital in the umbrella sector. First consider the intuition for this result. We know that the increase in p causes labor to leave the umbrella sector and to enter the corn sector; this reallocation increases C_k and decreases U_K, because the cross-partials are positive (see equation (40)). Therefore r_u falls in absolute terms. Since p rises, r_u/p obviously falls, so the real return to capital in the umbrella sector falls. Remember, r_u is the number of units of umbrellas that a owner of umbrella capital can purchase by renting one unit of umbrella capital; r_u/p is the number of units of corn that he can obtain by renting one unit of his capital. Since he can buy less of both commodities, by renting one unit of his capital, the real return to umbrella-specific capital has decreased.

The increased labor in the corn sector caused by the increased price, p (see equation (44)) increases $C_K = r_c/p$. The fact that this ratio increases and the fact that p increases means that r_c must increase. Thus, the real return to capital in the corn sector increases.

To obtain this result mathematically, rewrite the first line of equation (49) as

$$\frac{r_c}{p} = C_K.$$

Differentiate this equation, using equation (44) and the assumption that $C_{KL} > 0$ to obtain

$$\frac{d\left(\frac{r_c}{p}\right)}{dp} = \frac{dC_K}{dp} = C_{KL}\frac{dL_c}{dp} = C_{KL}\frac{dL_c}{dp} = -C_{KL}\frac{dL_u}{dp} > 0.$$

The effect of the supply of labor Now we consider the effect on the equilibrium real factor returns of a change in factor supplies, holding fixed the commodity price. Again, you should be able to determine this effect using geometric or mathematical methods. For example, an increase in labor causes the width of the box in figure 1 to increase, leading to a fall in the absolute value of w, and thus a fall in the real wage. Since there is then more labor in both sectors, the marginal product of the fixed factor in each sector rises, as does the real return to capital in that sector.

In contrast, an increase in the amount of capital in a sector causes the marginal product of capital in that sector to fall and the marginal product of labor in that sector to rise; one of the curves in figure 1 rises, leading to an increase in w. Since p is constant, w/p rises. The real return to capital both sectors falls.

3.3 Trade

The Ricardo-Viner model is a special case of the model that we studied in Notes #2, and the trade equilibrium can be studied using the same graphical methods. Figure 1 can be used to derive PPF for this model. Figure 1 shows the relation between the relative commodity price and the allocation of labor. If I know the allocation of labor, then I know the amount of each good that is produced, since the other factors are fixed. Figure 2 shows the production possibility curve. I know that the slope of the production possibility curve is equal to the relative price, p, in an efficient competitive equilibrium. So I can pick a particular value of p, find out from figure 1 the output of each good, recognize that this is a point on the PPF; at that point the slope of the PPF is p. If I do this for many values of p, I can trace out the PPF. (See figure 2).

Using figure 2 (and indifference curves) we can find the autarkic price, and we can find the level of production, consumption and trade for an open economy. We can also ask the kinds of comparative static policy questions considered in Notes #2.

3.4 Imperfect property rights

Here we examine the effect of property rights in a general equilibrium model. We suppose that there are imperfect property rights to the specific factor in one sector. For example, suppose that one sector is natural resource-intensive (e.g. forestry) and the other sector is capital-intensive (e.g. cloth manufacturing). There are perfect property rights to capital used in the cloth sector but imperfect property rights to the natural capital in the forestry sector. For example, it may be relatively easy to keep unauthorized people from using a factory, but it is difficult to keep people from illegally using a forest. In this case, the competitive equilibrium is not first best. Opening such an economy to trade can lead to a loss in welfare.

To examine the role of imperfect property rights it is convenient to consider a situation in which two countries (North and South) are identical in every respect except that property rights to are weaker in South. In this case, South has an "apparent" comparative advantage in the forestry sector. The advantage is based on an institutional failure (imperfect property right) rather than a "real" comparative advantage. North always gains from trade, but South may either gain or lose. The leading paper in this literature is Chichilinisky's 1994 AER article.

These notes consider two types of imperfect property rights. I first consider a common property resource, and then a model with poachers. The two stories are slightly different, but the important properties of the models are the same.

3.4.1 A common property resource

Suppose that there are n firms in the forestry sector, where n is fixed. All firms can use the forestry resource (natural capital). Each firm's share of total forestry output is equal to their share of the purchased input (labor). Think of labor as being like "effort", so each firm's share equals their share of total effort. Here we have a common property resource, as distinct from an open access resource. Under open access, n is endogenous. With open access and free entry, all rents to the forest resource would be dissipated.

Let $F(L)$ be total output in the forestry sector, where L is aggregate labor in the forestry sector (not aggregate economy-wide labor here). We suppress the sector-specific argument, forestry capital, since that is held fixed for this static model. If firm i hires L_i units of labor, its share of total labor output is $\frac{L_i}{L}$ and its profits are

$$\pi_i = pF(L)\frac{L_i}{L} - wL_i,$$

where p is the price of forestry output. (We take the price of manufacturing as the numeraire.)

We want to obtain the Nash equilibrium in which each firm chooses the amount of labor it hires. Firm i chooses L_i taking $\sum_{j\neq i} L_j$ as given. Recall that$L_i + \sum_{j\neq i} L_j = L$. Using this relation, Firm i's first order condition in a Nash equilibrium is

$$p\frac{L_i F'}{L} + p\frac{(L - L_i)\,F}{L^2} = w.$$

In a symmetric Nash equilibrium $L_i = \frac{L}{n}$, so the first order condition can be written as

$$p\left(\frac{1}{n}F' + \frac{n-1}{n}\frac{F}{L}\right) = w. \tag{50}$$

Note that the function on the left side of equation (50) is a convex combination of the value of marginal product and the value of average product. The graph of this convex combination is shown as dashed curve in figure 3. The equilibrium amount of labor in the sector is L_{cp} (cp for common property), which is less than the open access level, L_{oa}, and more than the level under perfect property rights, L_{ppr}. For $n = 1$ we have perfect property rights, and for $n = \infty$ we obtain the open access equilibrium. With open access, all rents to the forestry resource are dissipated.

3.4.2 Poachers

In this model, owners of the forest resource have *de jure* but not *de facto* property rights, because they are not able to keep poachers from using the resource illegally. A poacher in forestry has to spend the fraction γ of her time in order to avoid detection. The amount of labor in the forestry sector is

$$L = L^e + (1 - \gamma) L^p, \tag{51}$$

where L^e is the number of legal workers (measured in hours or days or months...) employed by the legal owner, and L^p is the number of poachers (measured in the same units of time).

In this section I again assume that there are constant returns to scale in the both sectors. The owners of the sector-specific factors are the residual claimants. As we saw from equation (48), the assumption of constant returns to scale means that *when labor is hired up to the point that its value of marginal product equals the wage,* then the sector-specific factor earns its value of marginal product . We will see in the current model that the "wage equals value of marginal product of labor" condition does not hold in the forestry sector (i.e. it does not hold in the sector with imperfect property rights to the sector-specific asset). Instead of obtaining its value of marginal product, the legal owner of the forest (as the residual claimant) obtains the value of forestry output, minus the amount that was stolen by poachers, minus the wage bill paid to legal workers.

Workers decide whether to work in the cloth sector or in forestry, either as legal employees or poachers. While they are working (instead of avoiding detection) poachers are as productive as legal workers. Therefore, for each unit of time spent working (as distinct from avoiding detection) a poacher receives the value of average product of labor in Forestry, $\frac{pF(L)}{L}$.

As a legal worker, a worker receives the wage w. As a poacher, a worker obtains $(1 - \gamma) \frac{pF(L)}{L}$ per unit of time. If a worker is just indifferent between poaching and working legally, it must be the case that

$$(1 - \gamma) \frac{pF(L)}{L} = w. \tag{52}$$

We assume that in equilibrium production of cloth is positive. Therefore, workers never strictly prefer to poach rather than to work legally. (If they did strictly prefer to poach, no workers would be in the cloth sector, violating our assumption that production in the cloth sector remains positive.)

The parameter γ determines the extent of property rights. If $\gamma = 0$ there is open access; larger values of γ are equivalent to stronger property rights. If γ is greater than a critical level

$\bar{\gamma}$ (defined below) poaching is so unattractive that workers would rather work legally than as poachers. In this case, the Forestry firm ignores poachers. We take γ as exogenous and assume that it is positive but less than the critical level. Therefore, equation (52) holds in equilibrium.

An example may help to clarify this model. The activity in the forestry sector consists of gathering firewood. An increase in the number of workers in forestry lowers the marginal product of each worker, because with a larger number of workers there is more competition for finding the firewood. Suppose that an individual has 50 hours per week available, and suppose that $\gamma = 0.1$. A poacher needs to spend 10% of her time avoiding detection, so a full-time poacher can spend 45 hours a week looking for firewood. Suppose that the total number of person hours of labor (working) per week in forestry is 500; this is equivalent to 10 person-weeks (i.e., 10 people working full time for one week looking for firewood).[12] The total value of forestry output is $pF(10)$. The argument of F is person-weeks (rather than hours), so the wage in this example is the weekly (not the hourly) wage. A legal worker earns w per week. A poacher spending one week in the sector contributes the fraction $45/500 = 0.09$ of the total amount of labor time and therefore captures this fraction of the total value of the firewood. In a week, the poacher earns

$$(1 - 0.1)\,\frac{pF(10)}{10} = 0.09pF(10).$$

The agent is indifferent between working as a poacher or as a legal worker (in either sector) only if the return to poaching equals the legal wage.

Hotte et al (JDE, 2000) show that if it is optimal for the Forestry firm to operate, i.e. $L^e > 0$, and assuming that $\gamma < \bar{\gamma}$ (the critical value defined below) then in equilibrium $L^p = 0$ and $L^e = L$, the value that satisfies equation (52). The basis for this result is simply that legal workers are more productive than poachers, because legal workers can work full time rather than spending a fraction of their time evading detection. In addition, both workers receive the same wage, by virtue of the equilibrium condition (52). If poachers were present in equilibrium (i.e. if $L^p > 0$), the Forestry owner could induce one poacher to leave by hiring $1-\gamma$ additional workers. Hiring more legal labor, holding L^p constant, reduces the left side of equation (52). This makes poaching unattractive, so poachers leave the industry, until the number of remaining poachers is such that the equilibrium is restored. At a constant wage, hiring the additional $1-\gamma$ workers costs the firm $(1-\gamma)\,w$. However, the value of production of the additional worker

[12] The number of people involved is not important, only the total amount of time spent working.

is w. Therefore, when $L^p > 0$, hiring the additional $(1-\gamma)$ worker raises firm profits by γw. This opportunity to increase profits is exhausted only when $L^p = 0$; therefore, in equilibrium $L^p = 0$ must hold.

To establish this claim mathematically, define L as the solution to equation (52). Note that for $L^e < L$ (i.e. for $L^p > 0$), the amount of labor working in forestry is L. That is, when $L^e < L$ a change in L^e induces a change in L^p just sufficient to keep the value of L unchanged. For $L^e \geq L$ (i.e. for $L^p = 0$) the amount of labor working in forestry is L^e. The forest owner obtains profits

$$\pi = \begin{cases} \frac{L^e}{L} pF(L) - wL^e \text{ for } L^e \leq L \\ pF(L) - wL^e \text{ for } L^e \geq L \end{cases} . \tag{53}$$

The derivative of profits is

$$\frac{d\pi}{dL^e} = \begin{cases} \frac{pF(L)}{L} - w > 0 \text{ for } L^e \leq L \\ pF'(L) - w \text{ for } L^e \geq L \end{cases} .,$$

where the inequality in the first line uses the equilibrium condition (52). Thus, the graph of forestry profits, π, is an increasing function of L^e whenever $L^p > 0$. Therefore, the optimal level of L^e must drive L^p to zero.

We can use the previous result, that $L^p = 0$ in equilibrium, to find the critical value $\bar{\gamma}$. Recall that for $\gamma > \bar{\gamma}$ poaching is so unattractive that no one wants to poach, even when forestry owners do not behave strategically (by increasing the amount of legal labor beyond the point where their value of marginal product equals their wage). Define L^* as the socially optimal (equal to the profit maximizing) level of labor in forestry in the absence of poaching, i.e. the value that solves

$$pF'(L^*) = w. \tag{54}$$

The critical level $\bar{\gamma}$ is the solution to[13]

$$(1-\bar{\gamma}) \frac{pF(L^*)}{L^*} = w. \tag{55}$$

Provided that $0 < \gamma < \bar{\gamma}$, the equilibrium amount of labor in the forestry sector is greater than the first best level and less than the open access level. (The open access level of labor is

[13]Define $h(\gamma) \equiv (1-\gamma) \frac{pF(L^*)}{L^*}$; $h(\gamma)$ is a decreasing function of γ. In addition $h(0) = \frac{pF(L^*)}{L^*} > pF'(L^*) = w$, because average product is greater than marginal product for a concave function. Also, $h(1) = 0 < w$. Thus, by the mean value theorem there exists $0 < \bar{\gamma} < 1$ which satisifes equation (55).

the value of L that satisfies equation (52) when $\gamma = 0$.) That is, the graph of $(1-\gamma)\frac{F}{L}$ lies strictly above the graph of $F'(L)$. When $\gamma < \bar{\gamma}$, poaching is sufficiently attractive (i.e. property rights are sufficiently weak) that the forest owner wants to hire labor strategically, as a means of discouraging poachers from entering.

Figures 3*a and 3*b provide a graphical representation of the results above. Remember that L is the solution to equation (52) and L^* is the solution to equation (54). Figure 3*a shows the case where $L^* < L$ and figure 3*b shows the case where $L^* > L$. You should confirm that these two inequalities imply $\gamma < \bar{\gamma}$ and $\gamma > \bar{\gamma}$, respectively. The dashed curves are the graphs of $pF(L^e) - wL^e$ and the solid curves (which are defined only for $L^e \leq L$) are the graphs of $\frac{L^e}{L}pF(L) - wL^e$. Thus, the graph of the forest owner's profits, defined in equation (53), is the solid curve for$L^e \leq L$ and the dashed curve for $L^e \leq L$. Profits are continuous, but there is a kink at L. Figure 3*a shows that when $\gamma < \bar{\gamma}$ the forest owner's optimal level of L^e is L. In this case, the potential entry by poachers causes the forest owner to hire labor strategically, i.e. as a means of discouraging entry by poachers. Figure 3*b shows that when $\gamma > \bar{\gamma}$ the potential entry by poachers does not affect the owner's choice of labor; in this case, the owner wants to hire more than the amount of labor (L) needed to discourage entry by poachers.

The fact that there are no poachers *in equilibrium* does not mean that the possibility of poaching has no effect on the equilibrium. When property rights are sufficiently weak (i.e., when $\gamma < \bar{\gamma}$), the possible entry by poachers induces the forestry owner to hire more workers in order to make it less attractive to poach. In equilibrium poachers do not actually enter, but the threat of their entry causes there to be too much labor in the forestry sector (relative to the first best). By assumption, in this model, the forestry owner is not able to hire guards or to use any other means of protecting property rights; the only way that he can reduce the number of poachers is to reduce the return to poaching, i.e. to hire enough legal workers to drive the average product to a low enough level, so that poaching is unattractive.

3.4.3 Comparative statics of poaching model

The two models of imperfect property rights differ in their particulars, but in each model, for any commodity price too much labor is attracted to the forestry sector, relative to the first best (with perfect property rights). This feature is important for the results that we care about. We consider the poaching model.

At this point it is important to explicitly include the levels of sector specific capital in cloth and forestry. I previously suppressed these arguments in order not to encumber the notation. Now I need them because I want to determine the price of factors (the wage and the rental rates for the two sector specific factors. Denote these factors as K and f, the factors that are specific to the cloth and forestry sectors, respectively. Normalize the amount of labor to 1; let L be the amount of labor in forestry, so that $1 - L$ is the amount of labor in the cloth sector. We assume that the economy is incompletely specialized, i.e. that $0 < L < 1$.

Recall that I assumed constant returns to scale in both sectors. This assumption allows me to normalize the number of firms in each sector to 1. If the total amount of forest land is f, it does not matter if there is a single firm that owns the entire stock, or a number of firms who own possibly different amounts of the stock. The rate of return to capital in the cloth sector (i.e. the amount that a firm would pay to rent a unit of that capital) equals its value of marginal product (which also equals the share of residual value, i.e. the value of cloth production minus the wage bill):

$$r_c \equiv C_K(K, 1 - L) = \frac{C(K, 1 - L) - w\,(1 - L)}{K}. \tag{56}$$

The rental price of each unit of the forestry asset is

$$r_f \equiv \frac{pF(f, L) - wL}{f}. \tag{57}$$

Note that the rental value of the forestry asset is not equal to the value of marginal product of this asset. The reason is that in the forestry sector, labor is paid more than its value of marginal product. Labor earns its value of marginal product in the cloth sector. The equilibrium condition for labor employment in the cloth sector, and the equilibrium condition for labor employment in the forestry sector (equation (52)), together with the fact that $L^p = 0$, imply

$$w = C_L(K, 1 - L) = (1 - \gamma)\, p \frac{F\,(f, L)}{L}. \tag{58}$$

These two equations determine the labor allocation and the wage.

The effect of property rights on real returns to factors, at constant p We continue to assume that $\gamma < \bar{\gamma}$. At a constant relative commodity price, p, stronger property rights (larger γ) lowers the wage and increases the returns to the sector-specific factors in both sectors. Since the commodity prices are held fixed for this experiment, stronger property rights decrease the real return to labor and increase the real return to both sector-specific factors.

Figure 4 graphs the equilibrium condition (58). The dashed line shows pF'. Since $(1-\gamma)\,p\frac{F}{L} > pF'$ (because $\gamma < \bar{\gamma}$) as noted above, the amount of labor in the forestry sector (L) is greater than the amount under perfect property rights. From figure 4 it is clear that decreasing γ (weakening property rights) increases the amount of labor in forestry. (In figure 4 as we move right to left on the horizontal axis, the amount of labor in forestry increases.) Using either figure 4 or the equilibrium conditions (58), we have

$$\frac{dL}{d\gamma} < 0; \quad \frac{dw}{d\gamma} < 0. \tag{59}$$

Now consider the effect of property rights on the return to the sector specific factors. Again, it is clear from figure 4 that the larger amount of labor in cloth ($1-L$) caused by stronger property rights (and lower w) increases the marginal productivity of capital in the cloth sector, $C_K(K, 1-L)$, leading to a higher r_c (using equation (56)). Formally, we have

$$\frac{dr_c}{d\gamma} = -\frac{\partial C_K(K,1-L)}{\partial L_c}\frac{dL}{d\gamma} > 0.$$

For the forest sector we have

$$\frac{dr_f}{d\gamma} = \frac{(pF_L(f,L)-w)\,\frac{dL}{d\gamma} - L\frac{dw}{d\gamma}}{f} > 0.$$

The last inequality uses the fact that in equilibrium $pF_L(f,L) - w < 0$ (see figure 4) and inequality (59).

Since this experiment holds fixed the price of the numeraire and p, the price of the forestry good, the conclusions

$$\frac{dr_f}{d\gamma} > 0, \frac{dr_c}{d\gamma} > 0 \text{ and } \frac{dw}{d\gamma} < 0$$

mean that an increase in property rights lowers the real return to workers and increases the real return to owners of the sector-specific factors. Owners of the two specific factors always prefer stronger property rights to the forestry resource, and workers prefer weaker property rights. In this sense, owners of forestry and cloth capital are "natural allies" against labor. Both this result, and the conclusion that weaker property rights confers a comparative advantage on the forestry sector, can be overturned in a slightly more general setting.

The effect of property rights on the autarchic equilibrium Figure 5 graphs the production possibility frontier. Since there is a single mobile factor, and that factor is fully employed, production always takes place on the PPF. (In a more general model, a competitive equilibrium is consistent with production inside the PPF.) Figure 5 shows that under perfect property rights, the autarkic equilibrium would be at point A, where the indifference curve is tangent to the PPF. Under perfect property rights the autarkic price is p^*. The figure shows the IEP associated with p^*.

Suppose that property rights are imperfect. We saw above that weak property rights increases the supply of forestry for any price. In particular, at price p^*, production occurs at a point such as B, i.e. a point associated with more forestry and less cloth production, relative to A. When price is p^*, production is at B; the national income accounting identity (which states that the value of production equals the value of consumption) means that the economy wants to consume at C (the intersection of the economy-wide income constraint and the IEP associated with p^*. Therefore, at p^* there is excess supply of forestry. Consequently, with imperfect property rights, the equilibrium autarkic price must be lower than p^*. The production point must be between points A and B.

3.4.4 Trade in the poaching model

Suppose that two countries, North and South, are identical in their tastes and technology, but North has perfect property rights and South has imperfect property rights. (Of course this is not a descriptive model, since the two regions differ with regard to many features.) The autarkic price in North is p^* and the autarkic price in South is $p^s < p^*$. In this situation, South's comparative advantage in forestry is due entirely to an institutional failure, its weak property rights. If the two countries trade, South increases its forestry production and exports the forestry product. (In order to confirm this, use the excess supply and demand curves developed in Notes #2).

The market failure (imperfect property rights) causes South to have too much labor in the forestry sector. Opening up to trade causes even more labor to move to the forestry sector, exacerbating the market failure, tending to reduce South's welfare. On the other hand, South enjoys the usual sorts of gains from trade, which tends to increase it welfare. Thus, the net welfare effect (for South) of opening up to trade is ambiguous.

However, if opening up to trade causes only a small increase in the relative price of forestry (for South), then trade is unambiguously harmful to South. (Problem set #3 shows an analogous result in a partial equilibrium setting.) This result is easiest to show using mathematics.

Denote $V(y,p)$ as South's indirect utility function (i.e. the indirect utility function for the representative agent in South), where $y = pF + C$ is South's income, evaluated at the world price p. The differential of income is

$$dy = Fdp + pdF + dC \tag{60}$$

In the absence of trade restrictions or tax policies, world price equals domestic prices. By Roy's identity, consumption of the forestry product in South is

$$F^{con} = -\frac{V_p}{V_y}. \tag{61}$$

The superscript "con" denotes consumption; F and C denote production of forestry and cloth. From figure 5, production occurs at a point where the PPF is steeper than the relative price that producers face p. Therefore, at the production point

$$\frac{dC}{dF} < -p. \tag{62}$$

We want to examine the welfare effect of an increase in p; that is, we assume that $dp > 0$ Totally differentiate the indirect utility function to obtain

$$dV = V_p dp + V_y dy.$$

Now divide by V_y and use equations (60) and (61) to obtain

$$\begin{gathered} \tfrac{dV}{V_y} = \tfrac{V_p}{V_y} dp + dy = \\ -F^{con} dp + Fdp + pdF + dC = A + B \end{gathered} \tag{63}$$

where I used the definitions

$$A \equiv (F - F^{con})\, dp > 0 \;\text{ and }\; B \equiv \left(\frac{dC}{dF} + p\right) \frac{dF}{dp} dp < 0.$$

First, note that $\frac{dV}{V_y}$ is a measure of the change in real income. To see this, identify the units of this ratio:

units V	:	utils
units dV	:	change in utils
units V_y	:	change in utils divided by change in income.

Therefore,

units $\frac{dV}{V_y}$: change in utils over change in utils divided by change in income equals change in income.

We know from equation (63) that the change in real income equals the sum of two terms, which I denoted as A and B. First, $A > 0$, because South exports forestry products ($F - F^{con} > 0$) and $dp > 0$. The fact that $A > 0$ simply says that an increase in the price of exports raises real income. We know that $B < 0$ because equation (62) implies that $\frac{dC}{dF} + p < 0$, and we know that an increase in the price of forestry goods increases production in that sector, i.e. $\frac{dF}{dp} > 0$. (Confirm this inequality using the equilibrium condition (58).) The lack of perfect property rights causes excessive production in the forestry sector, and a higher price of forestry exacerbates that imperfection.

If South begins to trade at a world price p that is only slightly higher than its autarkic price, then it unambiguously loses from trade. In this case, we can evaluate the total welfare effect, $\frac{dV}{dp}$ at South's autarkic price, where $F - F^{con} = 0$ (because domestic supply equals demand in autarky), so $A = 0$. However, even in autarky $B < 0$, so the total welfare gain is negative. Once again, I emphasize that Problem set 3 shows an analogous result in a partial equilibrium setting.

3.5 Minimum wage

Now we will use the specific-factors model to study the role of trade in a model with a labor distortion. In view of the theory of the second best, we know that opening an economy to trade in the presence of such a distortion can either increase or decrease welfare. The purpose of this exercise is to understand what determines the welfare effect, and to give you practice working with the specific-factors model. (We're back to umbrellas and corn.)

Suppose that for some institutional reason, the "umbrella wage", $w = \frac{W}{p_u}$, is not allowed to fall below w^*. We want to find the competitive equilibrium in the presence of the constraint $w \geq w^*$. The reason for this constraint is not important – we take it as exogenous. We can imagine that workers consume only umbrellas, so the utility of an employed worker depends only on the number of umbrellas s/he can buy. There exists an institutional requirement that employed workers receive the level of utility no lower than the level of utility achieved by consuming w^* umbrellas. Obviously, a more plausible model would define the minimum wage in terms of a bundle of commodities. However, we use models of this sort to improve our intuition – not to describe accurately the real world

As the relative price of corn (p) falls (i.e., the relative price of umbrellas rises), w will fall. If p falls to the level p^* in figure 6, the constraint becomes binding with full employment of labor.

Further decreases in p, below p^*, (e.g., to p') make it impossible to satisfy the wage constraint and also maintain full employment. The result is unemployment. The amount of labor in corn manufacturing must fall, in order to maintain the minimum wage.

Think of it this way: Suppose that the wage constraint is exactly binding, and suppose that the nominal price of umbrellas remains constant while the nominal price of corn falls, so p (the ratio of the price of corn to the price of umbrellas) falls. In order to maintain the wage constraint, the nominal wage must remain the same. Nothing has changed in the umbrella sector, so that sector continues to employ the same number of workers. However, since the nominal wage remains the same while the nominal price of corn falls, corn producers are willing to hire fewer workers. (The value of marginal product of workers in the corn sector falls). As the corn sector sheds workers, the amount of unemployment rises, and corn production falls. For example, at relative price p', the amount of unemployment is L'.

Figure 6 shows the PPF in a minimum wage economy ($A'BC$ rather than ABC). (At this point ignore all of the lines and curves in the figure with the exception of the "unconstrained" PPF ABC and the PPF in the presence of the wage constraint, $A'BC$.) Given the minimum wage, an economy in a competitive equilibrium (without some kind of intervention such as a subsidy or a tax) cannot produce at a point above B. At such points the minimum wage constraint is violated. However, any point between A' and B is feasible; such points involve different levels of unemployment.

3.5.1 Autarky with a binding minimum wage

Now we consider the autarkic equilibrium in the presence of the minimum wage. Price p^* is the price that induces full employment and at that price, $w = w^*$. That is, p^* supports production at point B in figure 7). Define p^{af} as the autarkic price in the absence of the minimum wage constraint. Recall that p^* is the price (not necessarily an equilibrium price) at which there is full employment and $w = w^*$ (i.e. the minimum wage constraint is exactly binding.) I want to consider two cases: first where the wage constraint is not binding, then where it is binding in autarky.

If the autarkic equilibrium lies at a point on BC, then the autarkic price in the absence of the constraint, satisfies $p^{af} > p^*$. In this case, the minimum wage constraint is not binding, and the constraint is therefore not interesting. The only situation that is interesting where the minimum wage constraint is binding in the autarkic equilibrium. Suppose then, that the autarkic equilibrium in the absence of the minimum wage constraint (p^{af}) is at a point on AB, such as point E in figure 8. At this point, the minimum wage constraint is not satisfied. That is, production and consumption point E, and associated price p^{af}, is not feasible in the presence of the minimum wage constraint.

When $p^{af} < p^*$ (so that the minimum wage is binding under autarky) there exists a unique equilibrium with production and consumption on $A'B$ (to the left of B). There is unemployment in the unique autarkic equilibrium.

Existence: By construction, at price p^* production occurs at point B and consumption occurs at a point such as D, to the left of B (figure 8) (Remember, $p^* > p^{af}$. At the higher relative price of corn, the IEP shifts away from the corn axis). Thus, at p^* there is excess supply of corn, as shown in figure 10: p^* is clearly not an equilibrium price. As p decreases, production of corn falls, so the supply curve is monotonic in price. (Here review the basic comparative statics of figure 3.) At a sufficiently low price (perhaps 0) supply of corn is 0. At a low price of corn, the demand for corn is strictly positive . Therefore, at a low price there is excess demand for corn, as shown in figure 10. Since the supply and demand graphs are continuous, there must be a point of intersection, i.e. an equilibrium price $p < p^*$. We know that at such a price there is unemployment.

If the demand curve were monotonic, uniqueness would be automatic (With monotonic demand, excess demand is also monotonic - because we know that supply is monotonic. Consequently there would be a single price at which excess demand is 0.) However, the demand curve is not necessarily monotonic, as indicated in figure 10. (The inverse demand curve may be a correspondence rather than a function.)

Nevertheless, for this economy there is a unique equilibrium autarkic price. We can show uniqueness using a proof by contradiction. The hypothesis that we want to falsify is that there are two equilibrium prices. Suppose that there exist two equilibrium prices, p' and p''(figure 6); both satisfy the wage constraint and they both equate supply and demand. We want to derive a contradiction. Price p' induces production at some point on AB, say point d'. Since p' is an equilibrium, supply equals demand, so the income expansion path must intersect AB at d', as shown. If $p'' < p'$ production of corn is lower at p'', i.e. production moves to a point such as e. The national budget constraint is the line through e with slope p'' Since $'' < p'$, the income expansion path IEP(p'') cannot lie above the IEP(p'). If there is some substitutability in consumption, IEP(p'') lies strictly below IEP(p') as shown in figure 6. Consumption occurs at point f, where there is excess demand for corn. Hence $p'' < p'$ cannot be an equilibrium. The same kind of argument shows that $p'' > p'$ cannot be an equilibrium.

3.5.2 Trade can lower welfare

Now consider an example where trade lowers welfare: Suppose that the autarkic equilibrium is at d' (figure 7) with autarkic equilibrium price p'. The country begins to trade, and the world relative price of corn is $p'' < p'$ (= the autarkic price). Then we know that when the country begins to trade (and the domestic price of corn falls) production shifts toward e, and the income expansion path shifts down, as shown. Consumption occurs at f, resulting in a loss of welfare (for this particular case).

Two things have happened here. Consumers face a world price different from the autarkic price. If production had remained at the autarkic level (e.g., if this was a "pure endowment" economy) welfare would have increased. (How do you find the level of consumption in this case? How do you show that welfare must have increased in this case?) However, the lower world price exacerbates the unemployment problem, leading to a production loss. The way I have drawn this, the production loss more than offsets the consumption gain, leading to a loss in welfare.

Is there a role for a tariff in this model? Do you think that is the first best policy? What would have happened if the world price had been above p'?

3.5.3 Adjustment of the specific factors can lower welfare

The situation I've just described is a short run model: labor adjusts, but the sector-specific factors don't move at all. Now we will see what happens in the long-run, when we allow the sector specific factors to adjust as well. In this model, the sector specific factors are "sector specific" only in the short term. These kinds of factors are sometimes called "quasi-fixed". In the long run, they are flexible.

There are a number of ways that we can view adjustment of the quasi-fixed factors. For our purposes, the particular adjustment process does not matter – I just want to compare a short run equilibrium in which capital earns a different return in the two sectors, and a long run equilibrium in which the return to capital is the same in both sectors. The point of this extension is to show that an increase in economic flexibility may increase the welfare loss resulting from trade (in the presence of a minimum wage constraint).

Here is a "putty-clay" model. Initially capital can be used in either sector. At this stage capital is putty. However, once invested in a particular sector (e.g. as an irrigation system in the corn sector or an umbrella machine in the umbrella sector) it becomes clay – it cannot be transformed back into putty. The flow of new capital (putty) is exogenous. This flow is allocated between the two sectors by investors who seek to maximize their return. All capital within a sector (that is, capital of all "vintages") obtain the same rental price Capital depreciates in both sectors. In a steady state (i.e., a long-run equilibrium), the amount of new investment in a sector equals the depreciation in that sector.

Suppose that we begin at a long run autarkic equilibrium (a steady state). There is no trade, capital earns the same return in the two sectors. The assumption that capital decays and the assumption that the capital stocks in the two sectors are unchanging means that both sectors must be receiving new investment. Consequently, the return to the specific factors in the two sectors must be the same. It would not pay to invest in a sector if the other sector offers a higher rate of return. Remember that capital's rental price equals its value of marginal product.

We'll assume that the minimum wage constraint is binding, so that there is unemployment in this long run equilibrium. Note that in the minimum wage economy, unemployment is consistent with long run equilibrium. In the "long run" capital adjusts to achieve equality of rental rates in the two sectors; there is no reason that this equality eliminates unemployment, which is caused by the minimum wage constraint. Assume also that the world relative price of corn is lower than the autarkic relative price. When trade begins, the domestic relative price of corn falls to the world price. As we have seen, this fall causes the corn sector to shed labor. The economy moves from d' to f in figure 7.

The price change and the resulting change in labor allocation both reduce the value of marginal product (which equals the rental price) of the corn-specific factor falls. The fall in the price of corn causes the value of marginal product to the corn-specific factor to fall, holding constant the amount of labor in the sector. As labor leaves the sector during the short-run labor adjustment process, there are fewer workers per machine in the sector, causing a further fall in the value of marginal product of the corn-specific factor. Consequently, the return on capital in the corn sector falls. No one invests new capital in the corn sector because the return is higher in the umbrella sector.

Since there is no investment in the corn sector, depreciation causes the stock of capital in the sector falls. The increased flow of investment in the umbrella sector causes the stock of capital in that sector to increase. Prior to beginning trade, the flow of new investment into the umbrella sector was just enough to offset depreciation. Since (by assumption) the aggregate flow of new capital is exogenous, the decreased investment in corn means that investment in umbrellas increases. As more capital moves to the umbrella sector, the flow of investment is greater than depreciation, so the stock of capital grows. With this putty-clay model, it is as if capital were leaving the corn sector and moving to the umbrella sector, although that is not literally occurring.

Capital flows to the sector where its marginal return is higher. That's a good thing, right? In a distorted economy, the answer is "maybe". As capital leaves the corn sector, that sector sheds more labour, worsening the unemployment problem. Of course, as capital moves into the umbrella sector, that sector hires more labor. The net effect on employment depends on whether the dying sector (corn) is labor intensive, relative to the umbrella sector. If the dying sector is relatively labor intensive, the welfare effect of the readjustment of capital is ambiguous. The increase in unemployment tends to offset the efficiency-increasing effect of having capital move to a sector where its marginal product is higher. If the growing sector is relatively labor intensive, then of course the adjustment of capital improves welfare, since both effects increase welfare.

Figure 9 shows two possibilities. After opening to trade, but before capital adjusts, the economy produces at point f. Its BOP constraint is the line through f with slope equal to the world price p''. The adjustment of capital draws capital into the umbrella sector, and the price of the mobile factor, labor, is constant; therefore, umbrella production increases. The economy moves to a point above the line $A'B$, It might move to a point such as r, where national income (and therefore welfare) has increased, or to a point such as s, where national income and welfare are lower.

3.5.4 An algebraic treatment

Here I specialize to constant returns to scale, in order to show in simple manner that adjustment of the quasi-fixed factor might lower welfare. (I use the assumption of constant returns to scale in order to write the production function in each sector as the product of the amount of capital in that sector times a function that contains a single argument, the capital/labor ratio in that sector.) The production functions are

$$\begin{aligned} C &= H\left(L_c, K_c\right) = K_c h\left(n_c\right) \\ U &= G(L_u, K_u) = K_u g(n_u) \end{aligned} \tag{64}$$

where labor/capital ratios in the two sectors are

$$n_c \equiv \frac{L_c}{K_c}, \quad n_u \equiv \frac{L_u}{K_u}.$$

Equality of wages fixes the two labor/capital ratios:

$$w^* = ph'\left(n_c\right) = g'\left(n_u\right). \tag{65}$$

After the economy opens to trade, p and w^* remain constant. Therefore, the capital labor ratios in the two sectors remain constant, as equation (65) shows. However, as capital adjusts from one sector to another (i.e., as the levels of K_c and K_u change), the production in the two sectors change. The change in output resulting from the reallocation of capital is

$$dC = h\left(n_c\right) \cdot dK_c \quad dU = g(n_u) \cdot dK_u. \tag{66}$$

Consistent with the scenario in the previous subsection, we assume that following trade liberalization the rental rate in the corn sector is below the rental rate in the umbrella sector. That is

$$r_c = p\left(h\left(n_c\right) - n_c h'\left(n_c\right)\right) < r_u = g\left(n_u\right) - n_u g'\left(n_u\right). \tag{67}$$

The total demand for labor is

$$L = K_c n_c + K_u n_u. \tag{68}$$

Suppose that dK units of capital move from umbrellas into corn, i.e. $0 < dK = -dK_c = dK_u$. Differentiate equation (68) to obtain the effect of capital reallocation on the total demand for labor:

$$dL = \left(n_u - n_c\right) dK.$$

Recall that the capital labor ratios are fixed, because these are determined by equation (65).

As capital moves from one sector to another, commodity prices do not change, since this is a small country. Therefore, welfare increases if and only if nominal income increases. Total (nominal) income in this economy is

$$y = pC + U. \tag{69}$$

Differentiating equation (69), and then using equations (66), (67), and (65), we can write the change in income as

$$\begin{gathered} dy = pdC + dU = \\ \left(-ph + g\right) dK = \\ \left[-\left(r_c + pn_c h'\left(n_c\right)\right) + r_u + n_u g'\left(n_u\right)\right] dk = \\ \left[r_u - r_c + n_u g' - pn_c h'\right] dK = \\ \left[\left(r_u - r_c\right) + \left(n_u - n_c\right) w^*\right] dK. \end{gathered} \tag{70}$$

The last line decomposes the income effect into two terms. The first term $\left(r_u - r_c\right) dK$, is positive, because both $\left(r_u - r_c\right)$ and dK are positive. This positive term reflects the fact that moving a factor into a sector where it obtains a higher return, increases national income. The second term, $\left(n_u - n_c\right) w^*$, is the employment effect. There is unemployment in this economy, so one additional unit of employment increases national income by w^*. As one unit of capital moves into the umbrella sector it employs n_u additional workers; however, as that unit of capital leaves the corn sector, it releases n_c workers. If the corn sector is relatively labor intensive, total employment falls, so the employment effect of the capital reallocation is $\left(n_u - n_c\right) w^* < 0$. In that case, the net effect of the capital reallocation can be positive or negative.

3.5.5 A different interpretation

Here's a slightly different interpretation of the results above, based on the concept of "shadow value". We noted above that in this small country model, a change in the allocation of capital increases welfare if and only if the change increases national income, which equals the value of production evaluated at world prices. The shadow value of capital in umbrellas (denoted sv_u) equals the change in the value of income due to a change in the stock of capital in umbrellas; the shadow value of capital in corn (denoted sv_c) equals the change in the value of income due to a change in the stock of capital in corn.

Recall that Chapter 2.1.3 showed that in the social planner's optimization problem (given in equation (16)), the Lagrange multiplier associated with the full employment constraint for capital equals the competitive rental rate of capital. The Lagrange multiplier for capital, also called the “shadow value” of capital, equals the marginal change in the social planner’s objective due to a marginal change in the amount of capital. In that setting – with no distortions – the private value of capital (the return to capital, it’s price) equals the social value of capital (the shadow value). A distortion in the economy drives a wedge between the private and social values of capital (and also of many other things).

In the current context, define the shadow value of capital (in each sector) as the change in national income due to a change in the amount of capital (in each sector). Repeating the steps used to obtain the last line of equation (70), we can write these shadow values as

$$
\begin{aligned}
sv_c &\equiv \frac{\partial y}{\partial K_c} = r_c + n_c w^* \\
sv_u &\equiv \frac{\partial y}{\partial K_u} = r_u + n_u w^*.
\end{aligned}
$$

Imagine that a social planner wants to choose the allocation of capital to maximize social welfare. Since the planner takes the world price as given, maximizing national welfare is equivalent to maximizing national income, evaluated a world prices. The social planner takes as given: (i) the world price (because this is a small country); (ii) the minimum wage constraint (because we treat this as exogenous); and (iii) the requirement that labor earn the same wage in both sectors. That is, the only variable that the planner can control is the allocation of capital between the two sectors. If we solved this problem to find the optimal allocation of capital, we could decentralize the optimum by means of a capital tax/subsidy in one sector.

At the social optimum, the shadow value of capital must be equal in both sectors; if these were not equal, the planner could increase income by moving capital to the sector where it has a higher shadow value. In the competitive equilibrium, however, capital moves to the sector where its *private value*, the rental price of capital r_u or r_c, is higher. The minimum wage creates a distortion in the labor market. This distortion drives a wedge between the private value of capital in a sector (its value of marginal product) and the social (shadow) value of capital in the sector. The social shadow value includes the employment-increasing effect of capital. The reallocation of capital causes the private value of the capital in the two sectors to approach each other (and eventually become equal); but as the difference in the private value of capital in the two sectors diminishes, the difference between the social value in the two sectors *might* actually increase. If the reallocation does cause a greater divergence in the shadow values of capital, then the reallocation decreases national income.

The general point of this example is that a distortion in one part of the economy (here, the labor market) creates a wedge between private values and social values in other sectors (here, the capital market). The intuition we develop from studying perfect markets may be misleading in a distorted market. Of course, this is just another application of the theory of the second best.