Chapter #13: Renewable Resources

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General Overview

Renewable resources are resources that *grow.* Unlike the case of nonrenewable resources in which a fixed stock is depleted over time, renewable resources are natural resources that can reproduce, grow, and die.

Economically important renewable resources include:
- Forests
- Fisheries
- Grasslands (used for grazing)
- Underground water (groundwater) fed by rainfall and surface water percolation

Issues for analysis
- Growth functions and equations of motion for renewable resource systems.
- Steady-state behavior of renewable resource systems.
- Open access, inefficient market outcomes, and policy corrections.
- Dynamic behavior of renewable resource systems.

A steady-state is a permanent level of stock that is maintained throughout time. An Equation of Motion is a formula that defines what happens to the stock over time. For example, when S_t denotes the resource inventory at period t:
- If $S_{t+1} - S_t > 0$, then the resource stock is growing over time
- If $S_{t+1} - S_t < 0$, then the resource stock is shrinking over time
- If $S_{t+1} - S_t = 0$, then the resource stock is in a steady state ($S_t = S_{t+1} = S_{t+2} \ldots$)

For clarity, we will consider only one stock variable (for example, only one species of fish) in each of our renewable resource models. More complex models consider multiple stock variables. Models with multiple stock variables may be required if, for example, two species interact in a predator-prey relationship.

We will also use a single indicator variable for the resource, which is Stock. Many times having only one indicator variable of a species is not effective. For example, when the renewable resource is trout in a lake:
> • Need to know the number of fish (**Stock**)
> • Also need to know the number of *juvenile* fish
> (**Cohorts**)

Yet, the use of a single indicator variable allows us to formulate tractable models that give us information about how we should manage the resource. These models can get quite complicated with multiple stock variables and more than one indicator variable.

For now, let us focus our analysis on *biological* renewable resources. We will consider non-biological renewable resources (e.g., groundwater) later in the course.

Economic models of biological resources combine biological models and economic models. For this reason, such combined models are called **bioeconomic models**. Shaefer's (1954) Biomass Model is a classic example of a biological model used in bioeconmic models. The following discussion of growth functions, steady-state, carrying capacity and equations of motion is based on Shaefer's model.

A Biological Model of A Fishery

Growth Functions
Let S_t represent the stock of a renewable resource at time t. For example, let S_t represent the biomass of a fish population at time t. Let $g(A_t, S_t)$ represent the **growth function** of the stock during time period t, where growth is a function of the level of the stock at the beginning of period t, S_t, and an exogenous parameter A_t, which represents factors other than stock which might affect growth in period t. This simple growth function neglects other biologically-important determinents of stock growth (e.g.'s: age distribution, sex distribution, size distribution), but its simplicity

allows us to concentrate on the dynamic interaction of economics and biology in renewable resource models. As previously mentioned, ore complex models take additional factors into account.

Carrying Capacity and MSS

Notice that growth is influenced by the level of the stock S_t, which represents fish biomass in our example. If the level of the stock is low, the number of births is greater than the number of deaths of fish so that the fish population grows. Yet, the growth of the fish population will be low, because few fish exist to reproduce. If the stock is high, food scarcity, predation, territorialism, and/or an increased incidence of disease may limit the growth of fish stock. If the stock is very high, growth may be negative for a period of time, that is there are fewer births than deaths, until the stock level falls to the **carrying capacity** of the environment.

The carrying capacity of an environment is the maximum level of stock that the environment can sustain indefinitely. *Carrying capacity is also called the maximum sustainable stock.* At the maximum sustainable level, births equal deaths, and the growth rate of the stock is zero (i.e., the fish population is constant, or in a steady-state). Without human intervention, the stock of fish will reach the carrying capacity of the lake and the population of fish will stay at the maximum sustainable stock indefinitely.

Steady-States

At the carrying capacity, the growth of the fish population is zero and the stock is in a **steady-state**. A steady-state is a situation in which the level of the stock is constant over time. There can be more than one steady-state. For example, a stock level of zero is also a steady-state, because at zero the stock level cannot rise (i.e., there are no fish to reproduce) and the stock level cannot fall below zero stock. Note that although the *growth* of the stock can be negative, the *level* of the stock cannot fall below zero.

The preceding discussion is often illustrated by using a graph like the one presented in Figure 13.1 below. For now, let's assume for simplicity that A_t does not influence growth, so we can drop A_t from the growth function.

Figure 13.1
Fisheries growth function g(S$_t$).
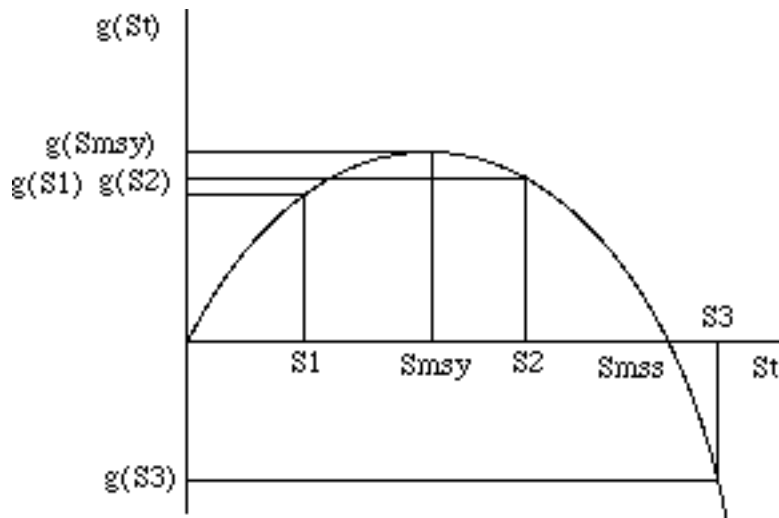
S_t = stock of fish (or other resource) at time t
$g(S_t)$ = biological growth rate of fish at time t

There are two steady-states in Figure 13.1: the origin and S_{mss}. To the right of the origin, the growth of the stock at first increases with the level of the stock (e.g., point S_1), but then food scarcity, disease, etc. cause growth to decline with further stock increases (e.g., point S_2). Eventually, carrying capacity S_{mss} is reached and growth falls to zero. To the right of S_{mss} (e.g., point S_3), the stock is at an unsustainably high level, and growth is negative (e.g., $g(S_3)$). To the right of S_{mss}, stock will fall until carrying capacity is reached. In order to conceptualize growth rate, think of it as fertility of a pool of fish over a period of time, minus the number of deaths in the pool of fish over that period.

Equation of Motion
    If we were to harvest an amount greater than the growth of the resource in a period, if $X_t > g(S_t)$, then the stock would be lower in the following period. Depending on the level of the stock, such a harvest could either decrease growth (e.g., if the level of the stock were S_1) or increase growth (e.g., if the level of the stock were S_2).

    We can express the relationships between stock level, growth and harvest in an **equation of motion** for the renewable resource:

$$S_{t+1} = S_t + g(S_t) - X_t$$

which states, in words, that "stock next period equals stock this period plus growth this period minus harvest this period." For example, a pond with five fish may each give birth to two more fish in a year. If five fish are caught by a fisher, ten fish will remain in the next year. It is important to note that in this formulation of the equation of motion, we are assuming that growth occurs before harvest in each time period.

We can rearrange the equation of motion as follows:
$$S_{t+1} - S_t = g(S_t) - X_t$$

In this form we see that the equation of motion says that "the *change* in the stock equals growth minus harvest."

If we are in steady-state, then we know that the change in the stock is zero:
$$S_{t+1} - S_t = 0 = g(S_t) - X_t$$
or, rearranging,
$$g(S_t) = X_t$$

Hence, in steady-state, growth equals harvest. Of course, if we are in steady-state and harvest equals zero, then growth $g(S_t)$ must equal zero, which is consistent with our earlier discussion about steady-states.

Sustainable Yields and MSY

We have yet to discuss the *optimal* harvest of fish. A sustainable yield of fish refers to the level of harvest which will result in a steady-state fish population. *Every point on the curve in Figure 13.1 represents a sustainable yield.* Of course, t is possible to harvest fish off the curve. Say we harvest a larger amount of fish than the growth of the fish stock, i.e., at a point above the curve. What happens then is that the overall stock decreases by the amount of the additional harvest and the system is not in a steady-state. The optimal policy, as we will show, is to maintain a steady-state, or sustainable, level of harvest through time.

Notice that if an amount of the stock X_t is harvested each period such that $X_t = g(S_t)$, then the stock will remain at level S_t. In this case the owner would be "harvesting only the growth" and leaving the stock at its original

level each period. Because the owner could then harvest $X_t = g(S_t)$ indefinitely, such a harvest level is termed a **sustainable yield**. There are many possible sustainable yields. In fact, each point on the $g(S_t)$ curve to the left of S_{mss} could be a sustainable yield if harvest were carried out such that $X_t = g(S_t)$. Note that the level of sustainable yield depends on the level of the stock S_t. That is, **every sustainable yield is associated with a steady-state stock of fish**.

The highest possible sustainable yield is called the **maximum sustainable yield**, denoted X_{msy}. In Figure 13.1, $X_{msy} = g(S_{msy})$. The level of the *stock* associated with X_{msy} is called "**the stock level that supports maximum sustainable yield**," denoted S_{msy} in Figure 13.1.

When biologists were first analyzing fishery dynamics, they noted that the largest level of sustainable harvest would be at the level X_{msy}. At last, they thought, we do not need economists to tell us what the optimal harvest level is for fish! The reasoning was that we would obviously be better off at the steady-state level associated with the highest annual harvest of fish. Needless to say, this reasoning was wrong.

Optimal Fish Harvest in Steady-State with Interest Rate of Zero

Before we develop a full, dynamic, economic model of a fishery, let's examine an example of fish harvest in steady-state. In this example, we will maximize the benefits minus the costs of fish harvest subject to the constraint that we maintain a steady-state. Note that this may not be socially optimal, because we still have not shown that it is socially optimal to be in a steady-state or described the conditions under which this premise holds. Thus, the problem we are about to solve is a second-best problem. Nonetheless, it is an important second-best problem because policy-makers and their constituents often want steady-state solutions due to the economic stability and predictability that they provide. Steady-state solutions are important to policy-makers because steady-state solutions are *sustainable* over the long run, i.e., the fish species do not become go extinct.

In steady-state, the fish stock is not changing, thus harvest must equal growth:
$$g(A_t, S_t) = X_t$$

When we examine a fishery in steady-state, the steady-state equation above serves as a *constraint.* Let's assume that our objective is to maximize the total benefits, $B(X_t)$, derived from fish harvest minus the total cost associated with fish harvest. Suppose that the total cost of fish harvest is $C(X_t, S_t)$.

> • $C(X_t, S_t)$ depends on the level of fish harvest, X_t, *and* on the stock of fish, S_t (this is because the fewer the number of fish, the more expensive it becomes to locate and catch them in the lake).
> If $C_x > 0$ (the more you fish, the more it costs, i.e., positive MC)
> If $C_s < 0$ (the cost of fishing declines as the level of stock increases)

Our choice variables are:
> • the level of harvest X_t, and
> • the stock of fish S_t.

We indirectly control S_t by changing X_t. To keep the example simple, let's assume a zero interest rate, so that we do not need to discriminate between different periods in time. Our problem is now:

$$\max_{X_t, S_t} \quad B(X_t) - C(X_t, S_t), \text{ subject to: } g(A_t, S_t) = X_t$$

Because we will be in steady-state, the fish stock and the harvest level will not be changing, hence the (second-best) levels of X_t and S_t will be the same for *every* time period. Thus, we only need to examine *one* of the time periods. So, the dynamic problem above becomes the following static problem:

$$\max_{X, S} \quad B(X) - C(X, S), \text{ subject to: } g(A, S) = X$$

where we have dropped the time subscripts because we only need to examine one of the time periods if we are in steady-state.

The Lagrangian expression for this problem is:

$$L = B(X) - C(X, S) + \lambda [g(A, S) - X]$$

and the FOC's are:

(1) $\quad \dfrac{dL}{dX} = \dfrac{dB(X)}{dX} - \dfrac{dC(X,S)}{dX} - \lambda = 0$

$$(2) \quad \frac{dL}{dS} = -\frac{dC(X,S)}{dS} + \lambda\frac{dg(A,S)}{dS} = 0$$

$$(3) \quad \frac{dL}{d\lambda} = g(A,S) - X = 0$$

FOC (1) says that price (price = MB = $B_X(X)$) equals the marginal cost of changing the harvest level plus the user cost of changing the harvest level.

FOC (2) says that the value of marginal product of the stock (g_S) equals the marginal cost of changing the stock level. The marginal cost associated with changing the stock level is the change in the cost of finding the fish in the ocean as the stock level declines.

In such a problem, how does the steady-state stock level, S_{ss}, compare with maximum sustainable yield stock level, S_{msy}? From FOC (1), = price - marginal cost with respect to X. Hence, FOC (2) can be rewritten as:

$$(P - Cx)g_S = C_S$$

Now, because C(X, S) falls as S increases, (i.e., $C_S < 0$), we know that $(P - Cx)g_S$ must be negative for FOC (2) to hold. Given P - Cx 0 in steady-state (which must hold for   0), then it follows that $g_S < 0$ in steady state. Look at Figure 13.1:
$$\text{if } g_S < 0, \text{ then } S_{SS} > S_{MSY}.$$

Thus, the steady-state (second-best) level of fish stock, S_{SS}, is greater than the level of fish stock that supports the MSY, S_{MSY}. This is because maintaining a larger stock of fish has the additional benefit of reducing the cost of fishing. Note that the steady-state (second-best) level of fish harvest, X_{SS}, is less than X_{MSY}.

Were the Biologists wrong?
The factor that biologists did not take into consideration is the economic reality that the cost of fishing decreases as the fish population becomes larger. We can reconcile the biologists' notion in the case where $C_S = 0$ (i.e., no stock effects in the cost of fishing).

When $C_S = 0$, FOC (2) is revised to read $g_S = 0$, which implies $g_S = 0$ since > 0. As we can see, the harvest associated with the maximum growth rate of fish, $g_S = 0$, is X_{MSY}.

## Open Access and Competitive Behavior

Without barriers to entry, competitive fishermen will not consider the future implications of today's fishing. Instead, they will maximize profit and, if there is a steady state, will harvest the resource according to:

Price = $B_X = C_X$ price = MC
$g(A,S) = X$ steady-state

Competitive industry members will not recognize that their harvesting depletes the resource, thus increasing production cost. When a resource has open access, each fisher believes that any fish he does not catch will simply be caught by another fisher, so that the gains to keeping higher stocks will not be realized anyway. Therefore, *under competition:*

(1) Stock is smaller than optimal.
(2) Output may be greater or smaller than is optimal.
(3) Social cost associated with extra harvesting will outweigh benefits from extra fish in cases where open access output is greater than optimal output.

Figure 13.2: There can be more than one steady state.

Steady State Outcomes: Optimal vs. Competition
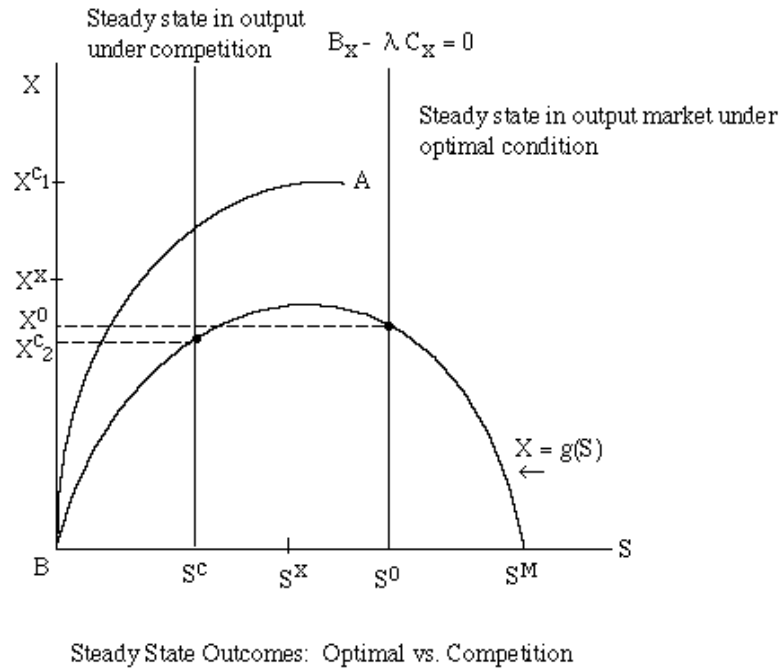
Referring to Figure 13.2, under competition, if C_X is relatively small and demand is big, there may be a period when $X^c > X^x$ and the resource can be depleted, for example, when $X^c = X^{c1}$. The marginal cost may be increasing as S declines and the possible output stock combinations under competition will follow the line AB and the steady stock with $X = S = 0$ is at B.

When the open access outcome does not involve a steady state, the fish stock can be sent to zero through extinction. This has come very close to occurring in ocean resources where many countries have competed against each other to catch certain species and have driven many of them near extinction. Under open access, each fisher seeks to extract every profitable unit he can, but is immizerized by the entry of new fishers that always drive the profits of fishing to zero. Hence, the fisher is always trying to reduce the costs of extracting the resource in order to gain temporary profit (private individuals are price-takers and only care about costs). The best way to lower extraction costs is to simply catch

everything, then trash unwanted fish species, and this is largely what we have seen:

> - **Drag Netting** is often used to dredge up all fish in an area
> - **Clear-cutting** of forests is another example

If the marginal harvesting cost approaches infinity as the stock approaches zero, a steady state may be reached with a very small stock level. The elimination of buffalo from the American West between 1860-1870 is an example of when open access and low C_X lead to extinction.

If C_X is substantial and under the optimal solution λ/C_x is small, competitive outcomes will not be far away from optimal outcome. Several intervention policies alter the open-access outcome including:

- Standards limiting the catch. This can be performed by:
  - Exhaustive capacity restrictions (i.e., limited the number of fish per boat)
  - Time restrictions.
- An output tax of   . Recall that the only difference between open access and the socially optimal solution is that open access fishers do not recognize the shadow value of maintaining the stock.
- Tax on fishing effort placed on labor or on the number of boats
- Moratorium on Fishing
- Change Length of Fishing Season (only encourages highly concentrated effort)
- Regulations on Technology (i.e., no gill-netting; no using "fish finders", etc.). Regulating technology, of course, is incredibly inefficient.

The Need For International Agreement

One of the things that is necessary to control open access problems is an International Authority to Monitor and Enforce controls on fishing so that fish populations are not unsustainably harvested by open access.

Renewable Resource Management in Steady-State

A Simple Bioeconomic Model of a Steady-State Fishery

Consider a price-taking fishing industry with:

> Fish Stock (in biomass units): S
>
> Growth Function of Fish Stock: $g(m, n, S) = mS - nS^2/2$.

Note that m and n are two parameters which we represented with parameter A in the last lecture. In this case:
- m might represent availability of food in the ecosystem
- n might represent the number of predators, but is constant, i.e., n n(S)

Fish Harvest (in biomass units):   X
Total Benefits of Harvest: $B(X) = P*X$

Total Costs of Harvest: $C(X, S) = \dfrac{bX}{S}$

We need to keep in mind that the larger the fish stock, the lower the cost of fish harvest. Assuming that the fishery will be in steady-state, the fishery's economic problem can be stated as:

$$\max_{X,S} \quad B(X) - C(X, S), \text{ subject to: } g(m, n, S) = X$$

First, note that we can find several important quantities without solving the optimization problem.

Maximum Sustainable Stock

Maximum sustainable stock S_{mss} occurs where $g(m,n,S) = 0$, hence:
$$S_{mss} = 2m/n$$

Of course, S_{mss} will only be achieved if the harvest $X = 0$.

Stock Level that Supports Maximum Sustainable Yield

The stock level that supports maximum sustainable yield, S_{msy}, occurs where $dg/dS = 0$:
$$S_{msy} = m/n.$$

Maximum Sustainable Yield

In steady state, $X = g(m,n,S)$. So, to find maximum sustainable yield, substitute S_{msy} into $g(m,n,S)$:
$$X_{msy} = g(m,n,S_{msy}) = m^2/2n$$

Optimal Steady-State Stock and Harvest Levels

To find the optimal steady-state stock and harvest levels, we need to solve the optimization problem outlined above. The optimal choice

problem can be solved using the Lagrange multiplier method. The Lagrangian expression for the problem is:

$$L = B(X) - C(X, S) + \lambda[g(m, n, S) - X]$$

The FOC's are:

(1) $\dfrac{dL}{dX} = \dfrac{dTB(X)}{dX} - \dfrac{dTC(X,S)}{dX} - \lambda = 0$

       or by substituting, $P - b/S - \lambda = 0$

FOC (1) says: price - marginal cost of harvest - user cost = 0

(2) $\dfrac{dL}{dS} = -\dfrac{dTC(X,S)}{dS} + \lambda\,\dfrac{dg(m,n,S)}{dS} = 0$

or, substituting, $-(-bX)/S^2 + \lambda[m - nS] = 0$

FOC (2) says: marginal benefit of - marginal cost of   = 0
               decreased harvest   decreased growth
                  costs   due to overcrowding

(3) $\dfrac{dL}{d\lambda} = g(m,n,S) - X = 0$

or, substituting, $\lambda[mS - nS^2/2] - X = 0$

FOC (3) says: growth - harvest = 0, which is simply the condition for a steady-state.

Solving for Steady-State Stock Level

       We can solve the system of FOC's for the optimal steady-state stock, S*, as follows:

    Solve FOC (3) for X. Plug the resulting expression for X into FOC (2) to get:

(4) $-[-b(m/S - n/2)] + \lambda[m - nS] = 0$

Putting aside expression (4) for the moment, solve FOC (1) for λ:

(5) $= P - b/S$

Now plug   into (4) and solve for S* to get:

(6) $S^* = m/n + b/2P$

Solving for Steady-State Harvest Level

To find the steady-state harvest level, X*, plug the expression for S* into FOC (3) and simplify:

(7) $X^* = \dfrac{m^2}{2n} - \dfrac{nb^2}{8P^2}$

Notice that the optimal harvest level X* increases with price P, decreases with the harvest cost parameter b, decreases with growth function parameter n, and increases with growth function parameter m. That is:

- As the price of fish increase, the optimal harvest level increases towards X_{MSY}

- As the cost of harvesting decreases through improved technology, X* increases

- As the number of predators increases, the fish stock decreases, which implies that the optimal harvest, X* decreases

- As the available food in the ecosystem increases, X* increases

If we attempt to increase our fish harvest we can use several means:

- Subsidize investment in new technology

- Remove predator species (this may lead to other, unintended consequences)

- Add fish food into the ecosystem.

Solving for Steady-State User Cost

The user cost in steady-state, λ*, can be found by substituting S*
back into FOC (1):

$$(8) \quad \lambda^* = P - \frac{b}{\dfrac{m}{n} + \dfrac{b}{2P}}$$

Numerical Example

Suppose P = 1, b = 10, g(S) = .2S -.002 S^2 (m = .2 and n = .004)
  Maximum sustainable stock is S_{mss} = 100.
  The stock level that supports MSY is S_{msy} = 50.
  Maximum sustainable yield is X_{msy} = 5.
  Optimal steady-state stock S* = 55.
  Optimal steady-state harvest X* = 4.95

Notice how changing the parameters would affect S* and X*. For
example, if harvesting cost parameter b were to increase to b= 50, then the
optimal steady-state stock would increase to S* = 75 and the optimal steady-
state harvest would fall to X* = 3.75.

A common policy goal of fishery managers for many years was to attain
steady-state stock S_{msy} and to set steady-state harvest equal to X_{msy}. This
policy maximizes biological growth and steady-state harvest. However, we
often wish to maximize the *value* of the biological growth and harvest. S*
and X* maximize the *value* of the biological growth and harvest. Notice
that S* > S_{msy} and X* < X_{msy}, i.e., the value of the biological growth and
harvest is maximized by harvesting less and leaving a larger stock in the
ocean. *Note: this conclusion may not hold in cases where the interest rate
is non-zero.*

Open Access Market Failure

Suppose we have a competitive fishery with open access to the fish
stock. Under open access, each fisherperson will ignore user cost, and the
resulting steady-state will occur at an inefficiently low stock level. Under
open access competition, fishers ignore the user cost component of FOC (1)
and instead harvest until:

(9)   P - dTC/dX = 0

or, substituting and rearranging,

(10)   P = b/S

This implies that the steady-state stock under open access competition, S_c, is:

(11) $S_c = b/P$

Given growth function $g(m,n,S) = mS - nS^2/2$ and the fact that we are in steady-state (i.e., $X = g(m,n,S)$), we know that:

(12) $X = mS - nS^2/2$

Substituting equation (11) into equation (12), we get an expression for the harvest level under open access competition, X_c:

$$(13) \quad X_c = \frac{mb}{P} - \left[\frac{nb^2}{2P^2}\right]$$

In terms of our numerical example, $S_c = 10$ and $X_c = 1.8$. Hence, open access competition results in too little stock (i.e., $S_c < S^*$), and too little harvest (i.e., $X_c < X^*$). If the stock were allowed to increase, then larger harvests could be supported. However, under open access competition, no fisher has the incentive to reduce current harvest in order to allow the stock to increase and harvests to be larger in the future. This is because, under open access competition, if a given fisher reduces her current harvest, then some other fisher would harvest the fish in the current period and the "tragedy of the commons" result occurs.

A Harvest Tax to Correct Open Access Market Failure

From FOC (1) we know that "price = extraction cost + user cost" at the optimal steady-state harvest level. However, if open access competition exists, market failure will occur because private firms will have no incentive to consider user cost when setting harvest levels.

A policy to correct this market failure is a tax per unit of harvest equal to the user cost at the optimal harvest level. In our case:

(14)   tax per unit = $\lambda^* = P - \dfrac{b}{\dfrac{m}{n} + \dfrac{b}{2P}}$.

Shifting the Growth Function: Fish Feeding

Various management activities can be undertaken to influence the growth of resources. For example, the growth function of a fishery may be shifted upwards by feeding the fish, or the growth function of a forest may be shifted upwards by fertilizing the forest. Private firms may undertake these activities, as in the case of salmon raised in pens off the coast of Norway, or Christmas trees grown by farmers in the mountains of North Carolina. Alternatively, when the resources are public goods, it may be efficient for these activities to be undertaken by the government. Indeed, in many countries, government-financed fish hatcheries feed and raise fish in order to maintain high commercial or recreational fishing levels. In either case, the economic issue is to find the optimal level of feeding.

Let the growth equation for a renewable resource be denoted g(S, A), where 'A' is a variable representing the level of feeding. We assume that feeding increases growth, i.e., dg/dA > 0.

Assuming that the price per unit of feed is denoted by v, total cost becomes:

(15)   C(X,S,A) = C(X,S) + vA

And the optimization problem is:

(16) $\underset{X,S,A}{\max}$ B(X) − C(X, S) − vA

subject to: g(S, A) = X

The Lagrangian expression for this problem is:

(17) $\underset{X,S,A,}{\max}$ L = B(X) − C(X, S) − vA + λ $\big[$g(S, A) − X$\big]$

The first-order conditions include the familiar conditions:

$$(18) \quad \frac{dL}{dX} = \frac{dB}{dX} - \frac{dC}{dX} - \quad = 0$$

$$(19) \quad \frac{dL}{dS} = -\frac{dC}{dS} + \frac{dg}{dS} = 0$$

$$(20) \quad \frac{dL}{d} = g(S,A) - X = 0$$

but we also have a new first-order condition:

$$(21) \quad \frac{dL}{dA} = \frac{dg}{dA} - v = 0 \, .$$

This new condition says that we should increase feeding until the value of marginal product of feeding (dg/dA) is equal to the price of feed v. FOC equations (18)-(21) represent a system of four equations in four unknowns that can be solved for the optimal levels of X, S, A, and   in a similar fashion as in the last example.

Although the preceding steady-state models are elegant, they ignore several important issues. For example:
- Resource populations can be driven to **extinction** (not just to a lower steady-state stock level) under open access competition.
- **Interest rates** play a very important role in the management of renewable resources and should be included in dynamic model
- Managers are often interested in what stock and harvest levels will be *on the way* from current conditions to an eventual steady-state. The way the stock or harvest level changes from one state to another in a dynamic system is referred to as **transition dynamics**.

In the next lecture we investigate a more general model that can address these issues.