Forestry Economics

The economics of forest resources are very similar to the dynamic management of a fishery:

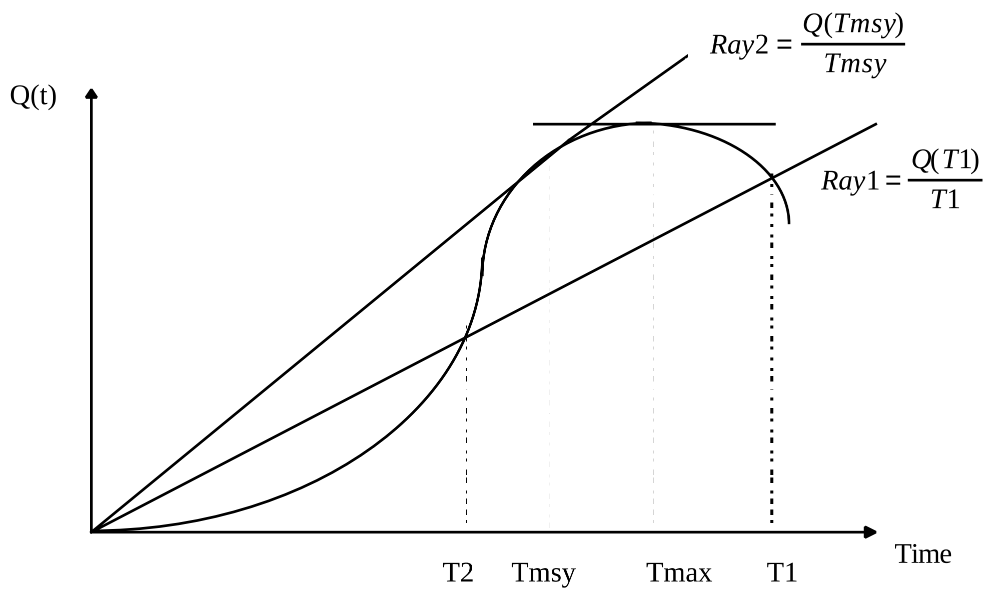
- Both forests and fisheries are renewable resource systems
- The economic principles that determine optimal management are very much the same

How Is The Forestry Problem Different From a Fishery?

- Forest Solutions Determine "When" Rather than "How Much"
- Growth Occurs over Long Time Periods and Can be Measured
- The Forest Problem Solves For the Optimal Time to Harvest Entire Stock
  - the solution gives the **optimal length of each rotation** of stock
- Property Rights are Secure (No Open Access Problems)

In the Forestry Problem, **the critical element is that the Growth Function is a Function of Time; not a function of stock**.

The Forestry Growth Function



Q(t) = Volume of Timber (i.e., Board Feet or Cubic Feet)

The growth function of a typical stand of trees looks like this.

- At first, volume increases at an increasing rate for young trees.
- Then growth of volume slows and increases at a decreasing rate
- Finally, when the trees are very old, they begin to have negative growth as they rot, decay, etc.

- The volume of a stand of trees is maximized at time Tmax
  -this volume is not the Maximum Sustainable Yield
- **MSY** is **where the growth rate = Avg. Growth *per rotation***
- The average growth rate of a stand, at any time, t, is:

$$A.G. = \frac{Q(t)}{t}$$ (shown by a ray through the origin)

- Ray 1 shows that the avg. growth is achieved by cutting at T1 or T2, but neither time gives the Maximum Average Growth.
  - we want the highest avg. Growth over all harvests

How Do We Find T(msy)?

The MSY occurs at a rotation length that maximizes the average annual growth of the stands through time.

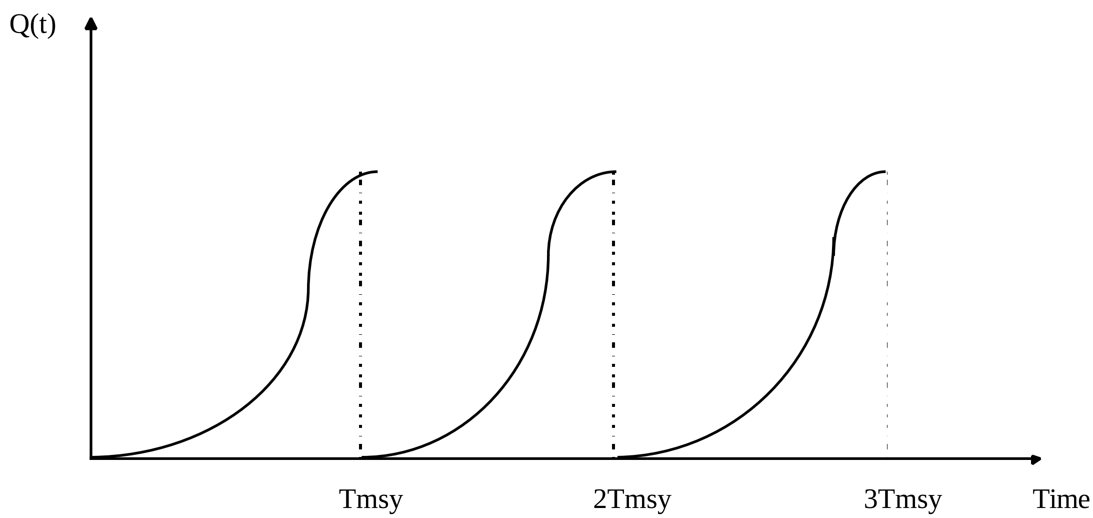
• Max. { Q(t)/t } =>
$$\frac{d\left(\frac{Q(t)}{t}\right)}{dt} = \frac{Q'(t)t - Q(t)}{t^2} = 0$$

Rearranging terms, we get: $\frac{Q(t)}{t} = Q'(t)$

to harvest the MSY, we should cut the stand of trees when marginal growth equals average growth of the stand.

- Ray 2 shows where this condition is met, where the average growth is tangent to the growth function

Harvest Pattern Over Time



As in the case of the fishery when moving from 2 time periods to T periods, the optimal dynamic solution is to replicate a single optimal decision many times.

The Economic Decision to Harvest a Stand

- we must include the discount rate of the forester

• In discrete time, we find that $P_t = \left(\frac{1}{1+r}\right)^t$

• In continuous time, $as\ \partial t \rightarrow 0,\ \left(\frac{1}{1+r}\right)^t \rightarrow e^{-rt}$

The Forestry Optimization Problem:
- we want to find the point in time where the NPV is maximized.

The Optimal Single Rotation:
- there is no opportunity cost incurred by failing to plant the next stand of trees at the optimal time.
- Let P = constant price per pound of the crop.
- There are no harvesting costs, so that π = TR

$$\underset{t}{Max.}\left\{\pi = e^{-rt} PQ(t)\right\}$$

FOC:

$$\frac{d\pi}{dt} = PQ'(t)e^{-rt} + PQ(t)e^{-rt}(-r) \quad = \quad 0 \Rightarrow \; PQ'(t) = rPQ(t)$$

MB of waiting (value of new growth) = MC of waiting (lost interest on TR)

If the forest manager delays the harvest, she will not earn interest on revenues PQ(t).

If the forest manager delays the harvest, she will gain the value of new growth Q’(t).

Single Rotation (cont.)

We can rearrange the optimality condition to get:

$$\frac{Q'(t)}{Q(t)} = r$$

the percent rate of growth should equal the discount rate.

• If $\frac{Q'(t)}{Q(t)} > r$, the crop is increasing in value quicker than market investments and the farmer should delay the harvest decision.

• If $\frac{Q'(t)}{Q(t)} < r$, market investments are increasing in value quicker than the growth in value of the crop (harvesting should have already occurred)

The Case of An Infinite Forest Rotation (Faustmann Rotation)

must consider the opportunity cost of future rotations.

Preliminaries:

· review a calculus identity that we will use:

- The sum of an infinite series is: $For\ |X| < 1,$

$$\sum_{i=0}^{\infty} X^i = \left(1 + X + X^2 + X^3 + \ldots\right) = \frac{1}{1 - X}$$

- In our problem of infinite rotation we obtain:

$$\sum_{i=0}^{\infty} e^{-irT} = \left(1 + e^{-rT} + e^{-2rT} + e^{-3rT} + \ldots\right) = \frac{1}{1 - e^{-rT}}$$

where T is the length of each rotation.

Infinite Rotation (cont.)

- Net Price = Price - per unit harvesting and replanting costs.
- P = constant net price per cubic foot of timber
- Q = the volume of timber (in cubic feet)

$$\pi = PQ(T)e^{-rT} + PQ(T)e^{-2rT} + PQ(T)e^{-3rT} + \dots$$
$$= PQ(T)\left[e^{-rT} + e^{-2rT} + e^{-3rT} + \dots\right]$$
$$= PQ(T)e^{-rT}\left[1 + e^{-rT} + e^{-2rT} + \dots\right]$$
$$= PQ(T)\frac{e^{-rT}}{1-e^{-rT}} = \frac{PQ(T)}{e^{rT}\left(1-e^{-rT}\right)}$$
$$= \frac{PQ(T)}{e^{rT}-1}$$

The Optimization Problem is: $\underset{T}{Max.}\left\{\pi = \frac{PQ(T)}{e^{rT}-1}\right\}$

FOC:

$$\frac{d\pi}{dT} = \frac{PQ'(T)}{e^{rT}-1} + \frac{PQ(T)(-1)(r)(e^{rT})}{\left(e^{rT}-1\right)^2} = 0$$

$$\Rightarrow \quad PQ'(T) = \frac{PQ(T)re^{rT}}{e^{rT}-1} = \frac{rPQ(T)}{1-e^{rT}}$$

cross-multiply $\Rightarrow \quad PQ'(T) = rPQ(T) + PQ'(T)e^{-rT}$

MR of delaying = MC of waiting + MC of delayed future income

- The last term shows that delaying the current harvest also delays income received from future harvests

Infinite Rotation (cont.)

the optimal rotation time, T, requires the forester to equate the marginal value of waiting to the marginal cost of delaying the harvest of current *and* future stands.

In general, T* < T* (single rotation) < Tmsy:

An Increase in the price of timber:

· tends to shorten the rotation length, because higher timber prices increase the profitability of each harvest
  - cutting trees earlier moves the profit of future harvests closer to the present

An Increase in the Interest Rate:

· tends to shorten the optimal rotation length, because the forest owner is now relatively more impatient.
  - more eager to move profit up into the present

An Increase in Harvesting Costs:

· Recall how we absorbed harvesting costs into the Net Price.
  • Thus, an increase in c is analogous to a decrease in Price

· An increase in c will tend to increase the rotation length, because cutting trees has now become less profitable
  - the owner wishes to delay paying future harvesting costs