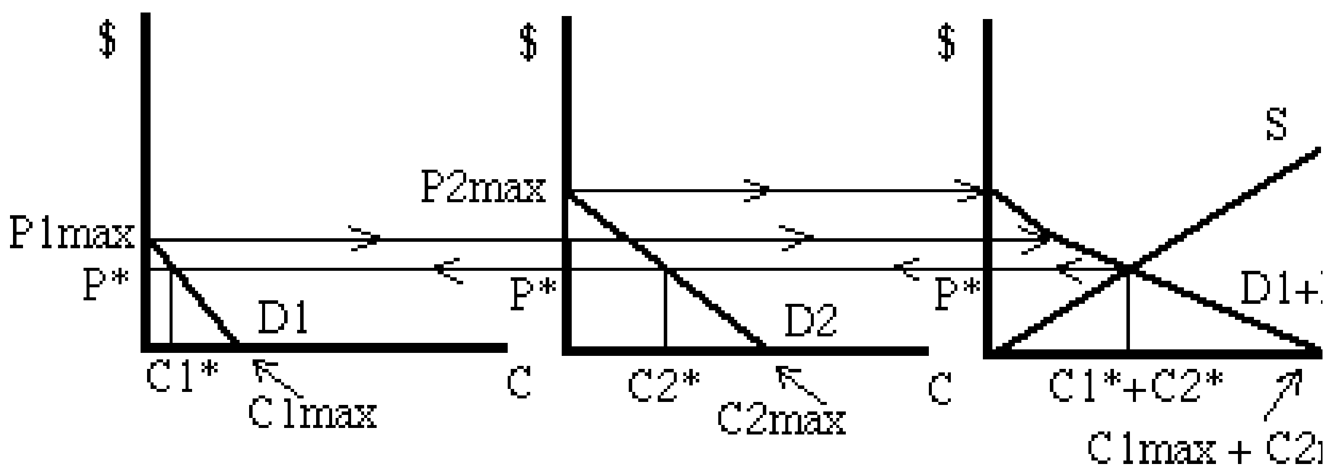
Public Goods

Public Goods have two distinct characteristics:
· **non-rivalry:** several individuals can consume the same good without diminishing its value
· **non-excludability:** an individual cannot be prevented from consuming the good

· nonrivalry =>
individual demand curves are summed **vertically** to get the aggregate demand curve for the public good.

· private goods =>
individual demands are summed **horizontally**.

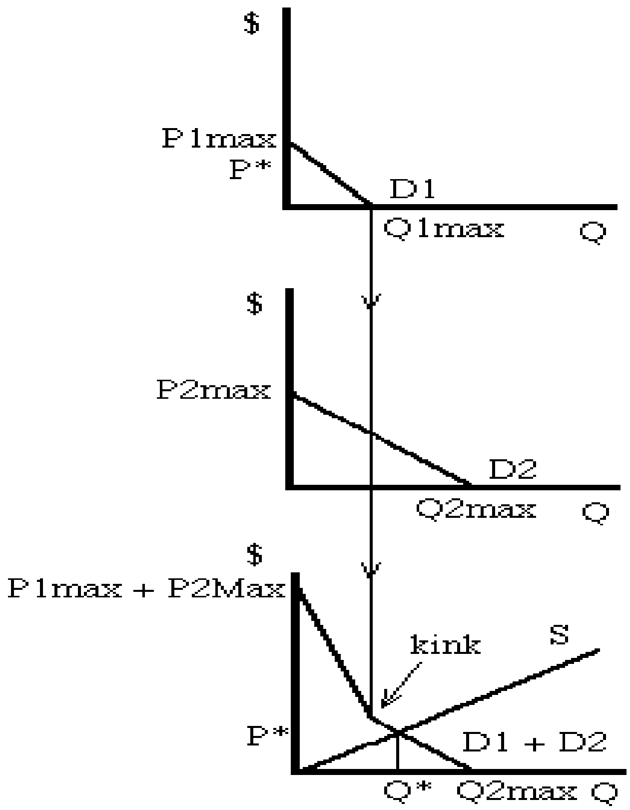
Deriving Aggregate Demand for Private Good



Why Private Goods Are Summed Horizontally:
• Exclusive: once you buy it, you own it and can consume it as you please.
• Rival: A good taken off the shelf isn't there for other people to consume.

Deriving Aggregate Demand for Public Good

(Recreational Demand for Water Quality at Mono Lake)



Demand is summed vertically:

- the same unit of water quality at Mono Lake can be enjoyed by all individuals.
- aggregate demand = the sum of individual *value* for the unit

- Note: almost no good or service is completely non-rival
  non-rivalry is a relative concept

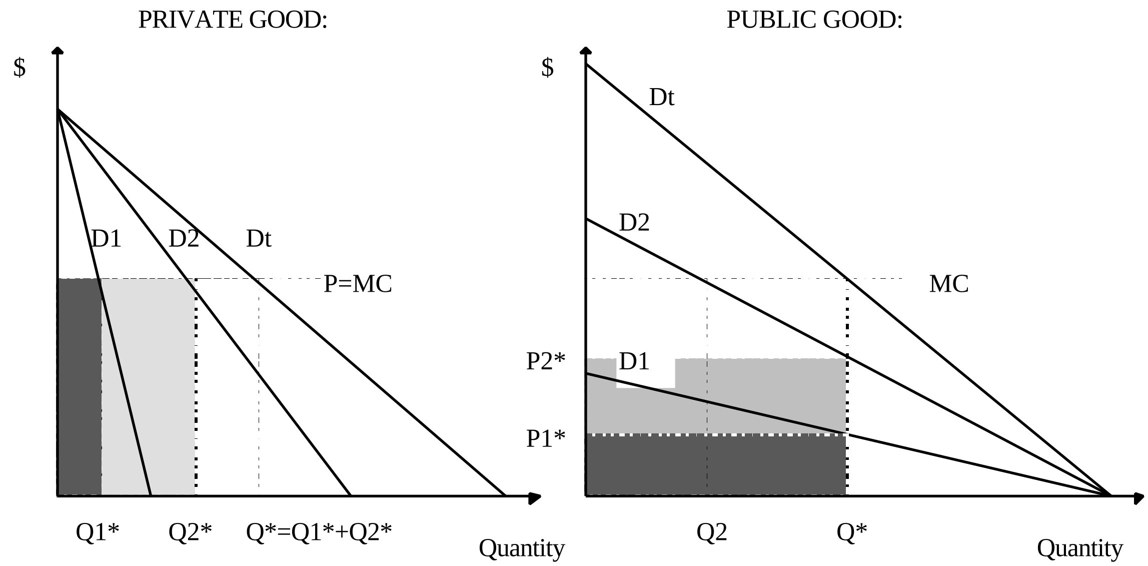
- for the purpose of discussion, we often use the notion of a **pure public good**

  a good or service that is *both* non-rival and non-excludable

Heterogeneity, Non-Rivalry and Market Failure

Consider Two Goods with Identical Aggregate Demand:

- The first good is a private good, (Chicken Sandwiches)
- The second good is a public good, (Water Quality at Mono Lake)



Private Good: market price is an efficient mechanism.

- equilibrium price of a chicken sandwich is P=MC
- consumer 1 eats Q1* sandwiches; consumer 2 eats Q2*
- total revenue paid by each is shown by the shaded regions.

Public Good: market price is not efficient mechanism
equilibrium price cannot be P=MC

- consumer 1 would not pay for any water quality improvements
- consumer 2 would pay for only Q2
- Q2 < Q*, the efficient level of water quality would not be met.
- if Q* is provided and each consumer pays his marginal value:
- total revenue paid by each is shown by the shaded regions

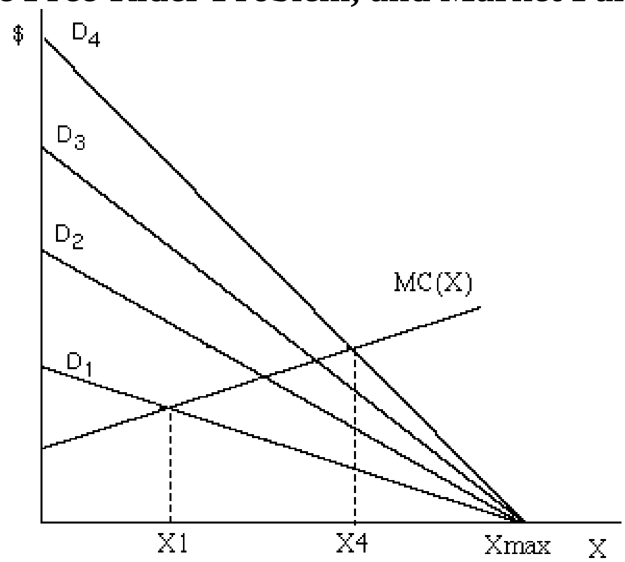
quantity is not an effective market mechanism

Non-Excludability and Market Failure

Private markets often underprovide nonexcludable public goods because individuals have the incentive to **free ride**

-- not pay for the benefits they receive from consuming the public good

Optimal Provision of a Nonexcludable Public Good, The Free-Rider Problem, and Market Failure



D1 = Demand of one individual for public good X.
D2 = Total Demand of two individuals for public good X.
D3 = Total Demand of three individuals for public good X.
D4 = Total Demand of four individuals for public good X.
MC= Marginal cost of providing the public good X.

- socially-optimal level of public good X with four consumers is X4

- if individual 1 decides to purchase (and the others free ride), the private market will provide a level of the public good equal to X1 (where MB_1 equals MC of provision)

The Socially-Optimal Provision of a Public Good

X = level of provision of a public good
n = number of homogeneous individuals in a society

(Inverse) demand of one individual: Di(X) = a - bX.

(Inverse) demand of n individuals ("aggregate demand"):

Dn(X) = n(a - bX) = na - nbX. ==>

$$TBn(X) = \int Dn(x)\,dx$$

(Inverse) Supply:

MC(X) = c + d X ==> $TC(X) = \int MC(x)\,dx$

The socially-optimal level of provision of X occurs where TBn(X) - TC(X) is maximized:

$$\underset{X}{Max.}\left\{ W(X) = \int_0^X Dn(x)dx - \int_0^X TC(x)dx \right\}$$

The FOC for this problem is:

Dn(X) = MC(X), or na - nbX = c + dX

Solving the FOC for X:

$$X^* = \frac{na - c}{nb + d}$$

- as n becomes very large, X* approaches the value a/b (the X intercept of aggregate demand)

Private Market Outcome for a Non-excludable Public Goods

- Private providers will provide public goods where the marginal benefit of one individual (the other individuals free ride) equals the marginal cost of providing the public good

$$\max \ \pi(X) = TB1(X) - TC(X)$$

The FOC for this problem is:

$$D1(X) = MC(X), \text{or} \qquad a - bX = c + dX$$

Solving for the level of the public good provided by the private market:

$$X^{Comp.} = \frac{a-c}{b+d}$$

- note that $X^{comp} < X^*$ (the private market under-provides the public good)

Mechanisms for Providing the Socially-Optimal Level of Public Goods:

- Civic responsibility, volunteerism, and donations
  --volunteer fire departments, donations to the arts
- Private provision of excludable public goods
  --movies, music concerts
- Public provision of excludable public goods through the use of entrance fees
  --entrance fees for a National Park
- Public provision of nonexcludable public goods through the use of general government tax revenues
  --taxes earmarked for National Defense
- Religious Beliefs
  --collection basket is passed around for donations

Gov't Provision of Non-Excludable Public Goods Through Taxes

- Public financing of public goods may be the only option in cases where the public good is non-excludable and, therefore, entry fees cannot be charged
  - --National Defense
  - --Public Education
  - --Social Welfare Programs.

- the Governments' problem
  - --there is only one public good
  - --gov't seeks to provide the public good in a budget-balancing, or **revenue-neutral** manner

If there are n individuals in the society, then:

$$\text{Total Tax} = TC(X^*) = \int_0^{X^*} MC(X)\, dx$$

so that the tax per individual = $TC(X^*)/n$.

Congestion Costs in Public Goods Models

· **congestion costs**:
an increasing number of users can reduce the benefits to each individual

· **negative congestion externalities**
the benefits to each viewer of a scenic vista may be reduced if the overlook site becomes crowded

· **positive congestion externalities**
"information highway": When the first individual subscribes to email, the value of the service is equal to zero, since there is no one out there to send messages to. As subscription to the service increases, however, the value of email increases due to the positive congestion externality.

X is the level of provision of a public good
N is the number of people consuming the public good
Bi(X,N) is the benefit to individual i from the public good at a level of X when N individuals are using the public good
the existence of congestion costs implies that:

$$dBi/dN < 0$$

benefit to an individual of consuming the public good decreases as the number of individuals consuming the public good increases.

consider the following functional form:

$$Bi(X,N) = \frac{a + bX - cX^2}{N},$$

where the parameters a,b,c > 0.

When we maximize Benefits with respect to N, we find that:

$$\frac{dBi}{dN} = \frac{-(a + bX - cX^2)}{N^2},$$

the expression is negative => a negative congestion externality

EXCLUDABLE PUBLIC GOODS

- with **excludable public goods**, private markets may either provide the efficient level or inefficient level of public goods

- two key issues determine whether the private market will provide the efficient level of public goods:
  - **heterogeneity of consumer demand,** and
  - the ability of private providers to **price discriminate**.

The Socially-Optimal Level of an Excludable Public Good

- the socially-optimal level of provision of an excludable public good is **the same as it is for a non-excludable public good**, namely, X*.

- when public goods are excludable, a private firm can build some kind of barrier to prevent consumers from free-riding

- private owner of the resource will charge each consumer his/her willingness to pay

- different cases arise depending on whether consumers are homogeneous or heterogeneous

Excludable Public Goods with Homogeneous Consumers

Private firm will build a fence and act as a monopoly by charging each individual their maximum willingness to pay:

$$TR(X) = \int_0^X n\, Di(X)\, dx = \int_0^X Dn(X)\, dx$$

The monopolist maximizes profits:

$$\underset{X}{Max.}\left\{\pi = \int_0^X Dn(x)dx - \int_0^X MC(x)dx\right\}$$

When Dn(X) = n(a - bX) and MC(X) = c + d X, as in the social problem, the FOC is:

$$Dn(X) = MC(X) \quad \text{or} \quad n(a - bX) = c + dX$$

Solving for Xm, we get:

$$Xm = (na - c)/(nb + d)$$

and find, comparing Xm with X*, that Xm = X*.

Thus, **in the case of homogeneous consumers, we get the surprising result that the monopolist provides the optimal level, X*!**

--the distribution of welfare is very different

The monopolist would set the entry fee Em equal to the maximum willingness to pay of each individual at X*, which can be found using conventional methods of integration on individual demand,

$$Em = \int_0^{Xm} Di(x)dx = aX - \frac{bX^2}{2}$$

then substituting in for X = X* to get:

$$E_m = \frac{(na - c)(anb + 2ad + bc)}{2(nb + d)^2}$$

Government Provision of Excludable Public Goods

- If entrance can be controlled, public provision of public goods can be financed through **entry fees**.
  --national parks, toll bridges

- government can build a fence and charge an entry fee to cover costs:

  Egovt = TC(X)/n

- using the functional forms from the earlier example, the shadow price associated with X*, λ = MC(X*) = Di(X*):

$$\lambda = Di(X^*) = a - b\left(\frac{na - c}{nb + d}\right)$$

$$= \frac{ad + bc}{nb + d}$$

Note that consumers will now receive a welfare surplus from entry:

$$E = \frac{\lambda X^*}{2n} = \frac{(na - c)(ad + bc)}{2n(nb + d)^2}$$

It is easy to show that Em > E, since,

$$\frac{(na - c)(anb + 2ad + bc)}{2(nb + d)^2} > \frac{(na - c)(ad + bc)}{2n(nb + d)^2}$$

- for any a, n, b > 0:

The difference Em - E is consumer surplus.

Concessionaire Provision of Excludable Public Goods

- The government can grant a license to a private firm (the "concessionaire") to build a fence, provide the public good and charge an entry fee.

- the government regulates the level of the entry fee to ensure a more equitable distribution of welfare

- One measure of the level of competitive profits is the producer surplus the firm would make if the public good were in fact a private good with market demand Dn(X) and the private good were produced and sold at level X*.

- This outcome would allow the concessionaire to charge the unit price, P = MC(X*) = λ, which is the shadow price of providing the public good.

$$PS(X^*) = \lambda X^* - TC(X^*)$$

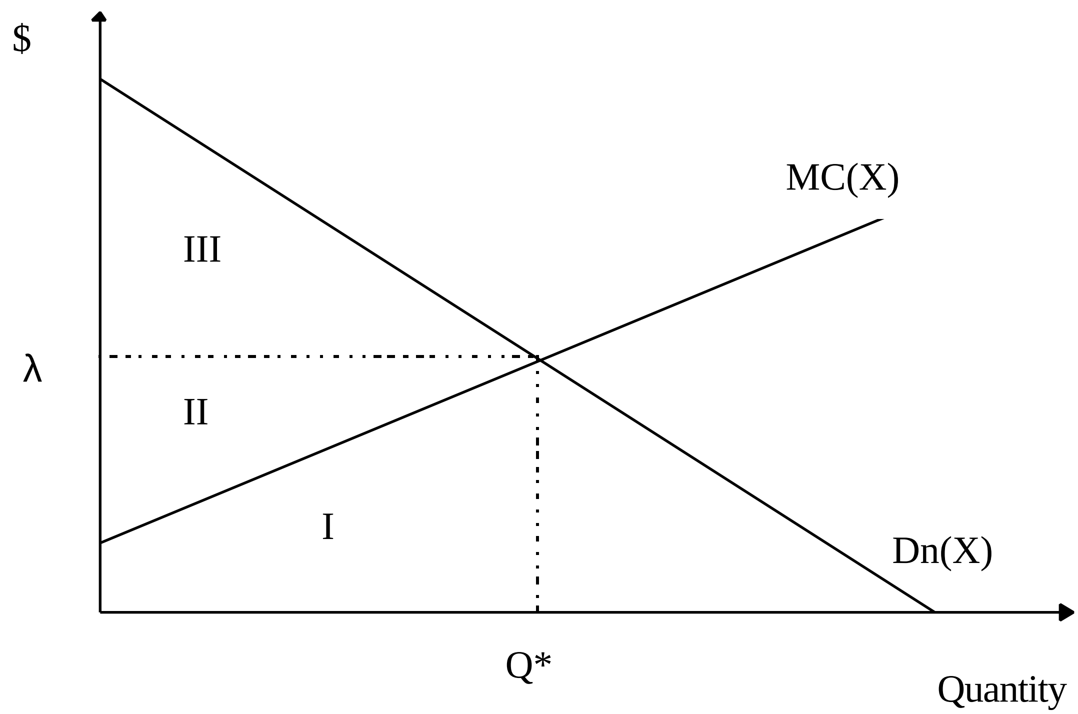
- For the private firm to make PS(X*) in profits, the government must allow the firm to charge each consumer an entry fee, Ec, where:

$$Ec = \lambda X^* = \frac{(ad + bc)(na - c)}{(nb + d)^2}$$

Excludable Public Goods (cont.)

Graphically,

- A Benevolent Government charges the entry fee, E $= \frac{I}{n}$
- A Concessionaire charges the entry fee, Ec = $\frac{I + II}{n}$
- The Monopolist charges the entry fee, Em= $\frac{I + II + III}{n}$



Club Provision of Public Goods:

- club provides access to a public good to a restricted number of members.

the objective function of the club is:

$$\underset{n,X}{Max.}\left\{nB(n,X)-C(X)\right\}$$

where B(n,X) is the individual total benefit function from the public good.

the FOCs are:

$$(1)\quad n\frac{dB(n,X)}{dn}+B(n,X)=0$$

$$(2)\quad n\frac{dB(n,X)}{dX}-\frac{dC(X)}{dX}=0\,.$$

- Equation (1) states that the optimality condition for club size

members should be added to the club until the MB to current members adding an additional member is equal to the marginal congestion cost of adding an additional member.

- Equation (2) states the marginal condition for the level of public good provision, X the MC of providing the public good is equal to the MB received by all of its members.

- equations (1) and (2) define the optimum:
  - --the larger the membership size, the greater X is.

Heterogeneous Demand for a Public Good

If firms are heterogeneous, two cases arise:

- private firms can price discriminate
- private firms cannot.

- Suppose there are two people in your district with different (heterogeneous) marginal benefits from improved air quality

- the marginal willingness to pay for improved air quality is:

  (1) $p_1 = 100 - 10Q$ for the first person
  (2) $p_2 = 40 - 2Q$ for the second person.

Here Q refers to the level of air quality

Finding the Aggregate Demand for a Public Good with Heterogeneous Consumers

- To find the aggregate demand for pollution reduction, you must add the individual demand curves vertically.
- person 1 is willing to pay positive amounts for Q up to 10 units of improved air quality
- person 2 is willing to pay for improvements up to 20 units.

the aggregate demand for improved air quality is

(1) $p = p_1 + p_2 = 140 - 12Q$ for $0 \leq Q \leq 10$

(2) $p = p_2 = 40 - 2Q$ for $10 \leq Q \leq 20$

Notice that the aggregate demand has a **kink** in it.

Calculating the Socially-Optimal Level of a Public Good with Heterogeneous Consumers

- the MC of providing improved air quality: MC = $68/(g/$m^3$)

- to find the efficient level of air quality

Note: kinked demand => look at both segments of the demand curve separately

Setting (1) equal to the MC curve:

$$MC = 68 = 140 - 12Q^* = p \qquad \text{for } 0 \leq Q \leq 10,$$

which solves for $Q^* = (140 - 68)/12 = 6$

which is consistent with the range $0 \leq Q \leq 10$. This is the correct value.

Setting (2) equal to the MC curve:

$$MC = 68 = 40 - 2Q^* = p \qquad \text{for } 10 \leq Q \leq 20,$$

which solves for $Q^* = (68-40)/-2 = -14$,

which is clearly outside the range $10 \leq Q \leq 20$ (doesn't make sense!).

Therefore, $Q^* = 6$ is the efficient level of air quality improvement

Each person should be charged an amount such that, for them, the MB of air quality improvement just = MC that is charged to them (**Lindahl Tax**)

$p_1^* = 100 - 10(6) = \$40$ per unit of cleanup.

$p_2^* = 40 - 2(6) = \$28$ per unit of cleanup

total receipts ($408) = total cost ($408)

The Case of Increasing Marginal Costs

suppose you have increasing MC:

$MC = 7 + 7Q$,

solve for the new efficient level of cleanup using segment (1)

$MC = 7 + 7Q^* = 140 - 12Q^* = p$

which solves for

$Q^* = (140-7)/(7+12) = 7$ units

Person 1 should be charged

$p_1^* = 100 - 10(7) = \$30$/unit,

while person 2 should be charged

$p_2^* = 40 - 2(7) = \$26$/unit.

Receipts will be: (7 units)($30/unit +$26/unit) = $392.

Now, because MC are increasing, the TC of cleaning up this amount (the area under the MC curve between 0 and 7) will be less than this.

The total cost is

(1/2)($56/unit-$7/unit)(7 units) + ($7/unit)(7 units)
= $171.50 + $49 = $220.50.

Heterogeneity and Exclusion from the Market

Suppose there are three individuals:

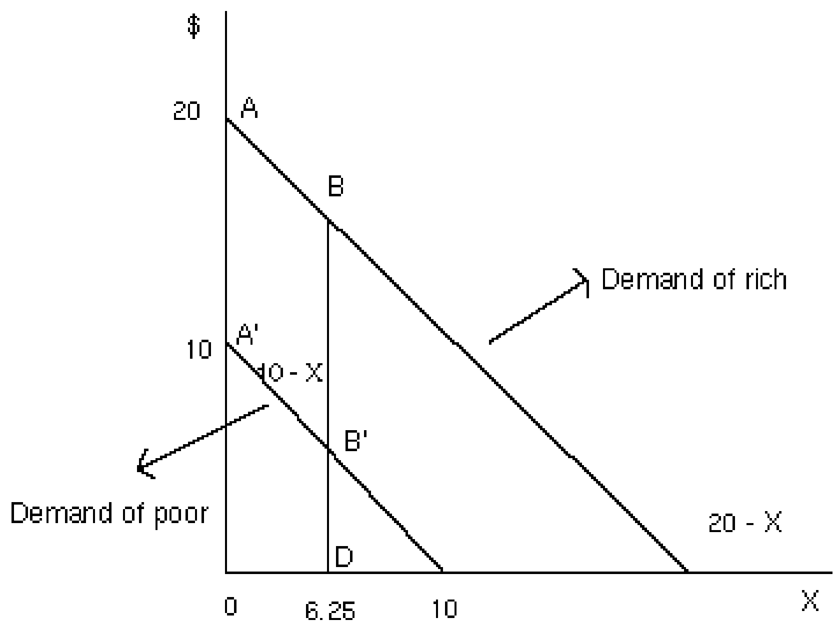
- Two rich with $D_i(X) = 20 - X$, $\quad i = 1, 2.$
- One poor with $D_3(X) = 10 - X$.

Total Demand = $\begin{cases} 50 - 3X & \text{for } X \leq 10 \\ 40 - 2X & \text{for } X \geq 10 \end{cases}$

If Marginal Cost = 5X, then, at the social optimum, 50 - 3x = 5X,
=> X* = 6.25 and a shadow price = $P_3^* = 50 - 3(6.25) = 31.25$.

<u>Benevolent Government</u>: $E = \frac{TC(X^*)}{3} = \frac{5(X^*)^2}{2(3)} = \$32.55$

<u>Concessionaire</u>: Ec = X* (P_3^0/3) = (6.25)(31.25)/3 = $65.10



- individual will pay if benefits from X is greater than the fee
- Benefits of a rich person = area of 0ABD

  = [20 + (20 - 6.25)] · 6.25/2 = $105.47 > 65.10.

- Benefit of poor people = area of 0A'B'D

  = [10 + (10 - 6.25)] · 6.25/2 = $42.97 < 65.104.

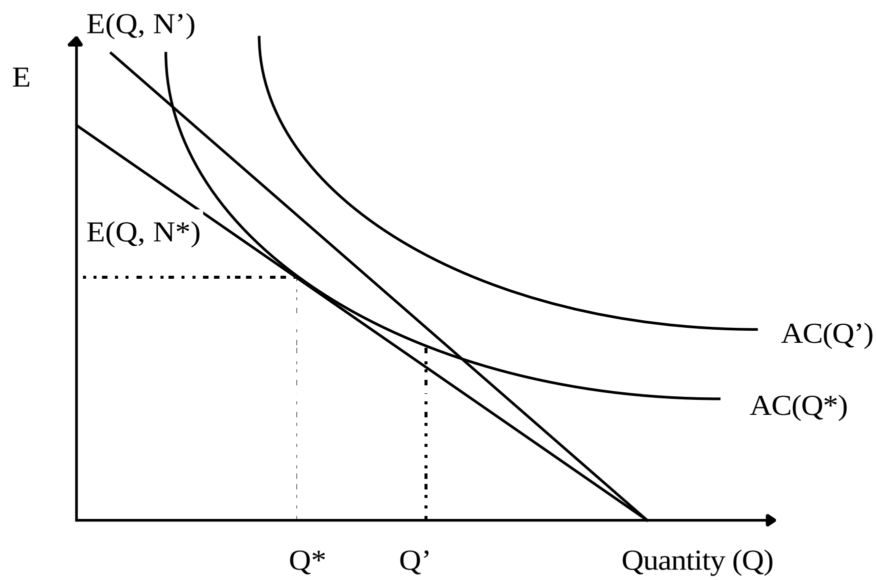
  => poor cannot pay entry fee if provided by concessionaire

The Solution to the Second-Best Problem of Balanced-Budget Provision:

- low-demand individuals are not able to afford the Benevolent Gov'ts' Entry Fee

- the regulator must choose the level of provision subject to the balanced-budget constraint that the sum of revenue from entry fees equals the Average Cost of provision.

- Imagine a continuum of individuals, ordered from highest to lowest demand for the public good.
- Notice that the *y-axis* does not give a unit price, but gives the marginal entry fee per unit of output supplied, or the *Marginal Revenue collected by government*.



The regulator begins with the lowest demand individuals and, provided AC can not be recovered through Entry Fees for all individuals, begins to discard them from the market. Each time, the regulator re-calculates the residual demand with a smaller and smaller group, until finally, at point Q*, the highest quantity of provision is found for which entry fees can just cover the AC of providing the good.

Public Goods, Environmental Amenities and Nonuse Benefits:

- Environmental amenities provide both use and **nonuse** benefits

- **Nonuse** benefits reflect benefits that are derived from the simple existence, rather than use, of certain environmental goods

- Example: We all benefit from *maintaining a healthy Rainforest*, since the Rainforest ecosystem is critical to maintaining a *healthy atmosphere*, and also because much of the *new medicine* that is developed is derived from tropical plant species. Yet, these are nonuse values, because they do not depend on us ever visiting the Rainforest.

- use benefits may be provided by the private sector:
  --need to prevent monopolistic outcomes
  --cross-subsidization may be required to make environmental amenities accessible to low income individuals.

- Access to many environmental amenities can be restricted by travel cost. Even when physical entry is free, transaction cost prevents many from enjoying faraway environmental amenities.