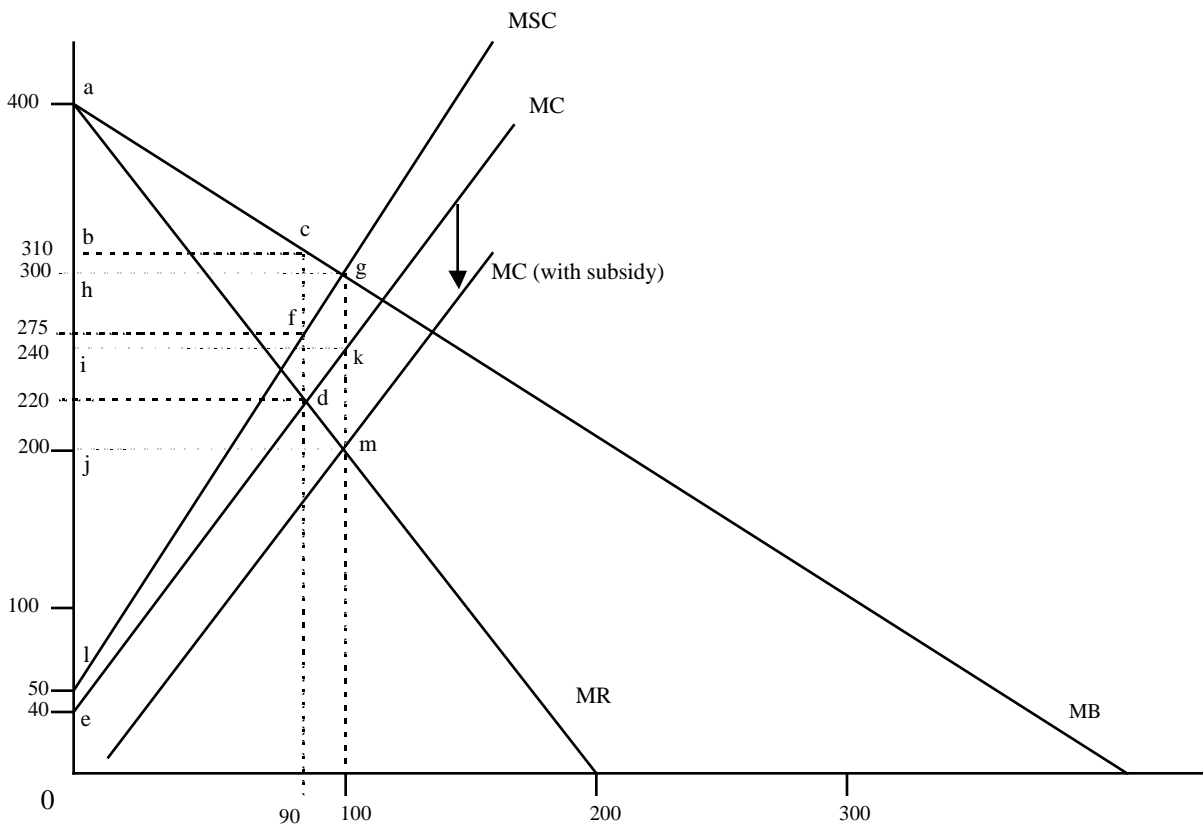


**Key to PROBLEM SET 1**

1. The graph below illustrates the case where a monopolist supplies the market. (Note that the graph is not exactly to scale, so coordinates should be calculated as the intersection of curves, or by substituting values in the relevant functions. Ex: at point “g”  $MB = MSC$ ; at point “k”,  $MC = 40 + 2 \cdot 100 = 240$ )



a) Social Optimum is attained at a point (g on the graph) where the marginal social cost (sum of private and external marginal costs) is just equal to the marginal social benefit (simply the inverse demand in this case).

$$MSC = MC + MEC = (40 + 2Q) + (10 + 0.5Q) = 50 + 2.5Q$$

$$\text{Social Equilibrium} \Rightarrow MSC = MB \Rightarrow \text{Point } g \Rightarrow 50 + 2.5Q = 400 - Q \quad \Rightarrow \quad \Rightarrow Q^* = 100$$

$$\text{Total External Cost, TEC, at } Q^* \Rightarrow \text{Area } ekg \Rightarrow 100(10 + 60)(1/2) = 3,500 \quad \Rightarrow \text{TEC}^* = 3,500$$

$$CS^* \text{ (varies, but assuming quota)} \Rightarrow \text{area } agh \Rightarrow (1/2)(400-300)(100) = 5,000 \quad \Rightarrow CS^* = 5,000$$

$$PS^* \text{ (varies, but assuming quota)} \Rightarrow \text{area } ekgh \Rightarrow (1/2)(260+60)(100) = 16,000 \quad \Rightarrow PS^* = 16,000$$

$$\text{Total Welfare, } W^* = CS^* + PS^* - TEC^* = 5,000 + 16,000 - 3,500 = 17,500$$

but you should also see that  $W^* \Rightarrow \text{area lga} \Rightarrow (1/2)(350)(100) = 17,500 \Rightarrow W^* = 17,500$

A common mistake was “ $PS^* = 12,500$ ” (perhaps representing the area ghl). Recall that producer surplus is the difference between price received and marginal cost, *over the actual quantity sold*. It is often a triangle, but not always.

b) The monopolist produces at a level where the private marginal cost equals private marginal revenue:

$$\text{Total Revenues } TR = P(Q)Q = (400 - Q)Q = 400Q - Q^2.$$

$$\text{Marginal Revenue } MR = \frac{fTR}{fQ} = 400 - 2Q$$

$$\text{Equilibrium Quantity } \Rightarrow MR = MC \Rightarrow \text{Point d} \Rightarrow 400 - 2Q = 40 + 2Q \Rightarrow Q_m = 90.$$

$$\text{Equilibrium Price } \Rightarrow 400 - Q_m \Rightarrow 400 - 90 \Rightarrow 310 \Rightarrow P_m = 310$$

c) Remember that the dead-weight loss is the welfare deficit, relative to the socially optimal level.

$$\text{Consumer's Surplus, } CS_m \Rightarrow \text{Area abc} \Rightarrow (400 - 310)(90)(1/2) = 4,050 \Rightarrow CS_m = 4,050$$

$$\text{Producer's Surplus, } PS_m \Rightarrow \text{Area bcde} \Rightarrow (90)(270 + 90)(1/2) = 16,200 \Rightarrow PS_m = 16,200$$

$$\text{Total external cost, } TEC_m \Rightarrow \text{Area elfd} \Rightarrow (90)(10 + 55)(1/2) = 2,925 \Rightarrow TEC_m = 2,925$$

$$\text{Total Welfare, } W_m = CS_m + PS_m - TEC_m = 4,050 + 16,200 - 2,925 = 17,325$$

$$\text{also see that } W_m \Rightarrow \text{area acde minus elfd is alcfd} \Rightarrow (1/2)(350 + 35)(90) = 17,325 \Rightarrow W_m = 17,325$$

$$\text{Deadweight Loss } \Rightarrow DWL_m = W^* - W_m = 17,500 - 17,325 = 175$$

$$\text{also see that } DWL_m \Rightarrow \text{area agl minus alcfd is cgf} \Rightarrow (1/2)(35 \cdot 10) = 175 \Rightarrow DWL_m = 175$$

d) Correction of the externality

The government plans to intervene with a tax or a subsidy to shift the private MC curve so that it intersects the MR at the desired optimal level of output  $Q^*$ . Let's call "x" the amount by which the MC curve should change. The way we have written the equation, a negative value for x implies a subsidy (reducing MC), while a positive value implies a tax (increasing MC).

$$MC(Q^*) + x = MR(Q^*)$$

$$(40 + 2Q^*) + x = (400 - 2Q^*)$$

We know from a) that  $Q^* = 100$ , so x is the only unknown. Therefore

$$40 + 2(100) + x = 400 - 2(100)$$

$$240 + x = 200$$

$$x^* = -40.$$

The government should thus grant the monopolist a unit subsidy of \$40 per unit produced. This seems strange---subsidizing a polluter—but the intuition is simple. Relative to the social optimum, the market power of a monopolist make her produce “not enough” (and charge too high a price), but the unregulated externality makes her produce “too much”, so the two effects work in opposite directions. If the externality is relatively severe, the government should encourage the monopolist to restrict even further its production by charging a tax. But if the externality is relatively unimportant, the level of pollution is sub-optimal, and a subsidy should prompt the monopolist to increase its output. Recall that the optimal level of pollution is not necessarily zero!

The monopolist equates her subsidized MC with MR at point m in the graph, at the desired level of output ( $Q^*=100$ ). The new market price faced by consumers is determined by the demand:  $P^* = 400 - 100 = 300$ . The effective marginal cost to producers is the original MC minus the subsidy. (Note: we could have instead thought of the subsidy as increasing MR and MB, and end results would have been identical.)

e) Welfare implications of the government's policy

$$\text{Government Expenses } \Rightarrow x^* Q^* \Rightarrow \text{Area kmij} \Rightarrow (40)(100) = 4,000$$

Old Consumer Surplus ( $CS_m$ ) => Area abc =>  $(400 - 310)(90)(1/2) = 4,050$   
 New Consumer Surplus => Area ahg =>  $(400 - 300)(100)(1/2) = 5,000$   
 Change in Consumer Surplus => Area hbcg =>  $5,000 - 4,050 = + 950$

Old Producer Surplus ( $PS_m$ ) => Area bcde =>  $(90)(270 + 90)(1/2) = 16,200$   
 New Producer Surplus => Area hmg0 =>  $(100)(300 + 100)(1/2) = 20,000$   
 Change in Producer Surplus =>  $20,000 - 16,200 = + 3,800$

Old Total external Cost ( $TEC_m$ ) => Area elfd =>  $(90)(10 + 55)(1/2) = 2,925$   
 New Total External Cost => Area elgk =>  $(100)(10 + 60)(1/2) = 3,500$   
 Change in Total External Cost => Area dfgk =>  $3,500 - 2,925 = + 575$

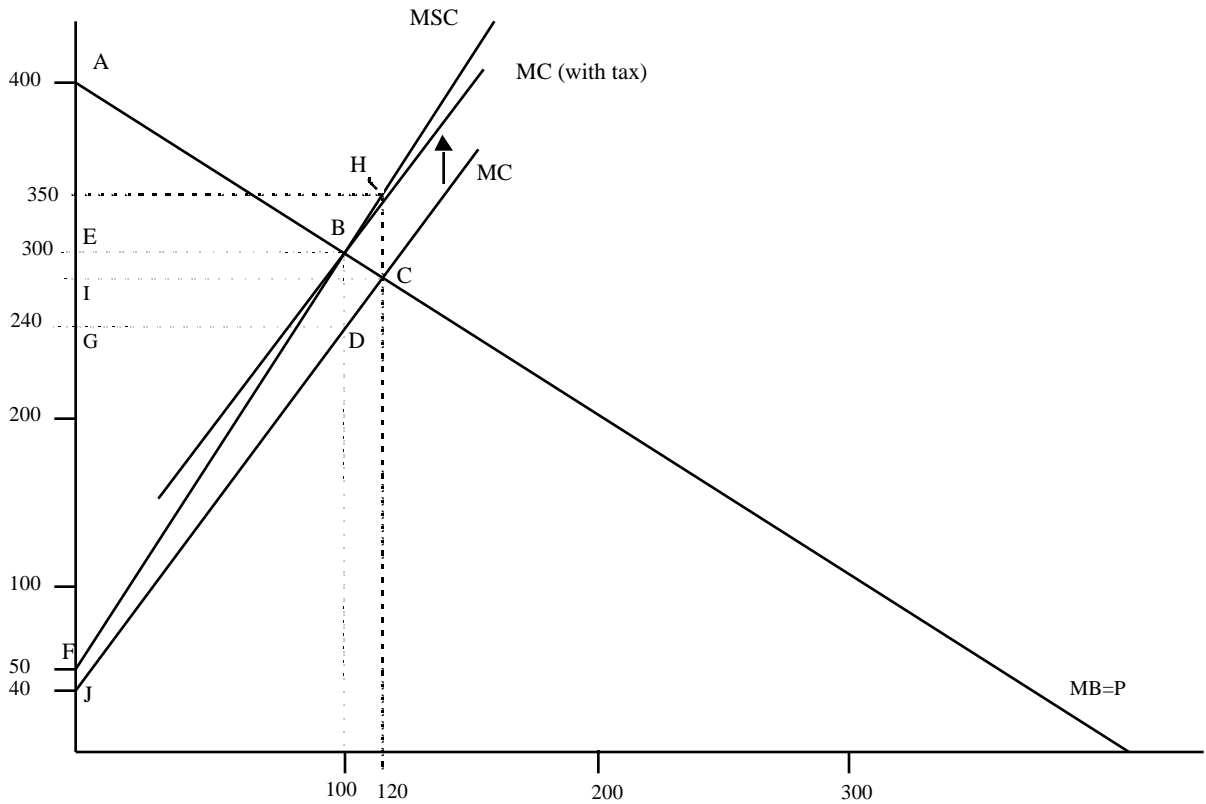
Summary of Welfare Changes:

Increase in Government Expenses (-)	- 4,000
Increase in Consumer Surplus	+ 950
Increase in Producer Surplus	+ 3,800
Increase in Total External Cost (-)	<u>- 575</u>

Net effect of subsidy + 175

The subsidy has a net benefit of 175, which is equal to the deadweight loss found earlier in part c). Hence, the subsidy eliminates the inefficiency created by the externality.

f) Assuming perfect competition instead, we can illustrate with :



Social Optimum is still attained at a quantity ( $Q^*=100$ ) where the marginal social is just equal to the marginal social benefit ( $MSC=MB=300$ ), at point B on the graph. So the calculations in part a) are still valid. Unregulated perfect competition (without concern for the externality) would result in an equilibrium at point C on the graph. As in the general case, we can use the condition  $MR=MC$  to calculate the correct quantity and price. But since the marginal revenue of firms in perfect competition is simply the price, we can set:

$$\text{Equilibrium Quantity} \Rightarrow MR = P = MC \Rightarrow \text{Point C} \Rightarrow 400 - Q = 40 + 2Q \Rightarrow \Rightarrow Q_C = 120$$

$$\text{Equilibrium Price} \Rightarrow 400 - Q_C \Rightarrow Q_C = 400 - 120 \Rightarrow \Rightarrow P_C = 280$$

Remember that the dead-weight loss is the welfare deficit relative to the socially optimal level, so:

$$\text{Consumer's Surplus, } CS_C \Rightarrow \text{Area ICA} \Rightarrow (400 - 280)(120)(1/2) = 7,200 \Rightarrow CS_C = 7,200$$

$$\text{Producer's Surplus, } PS_C \Rightarrow \text{Area CIJ} \Rightarrow (280 - 40)(120)(1/2) = 14,400 \Rightarrow PS_C = 14,400$$

$$\text{Total external cost, } TEC_C \Rightarrow \text{Area FJCH} \Rightarrow (90)[10 + (350 - 280)](1/2) = 4,800 \Rightarrow TEC_C = 4,800$$

$$\text{Deadweight Loss} \Rightarrow W^* - W_C = \text{Area BCH} = W^* - W_C \Rightarrow (350 - 280)(120-100)(1/2) = 700 \Rightarrow DWL_C = 700$$

Correction of the externality: Again, the government wants to shift the MC curve so that it intersects the MB at the desired optimal level of output  $Q^* = 100$ . In this case however, there is no ambiguity: unregulated perfect competition produces “too much” when there are negative externalities, so government will impose a tax. Let's call the amount of the tax “t”, which is the amount by which the MC curve will shift up:

$$MC(Q) + t = P(Q)$$

$$(40 + 2 Q) + t = 400 - Q$$

and we know from a) that  $Q = Q^* = 100$ , so

$$40 + 2(100) + t = 400 - 100$$

$$240 + t = 300$$

$$t^* = 60$$

The MC shifts up by 60, and a competitive market finds it profit-maximizing to produce the socially optimal level,  $Q^* = 100$ . The new equilibrium occurs at point B, and the new market price faced by consumers is determined by the demand:  $P^* = 400 - 100 = 300$ . The effective price received by producers is the market price minus the tax:  $P_p^* = P^* - t^* = 300 - 60 = 240$ .

#### Welfare implications of the government's policy

$$\text{Government Revenues} \Rightarrow t^* Q^* \Rightarrow \text{Area BDEG} \Rightarrow (60)(100) = 6,000$$

$$\text{Old Consumer Surplus (CS}_c) \Rightarrow \text{Area ICA} \Rightarrow (400 - 280)(120)(1/2) = 7,200$$

$$\text{New Consumer Surplus} \Rightarrow \text{Area ABE} \Rightarrow (400 - 300)(100)(1/2) = 5,000$$

$$\text{Change in Consumer Surplus} \Rightarrow \text{Area IEBC} \Rightarrow 5000 - 7200 = - 2,200$$

$$\text{Old Producer Surplus (PS}_c) \Rightarrow \text{Area CIJ} \Rightarrow (280 - 40)(1/2) = 14,400$$

$$\text{New Producer Surplus} \Rightarrow \text{Area DGJ} \Rightarrow (100)(200)(1/2) = 10,000$$

$$\text{Change in Producer Surplus} \Rightarrow \text{Area GDCI} \Rightarrow 10000 - 14400 = - 4,400$$

$$\text{Old Total external Cost (TEC}_c) \Rightarrow \text{Area FJCH} \Rightarrow (90)[10 + (350 - 280)](1/2) = 4,800$$

$$\text{New Total External Cost} \Rightarrow \text{Area FJDB} \Rightarrow (100)(10 + 60)(1/2) = 3500$$

$$\text{Change in Total External Cost} \Rightarrow \text{Area BDCH} \Rightarrow 3,500 - 4,800 = - 1,300$$

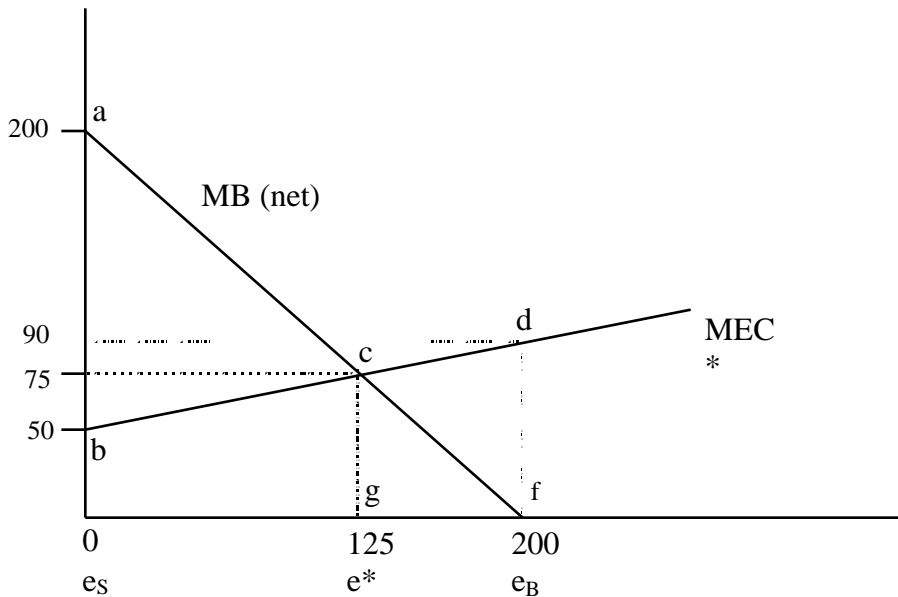
Summary of Welfare Changes:

Government Revenues	+ 6,000
Change in Consumer Surplus	- 2,200
Change in Producer Surplus	- 4,400
Change in Total External Cost (-)	<u>+ 1,300</u>

Net effect of subsidy + 700

The tax has a net benefit of 700, which is equal to the deadweight loss found earlier. Hence, the tax eliminates the inefficiency created by the externality.

2. This question illustrates issues raised by Coase's observation that, in certain circumstances, a socially optimal solution can be attained with little government intervention. Instead of introducing taxes or subsidies, a government can specify property right and let agents (boaters and swimmers in this case) negotiate among themselves.



- The boaters want to maximize their total benefits, and would thus like to keep boating as long as the marginal benefits are positive, so they would prefer  $e_B = 200$ . The swimmers want to minimize costs due to pollution, so they would prefer  $e_S = 0$ . If we are concerned about maximizing the social welfare, defined as the sum of all agents' welfare, the social optimum is at point c, where  $MEC = MB \Rightarrow 200 - e = 50 + 0.2e \Rightarrow e^* = 125$ .
- At social optimum ( $e^*$ ), benefits to boaters can be calculated by taking the area of the trapezoid a0gc. Costs to swimmers (a negative quantity of welfare), is given by the area b0gc, so net social welfare ( $W^*$ ) is given by the area abc =  $(200-50)(125)(1/2) = 9,375$ . The dead-weight loss is, by definition,  $DWL = 0$ . When  $e = 200$ , the DWL is given by  $W^* - W(e_B)$ , or area cdf =  $(90)(75)(1/2) = 3,375$ . When  $e=0$ ,  $W^* - W(e_S) = 9,375$  (area abc) is the DWL.
- The boaters will have the right to be located at  $e_B = 200$ . If a movement from  $e_B$  to  $e^*$  could be negotiated, the swimmers will benefit (their total "cost" is reduced) by area gcdf =  $(75)(75+90)(1/2) = 6187.5$ , so they would be willing to pay the boaters **as much as** 6,187.5 in order to secure this change. On the other hand, the boaters' benefits are reduced by area cge =  $(75)(75)(1/2) = 2,812.5$ , so they would be willing to accept **no less than** 2,812.5 to reduce emissions from  $e_B$  to  $e^*$ . Hence, the maximum gains from trade (if realized) are  $6,187.5 - 2,812.5 = 3375$ . The distribution of these gains will depend on the bargaining power of each group.
- In this case the swimmers have the right to be at  $e_S = 0$ . A movement from  $e_S$  to  $e^*$  will increase the swimmers' cost by area bc0g =  $(50+75)(125)(1/2) = 7,812.5$ , so they would be willing to accept **no less than** 7,812.5 as compensation for moving from  $e_S$  to  $e^*$ . On the other hand, the boaters' benefits will increase by area ac0g =  $(125)(200+75)(1/2) = 17,817.5$ , so they would be willing to pay **as much as** 17,817.5 to secure this change in emissions from  $e_S$  to  $e^*$ . Hence, the maximum gains from trade (if realized) are  $17,817.5 - 7,812.5 = 9,375$ . Again, the distribution of these gains depends on the bargaining power of each group.
- It may not be so easy to enforce property rights, or it may be expensive to negotiate (perhaps too many boaters—or swimmers—who do not agree on compensation). See Lecture Summaries for more on the Coase theorem.

Part B: Essay Question: "A Ban on Smoking in California's Public Bars"

- Smokers do not generally account for the full harm they inflict on others (an externality) through second-hand smoke. Enforcement is not perfect, but seems comprehensive.
- Even with the clear assignment of property rights that results from a smoking ban, there are still obstacles to negotiated solutions. The law does not explicitly make clean air rights transferable, so there is no easy

way for non-smokers to accept compensation in exchange for allowing smoke. There might also be too many parties involved.

The Coase Theorem may be more applicable to the situation of bar employees, since their number in each bar is relatively small, and they would only need to negotiate with bar owners. The employees may compensate an owner for disallowing smoking by accepting lower wages. Alternatively, they may request higher wages in exchange for compromising their health by working in a smoky bar. This solution of relatively high wages for workers in bars that allow smoking and lower wages for workers in bars that disallow smoking does not depend on the distribution of property rights.<sup>1</sup>

- c) The rationality of smokers (and others “addicts”) is a matter of dispute within the economic profession. Some feel that benefits derived from smoking cannot possibly outweigh possible health costs, and that the often stated “but I can’t stop” indicates that consumers are not maximizing their own welfare (or at the very least, there is some sort of time inconsistency). Others feel that smokers are perfectly aware of the addiction risks beforehand and are thus “rational”.
- d) Non-smokers—see part b).
- e) Even if the ban was imposed to give patrons a smoke-free environment, the market may come to some solution even without negotiation. If the harm to non-smokers is high enough, they should be willing to pay a premium for a non-smoking environment. This would give bar owners an incentive to voluntarily set and operate smoke-free bars. This kind of market segregation can make all sides happier.

While a tax on cigarettes addresses the externality problem caused by all smokers, it is non-specific. That is, it doesn’t address the issue at hand: second-hand smoke in public areas (non-smokers patrons of bars still have to breathe polluted air or leave the bar). Similarly, subsidies to help people quit, or for programs that help people quit, are non-specific. The tax in the case at hand would have to apply only in public areas. For example, in addition to a cover-charge, you may pay an additional fee when you go to a bar if you want to smoke. After paying the fee, you may get a ticket or hand stamp that shows you have a “permit” to produce second hand smoke. This fee-based approach would be easier to monitor and enforce than a cigarette-based maximum (i.e., than a standard).

Since the issue is one of air quality, clean air standards for bars have been proposed. In order to comply with the standard, bar operators would only have to curtail the amount of smoking, not necessarily eliminate it. Alternatively, they could install sufficiently effective ventilation systems. Owners may object to such a large expense.

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<sup>1</sup> The law had been criticized for its ambiguity. Bar owners aren’t required to actually kick out patrons that smoke, as long as they indicate that they are breaking the law. That meets the owners’ obligation for being compliant with the law. An employee may decide to waive his/her right, i.e., not request that it be legally enforced, in exchange for the aforementioned compensating wage. This flexibility makes the right transferable and enforceable. Thus, the law may have deliberately incorporated this ambiguity.