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EEP 101
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Chapter #14

Irreversibility and Option Value

A lot of times in economics, we analyze decisions as if they can be made continuously in very small increments. This procedure is what is referred to as **marginal analysis**, and the principle which always applies, in one form or another, at an economic optimum is MB=MC.

Yet, many decisions cannot be made continuously. Often, a decision involving natural resources is not a marginal choice of finding the optimal proportion of a resource to develop, but, instead, is an “all or nothing” proposition. Sometimes the decision variables are limited to **dichotomous (or discrete) choices**, such as when a certain parameter must take on either a value of zero or one.

For example: **Building a Dam** is a dichotomous choice, since you either build a dam or you don’t. *It is not possible to build half a dam.*

Similarly, when an urban area begins to encroach the boundary of a National Park, we may be faced with the decision of Preservation vs. Development. In some cases the decision to Develop a resource may be **irreversible**.

Definition of Terms:

Irreversibility: An irreversible action is one in which the effects of the action cannot be reversed, either absolutely or because the costs of doing so would be extremely high. If economic development leads to *extinction of species*, for example, then the extinction *absolutely cannot be reversed*. If a dam is built for a hydro-electric project the project can be reversed by removing the *dam* and making efforts to restore watersheds, *but can only be reversed at a great cost.*

Option Value: The value of retaining the option to make an irreversible decision until a future period in time. In a resource development context, the value of keeping a resource in a state of preservation as an option to develop at a later point in time. *The increase in expected net benefits as a result of making a decision after the uncertainty is resolved.*

The Benefit-Cost Method of Project Evaluation:

Project evaluation of Dichotomous Decisions has been based on Cost-Benefit analysis since the early 1900's:

- The basic premise is that if the Benefits of a proposed project are larger than the Costs, the project should be undertaken.

- When making a comparison between two projects, the project with the highest Benefit / Cost ratio should be undertaken.

The Benefits and Costs of a project are evaluated by the *expected present value*:

- PV(B) = $\int_0^\infty E[B(t)]e^{-rt}\,dt$ in the continuous time formulation, or

  $= \sum_{i=0}^{T} E[B_i(X)]\left(\frac{1}{1+r}\right)^i$ for some project X in discrete time

- PV(C) = $\sum_{i=0}^{T} E[C_i(X)]\left(\frac{1}{1+r}\right)^i$ for the project X

We Can Evaluate and Compare Projects Based on the Benefit-Cost Ratio:

- If $\frac{PV(B1)}{PV(C1)} > \frac{PV(B2)}{PV(B2)} > 1$, then Build Project 1.

For example, project 1 might be a Dam and project 2 might be a National Park.

In practice, implementing the Cost-Benefit criteria is not an easy task:

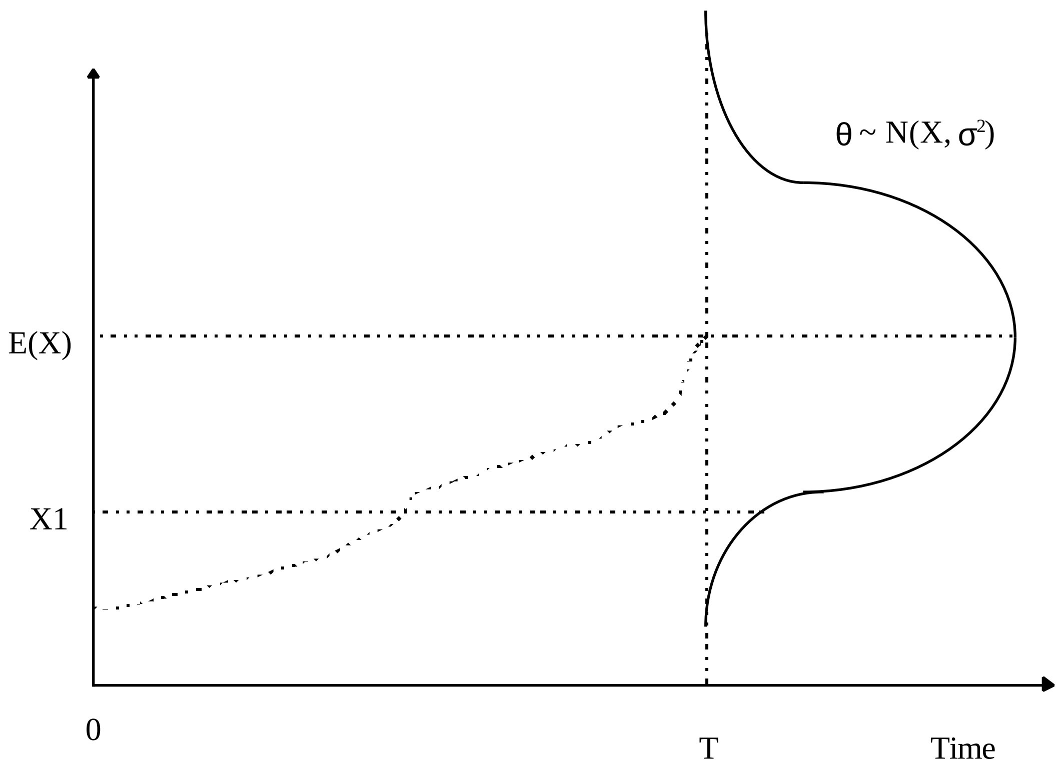
- It is difficult to identify Benefits and Costs before building the project
  - especially if nonmarket values exist for the project
- The choice of the discount rate is also a big issue
  - often the time horizon of Benefits and Costs differ, such as when costs are paid up front and benefits accrue over time.
  - high discount rates tend to make projects look unattractive

Yet, Cost-Benefit analysis has been widely used, particularly for evaluating Federal and State projects. There are three important issues Benefit-Cost analysis ignores:

1) The ability to delay an investment project until a later point in time
2) The potential irreversibility of the investment
3) The effect of uncertainty over future benefits

The existence of these 3 factors can introduce a value to delaying the investment until uncertainty has been sufficiently resolved: The value is called an **Option Value**.

Figure 12.1: The Value of A Stock Option



The concept of option value of a natural resource is closely related to the concept of a **Call Option** in the stockmarket.

Suppose as a shareholder of preferred stock, you are given the option to buy a given amount of stock in the company at some point in the future, such as at time T. Specifically, you are given the option to purchase the stock at a price of $X_T = E(X)$; that is, to buy the stock at the price that you and everyone else thinks the stock will be worth at time T. The distribution of prices might be known to be distributed normally, with an average price = E(X). Thus the expected value of the future stock purchase is $X_T - E(X) = 0$. Yet, the option is actually worth a positive amount of money because:

- If you get to time period T and the stock value is below E(X), such as at point X1, then you will not invest. Therefore, you lose nothing!

- But, if you get to time period T and the stock value is above E(X) you will buy it at a price below market value. Therefore you gain an amount of money equal to the difference between the stock value and the price you paid for it.

- **Stock options are worth a positive amount of money even though their expected value is zero, because they eliminate downside risk!**

Option Values in an Environmental Context

In order for an option value to exist in an environmental context:

- At least a portion of the investment must be irreversible
  - if the investment is completely reversible, there is no option value to waiting, since we could costlessly move back into the original state

- Uncertainty over future events must exist
  - with no uncertainty, we could accurately perform Cost-Benefit analysis as there would be no value to waiting for future information to occur.

- We must be able to delay investment as a viable policy choice

Option value is the result of an asymmetry between the decisions to invest or not invest

- If we choose to invest, we cannot reverse our decision
- If we do not invest, we can then decide to invest at a later time

Option Values occur when we wait before investing in irreversible projects because the decision to wait on investment eliminates downside risk!

How Investment Under Uncertainty Differs From Cost-Benefit Analysis:

Assume the Government can invest in a project with an initial cost of \$I in period t=0.

- The project will give expected benefits $E(B_t)$

- However, $E(B_t)$ is uncertain:
  - with Probability q benefits will increase by \$a in period t=1
  - with Probability (1-q) benefits will decrease by \$a in period t=1

- After period 1, the benefits will remain at that level throughout all time

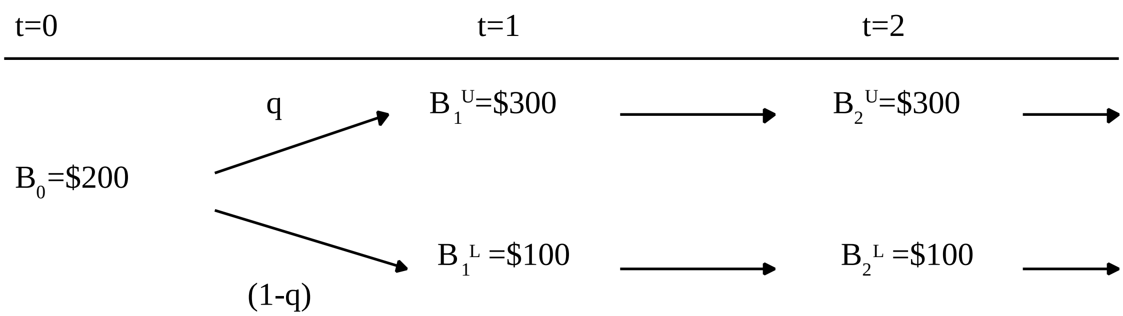
t=0	t=1	t=2
Pay \$I, receive B_0	$E(B_1) = q(B_0+a)+(1-q)(B_0-a)$ $= B_0 - a + 2qa$	$E(B_2) = E(B_1)$ $= B_0 - a + 2qa$

Example: (see Dixit and Pindyck, "Investment Under Uncertainty," Chapter 2).

Say,

- B_0 = \$200
- a = \$100
- I = \$1,600

then, we have the benefit matrix:



Say we also know that:

- q = 0.5, and
- r = 0.1

Using the Cost-Benefit Criteria,

- $\text{NPV} = -I + \sum_{t=0}^{\infty} \frac{E(B_t)}{(1.1)^t} = -\$1{,}600 + \sum_{t=0}^{\infty} \frac{\$200}{(1.1)^t}$

$$= -\$1{,}600 + \$200 \sum_{t=0}^{\infty} \left(\frac{1}{1.1} \right)^t ,$$

where we can now apply the rule for the sum of an infinite series, in which X = 1/(1.1), to get:

$$= -\$1{,}600 + \$200 \left(\frac{1}{1 - \left(\frac{1}{1.1} \right)} \right)$$

$$= -\$1{,}600 + \$200(110)$$

which implies that **NPV = \$600.**

It appears that, since the NPV of the project is positive the investment should be made.

However, we also know that, when we get to period t=1, benefits will either be \$100 or they will be \$300 indefinitely.

The Cost-Benefit figure of \$600 is not entirely correct, since it does not account for the opportunity cost of investing now instead of waiting and maintaining the option to not invest should the future benefits fall.

Now lets recalculate the NPV assuming we wait to see the outcome of benefits in t=1.

- If we wait there is no benefit and no expenditure in period t=0.

If the benefit decreases in period 1, then we have:

- $$\text{NPV} = (1-q)\left[\frac{-\$1{,}600}{1.1} + \sum_{t=1}^{\infty}\frac{100}{1.1^t}\right]$$

$$= 0.5\left[\frac{-1{,}600}{1.1} + \frac{1{,}100}{1.1}\right]$$

$$= \frac{-250}{1.1} < 0,$$

so the government should not invest in period t=1 when the value of the resource decreases.

If the benefit increases in period 1, then we have:

- $$\text{NPV} = q\left[\frac{-\$1{,}600}{1.1} + \sum_{t=1}^{\infty}\frac{300}{1.1}\right]$$

$$= 0.5\left(\frac{-1{,}600}{1.1} + \frac{3{,}300}{1.1}\right)$$

$$= \frac{\$850}{1.1} = \$773,$$

so the government should invest in period t=1 when the value of the resource increases.

If we wait to invest, the NPV *today* of the investment is \$773, while if we actually *make* the investment today according to Benefit-Cost estimations, the NPV is only \$600.

- In this case, **the Option Value is \$773 - \$600 = \$173!**

That is, we should be WTP \$173 to have a flexible investment plan instead of one that forces us to invest today.

- Waiting is valuable, because it allows us to eliminate downside risk.

Of Course, the **Value of the Parameters** is Really Important:

- Higher Investment Cost, \$I, increases the Option Value
  - This is because downside risk is larger
- Higher Initial Benefit, B_0, decreases the Option Value
  - This is because downside risk is larger

• Greater Uncertainty over Future Values Increases the Option Value
  - Uncertainty increases downside risk, spreads out tail of the distribution
• Larger Probability of Decreased Future Value Increase the Option Value
  - because downside risk is larger

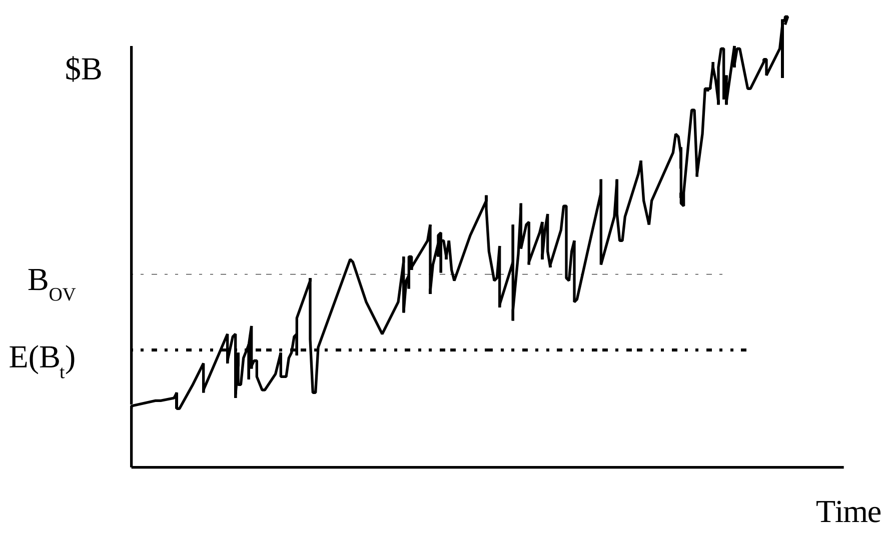
More Complicated Stochastic Processes

A **stochastic process** is a time related series of events which is subject to random variation. A stochastic process combines dynamics with uncertainty, so that the current state of the system determines only the probability distribution of future states, not the actual value.

The most simple formulation of a stochastic process is the one we just studied.
• we assumed a one-time binomial jump

Typically, a stochastic diffusion process is used, in which the effect of past disturbances are assumed to effect future outcomes according to some given rules of stochastic calculus.
• One of the most common stochastic diffusion processes used to model uncertain events is **Brownian Motion**, a process that **Einstein first proposed in 1905**. It is essentially a continuous time analog of the random walk process,

$$P_t = P_{t-1} + \varepsilon_t .$$



The idea is that Investment should be delayed until the project is **sufficiently "Deep in the Money",** so that the probability of the NPV dropping below zero is sufficiently small.

- The literature on Environmental Option Values calculates such Thresholds, called **hurdle rates**, shown as point B_{OV}, in which point a projects becomes sufficiently “Deep in the Money”

Option Values and the Choice Between Preservation and Development

Development, Preservation and Conservation areloaded terms;

- By **development**, I mean change to the environment made with intent for human good
- By **preservation**, I mean resources that are left for non-consumptive use
- By **conservation**, I mean resources that are applied to consumptive uses in a wise, economically beneficial manner.

Often development projects are dichotomous decisions; resources are either developed or not developed.

- This implies that there is an opportunity cost to develop in that you are giving up all future benefits to preservation.

The Benefit-Cost method of deciding between Development and preservation uses of a particular resource is to calculate Benefits and Costs to form the B/C ratio.

- The conventional approach has been to develop whenever B/C > 1

- The Option Value approach asserts that there is a value to the future information gained by observing future outcomes, and that considering Option Values:
    - We should wait until B/C > (1 + **hurdle rate)**

One of the major uncertain benefits of preservation is as a source of future medicine (the cure-for-cancer argument)

- Since most medicines used today are derived from tropical plants, and since each undeveloped acre of rainforest contains an estimated 1,000 new, undiscovered species. There is a high probability that the future value of rainforest in preservation will increase

Furthermore, as the tropical environment is continually transformed, acre by acre, from a state of preservation to a state of development, one might expect that the benefit from areas remaining in a state of preservation will increase.

Since the loss of preservation benefits on tropical lands is an opportunity cost of development, this is really the case of uncertain cost of investment:

- The E(Cost) of development in terms of the opportunity cost of foregone preservation will reflect only the current information available.
    - Hence, the true, uncertain value of preservation will only be known in future periods provided resources are left in preservation.

We will now turn to the case of the uncertain opportunity cost of investing in economic development.

The Case of Uncertain Preservation Benefits

Since the choice of whether or not to develop an area that is currently being preserved is an irreversible decision. If we develop a tropic rainforest, then we lose all future benefits that might arise from preservation, including the possibility of discovering new medicine. We can think of future preservation benefits as the uncertain opportunity cost of developing a natural resource area in terms of foregone future benefits of preservation.

Let:

I_0 = I = Cost of Developing an Acre of Rainforest (i.e., converting to farmland)

$$I_1 = \begin{cases} (I+a) & \text{with probability } q \\ (I-a) & \text{with probability } (1-q) \end{cases}$$

a = the value of new medicine discovered in an acre of Rainforest
q = the probability that new medicine is discovered in a given acre of Rainforest
r = the discount rate
D = annual profit from a developed acre of Rainforest, i.e., Net (π_d) = \$D

The question, again, is should we develop the resource today or hold the resource in a state of preservation until next year in order to assess whether or not an important medicine will be found.

- If we invest in development today:

$$\text{NPV}_0 = -I + \sum_{t=0}^{\infty} \frac{D}{(1+r)^t}$$

$$= -I + D\sum_{t=0}^{\infty}\left(\frac{1}{1+r}\right)^t$$

$$= -I + \frac{D(1+r)}{r}$$

$$\text{or,}\quad \mathbf{NPV_0} = \frac{\boldsymbol{D}(1+\boldsymbol{r}) - \boldsymbol{rI}}{\boldsymbol{r}}$$

The NPV is positive, but once again it ignores an opportunity cost. Thus, we still need to assess the value of the option to hold the area in a state of preservation until future information regarding preservation benefits is received.

Suppose that finding new medicine will make preservation the more viable option. Note that if it did not, that is, if development is *always* more highly valued that preservation, then there can be no option value, because behavior will never differ and a development outcome will arise regardless of whether or not new medicine is found. Thus, **there can only be an option value when the value of waiting to develop pays off by changing the optimal outcome to a state of preservation.**

Clearly, the regulator will only exercise the option to develop in period t=1 when development "is in the money" (i.e., when we know for certain that no new medicine will be found).

Let us now recalculate the NPV assuming that we wait until period t=1, in which case it will be *ex post* optimal to invest only when the opportunity cost of foregoing preservation is small, such as when I falls to I_1 = I - a.

$$\bullet\ \text{NPV}_{\text{OV}} = (1-q)\left[\frac{-(I-a)}{(1+r)} + \sum_{t=1}^{\infty}\frac{D}{(1+r)^t}\right]$$

$$= (1-q)\left[\frac{-(I-a)}{(1+r)} + \frac{D}{(1+r)}\sum_{t=0}^{\infty}\left(\frac{1}{1+r}\right)^t\right]$$

$$= (1-q)\left[\frac{-(I-a)}{(1+r)} + \frac{D}{r}\right]$$

$$\text{or,}\quad \mathbf{NPV_{OV}} = \frac{1-\boldsymbol{q}}{\boldsymbol{r}(1+\boldsymbol{r})}\left[(1+\boldsymbol{r})\boldsymbol{D} - \boldsymbol{r}(\boldsymbol{I}-\boldsymbol{a})\right]$$

We can now calculate the value of the option to delay development:

$$\bullet\ \textbf{Option Value} = \mathbf{NPV_{OV} - NPV_0}$$

$$= \frac{1-q}{r(1+r)}\left[(1+r)D - r(I-a)\right] - \frac{D(1+r)-rI}{r}$$

$$= \frac{(1-q)\left[(1+r)D - rI + ra\right] - (1+r)^2 D + r(1+r)I}{r(1+r)}$$

$$= \frac{rI(r+q) - (1+r)D(r+q) + ra(1-q)}{r(1+r)}$$

$$\textbf{or, Option Value} = \frac{(\boldsymbol{r}+\boldsymbol{q})\left[\boldsymbol{rI} - (1+\boldsymbol{r})\boldsymbol{D}\right] + \boldsymbol{ra}(1-\boldsymbol{q})}{\boldsymbol{r}(1+\boldsymbol{r})}$$

This expression gives the value of retaining a natural resource in a state of preservation for a single period before deciding whether or not to develop it.

The Effect of Alternate Parameters on Option Value:

The actual value of delaying development will depend on the parameters of a given problem, such as:

- the value of the resource in a perpetual state of development, D
- the opportunity cost of development in terms of lost medicine, a
- the physical cost of development, I
- the probability of finding new medicine, q
- the rate of time preference, r

We can analyze the effect of changes in the variables using Calculus:

The effect of a larger value for new medicine:

- $\frac{dOV}{da} = \frac{1-q}{1+r} > 0$

  - As the opportunity cost of development in terms of foregone medicine increases, the Option Value of delaying development (i.e., keeping land in preservation) increases.

As more of the surrounding countryside is developed, the price of new medicine will go up, since there will be a smaller supply of medicine found on other peoples land. Therefore, one might expect the Option Value of maintaining land in a state of preservation to increase as the world continues the process of development.

The Effect of Larger Development Benefits:

- $\frac{dOV}{dD} = \frac{-(r+q)}{r} < 0$

  - As the benefits from development increase, the option value of maintaining acreage in preservation decreases.

That is, poorer countries which stand to benefit greatly from engaging in development projects have a smaller option value of delaying economic development. Conversely, countries such as the U.S. with a small marginal value for an additional development project have much to gain from maintaining acreage in preserves, such as in National Parks and Wilderness Areas.

Unfortunately, much of the species diversity, and, hence, the potential for containing undiscovered new medicine is in areas of tropical rainforest, which are usually found in poorer countries. Also, much of the benefit of discovering a new medicine is embodied in consumer surplus, which will tend to have higher value in richer countries such as the U.S. This is an example of a case in which cross-subsidization might be beneficial, that is, rich countries might find it in their best interest to subsidize poor countries for acres of land left in preservation.

The Effect of Larger Development Costs:

- $\frac{dOV}{dI} = \frac{r+q}{1+r} > 0$

  - As the cost of development increases, the option value of maintaining acreage in preservation increases.

The reason for this is that downside risk is higher when there is a larger investment at stake. That is, the potential losses are greater when a larger amount is invested on land that might possibly contain new medicine.

The Effect of a Higher Probability of Discovering New Medicine:

- $\frac{dOV}{dq} = \frac{rI - (1+r)D - ra}{r(1+r)} = \left[\frac{I-a}{1+r} - \frac{D}{r}\right] < 0$ (from $NPV_{OV} > 0$)

  - as the probability of finding new medicine increases, the Option Value of maintaining acreage in preservation decreases.

Note: This is the same as stating that the Option Value increases as the probability that development will be profitable increases (i.e., preservation is not profitable when we get to period t=1). A higher value of (1-q), i.e., a lower value of q, implies that there is a greater chance that the cost of investment in terms of foregone preservation goes down so that the option to develop will be exercised.

The Effect of a Higher Discount Rate:

- $\frac{dOV}{dr} = \frac{r^2(1-q)(I-a) + (1+r)^2 Dq}{\left[r(1+r)\right]^2} > 0$, (using the quotient rule)

  - as rate of time preference increases, the Option Value of maintaining acreage in preservation increases.

A change in the discount rate has three effects;

1) The stream of development benefits is discounted for all future periods
2) The benefit of preservation is discounted for period t=1.
3) The cost of investing today is more important with higher discount rates

Since the benefit of preservation fully accrues in period t=1, while development benefits are discounted until the end of time, the higher the discount rate, the more period t=1 benefits are worth relative to development benefits that occur many periods from now. When the discount rate is high:

- The value of development benefits in the future disappears quickly.
- Preservation benefits gained next period are smaller
- It is more economical to defer development costs into future periods.

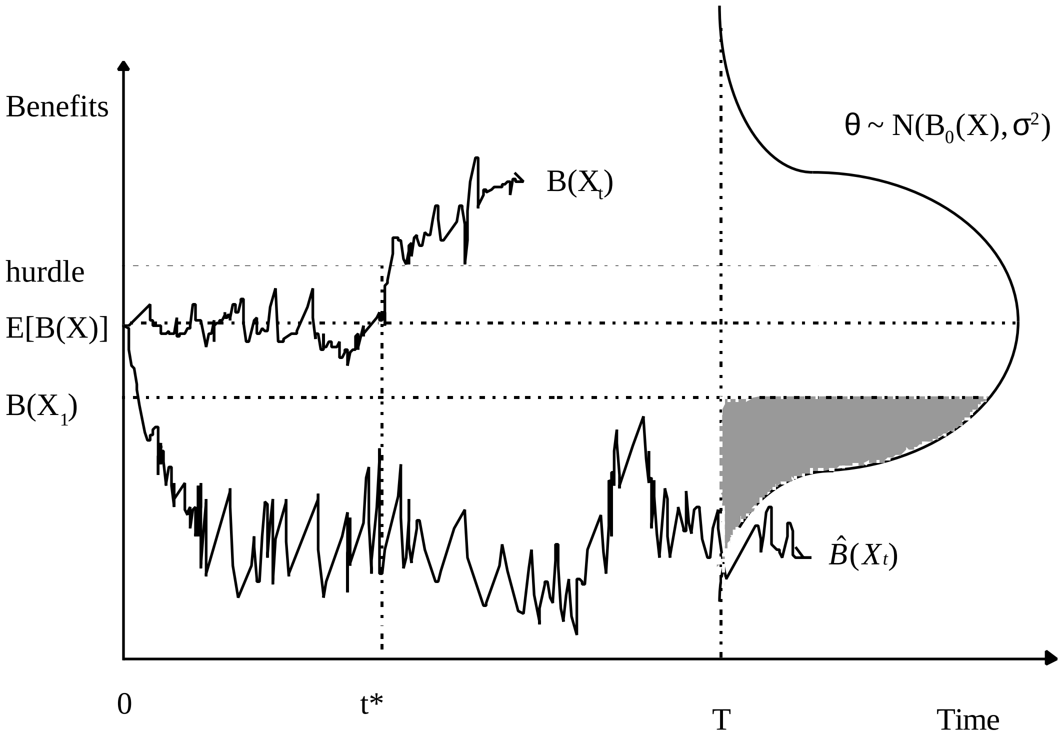
The effect of a higher discount rate is thus to increase the Option Value of preservation, since waiting defers uncertain expenses into the future.

Option Values In a More General Context

Unfortunately, we do not live in a world in which all of the uncertainty of today will be revealed in the very next time period. In actuality, uncertainty over benefits of preservation (and even development) tends to persist over time. Also, the benefits of a project generally fall into a range of values with a much richer distribution of outcomes than the binary case we have studied.

When uncertainty is slowly resolved over many periods of time and when there is a wide variety of possible benefits that might arise from a project, we might think about applying the Option Value context in the following way.

Consider a development project that has the following distribution of benefits at some time T.



Time period T is some hypothetical point far off into the future at which time the net benefit of development (Development Benefits less the opportunity cost of foregone preservation) will be known with absolute certainty.

- It does not matter how far into the future information is revealed
- The **important element is that the distribution of future NB is known**
  - If we don't know it, we use our best estimate.

Say we know that the distribution of future net benefit at time T will be:

- $B(X_T) \sim N(B(X_0), \sigma^2)$
  - That is; NB_T will be, on average, equal to NB today, but will be subject to some random variation given by σ^2.

The expected NB_T does not have to equal net benefit today and the basic concept of the problem will not change when $E(NB_T)$ takes on other values. We assume a constant NB to prevent having to calculate the effect of **drift**.

Since we know the distribution of development NB in period T, we can calculate the outcomes in this distribution in which development makes sense.

• i.e., development may not make sense for the realization: $NB_T \leq B(X_1)$

When development is non-optimal for all $NB_T \leq B(X_1)$, then development will not occur for any realization under the shaded portion of the distribution. We can now re-calculate the mean of the truncated distribution of NB_T, which only considers the cases in which development will occur.

- i.e., the mean of all possible values outside the shaded region of the distribution.

This **gives us our Expected Benefit of development if we waited until time period T**

- This **threshold value** is known as **the hurdle rate**

Intuitively, it is optimal to wait in every period in which a positive Option Value exists **(i.e., wait in period t whenever Option Value = NPV_{OV} - NPV_t > 0**).

- at some period t, however, $E_t[B(X)] = NPV_t = NPV_{OV}$ and Option Value = 0.
  - At this point we should develop, because it no longer pays to wait.

The Optimal Investment Time:

When we consider the Option Value of delaying development, the optimal time to develop the resource is at some point in the future when the realization of NB reaches the hurdle rate. At this point it is unlikely that the true realization of NB (if we were to wait until period T to find out) will be less than $B(X_1)$.

- The line $B(X_t)$ shows one possibility for the realization of benefits over time.
  - **Under $B(X_t)$, the optimal time to develop the resource is in period t*.**

- **For another possible realization of NB such as $\hat{B}(X_t)$, development will never be optimal, even if the expected NB_0 is positive under Cost-Benefit calculations**.