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EEP 101
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Chapter #12

Two-Period Renewable Resources Model With Non-Zero Interest Rate

Suppose that we are managing a renewable natural resource (e.g., a fishery) to maximize net present value, either for our firm or for society. For now, suppose that there is *not* open access to the resource, so that we will take into account all appropriate shadow costs associated with harvesting.

In the following model, we will not assume, prior to solving the model, that we are in steady-state. It may turn out that the solution to the problem is a steady-state solution. On the other hand, the solution may not be a steady-state solution. In general, the solution may give rise to a steady-state, or the solution may be to eventually harvest all of the resource (i.e., drive the resource to extinction), or the solution may involve a "cycling" of harvest and stock levels, where the optimal harvest and stock levels rise and fall over time, either converging to a steady-state, converging to extinction, or remaining in a perpetual cycle.

In general, we would solve the problem for all time periods (from time period zero to time period infinity) or we would solve the problem over the "**planning horizon**" of our firm or agency. A planning horizon is the number of time periods into the future for which the firm or agency makes plans. To keep the mathematics manageable, we will focus on a two time period planning horizon. These periods might be days, weeks, years, or even decades. There is really not much to gain from extending the model to a large number of time periods, T, since the optimal solution from period (T-1) to period (T) will mirror the optimal solution we find from period (T-2) to period (T-1) and so on, which, in turn mirrors the optimal solution we find from period 0 to period 1.

In planning horizon models, the issue arises of what to do with the "left-over" stock if any stock is in fact remaining at the end of the planning horizon. If demand is very low or harvesting costs are very high, then stock may indeed remain at the end of the planning horizon. This issue is dealt with by specifying a "**final-time function**" or "**salvage value function**."

A **salvage value function** gives the **value of any remaining stock at the** end of the planing horizon. For example, at the end of the planning horizon, we may wish to sell any remaining stock to someone else or else, perhaps, leave it as a bequest. The benefit we would get from selling the remaining stock or the utility we get from giving it away would be its salvage value. Usually the salvage value is a downward sloping function of the remaining stock level, because the more stock you try to sell at the end of the planning horizon, the lower the price you will receive at the end of the planning horizon for each unit of stock you try to sell. In other words, the final value function is often a demand function for the stock in the at the end of the planning horizon. Note that the salvage value is a function of stock, not harvest. In the case of a fishery, what would be sold at the end of the planning horizon is not caught fish (i.e., not "X"), but rather the right to catch the remaining fish in the sea, (i.e., the resource asset "S").

Let's lay out a **time line** to put these issues and terms in perspective:

```
initial stock                       stock at beginning              final stock
(stock at beginning                 of period 1                     (stock at end
of period 0)                                                        of period 1)
----------+--------------------------------+-------------------------------+----> time
         So                               S1                              S2
          |_______________________________|_______________________________|
                        X0                              X1
```

Now let's define terms, set up and solve the problem:

Definitions:

t	time period t = 0,1,2 note: (period 2 is the "final time"; no harvesting occurs in period 2)
S_t	resource stock at time t
$g(S_t)$	the growth function of the resource stock
X_t	harvest at time t
$B(X_t)$	total benefits from harvest at time t
$C(X_t, S_t)$	total costs of harvest at time t
$F(S_2)$	salvage value function
r	interest rate

The **objective** is to maximize the net present value of harvest in period 0, harvest in period 1 and salvage value in period 2. The **choice variables** are X0, X1, S0 and S1. There are two constraints, the equation of motion for the stock between periods 0 and 1, and the equation of motion for the stock between periods 1 and 2.

$$\max_{X_0, X_1, S_1, S_2} \text{NPV} = \underbrace{B(X_0) - C(X_0, S_0)}_{\text{net benefit period 0}} + \underbrace{\frac{B(X_1) - C(X_1, S_1)}{1+r}}_{\text{discounted net benefit period 1}} + \underbrace{\frac{F(S_2)}{(1+r)^2}}_{\text{discounted "salvage value" of final stock}}$$

subject to:

(1) $g(S_0) = S_1 - S_0 + X_0$, the equation of motion between periods 0 and 1

(2) $g(S_1) = S_2 - S_1 + X_1$, the equation of motion between periods 1 and 2

Also, So is the given, initial stock.

The **Lagrangian expression for this problem** is:

$$\max_{\substack{X_0, X_1 \\ S_1, S_2 \\ \lambda_0, \lambda_1}} L = B(X_0) - C\left(X_0, S_0\right) + \frac{B\left(X_1\right) - C\left(X_1, S_1\right)}{1+r} + \frac{F(S_2)}{(1+r)^2}$$

$$+\lambda_0\left(g(S_0) - S_1 + S_0 - X_0\right) + \frac{\lambda_1}{1+r}\left(g(S_1) - S_2 + S_1 - X_1\right)$$

where λ_0 and λ_1 are Lagrange multipliers.

<u>FOC's</u>

(1) $$\frac{dL}{dX_0} = B_{X_0}(X_0) - C_{X_0}\left(X_0, S_0\right) - \lambda_0 = 0$$

marginal benefit period 0	- marginal cost period 0	- user cost period 0	= 0

(2) $$\frac{dL}{dX_1} = \frac{B_{X_1}(X_1)}{1+r} - \frac{C_{X_1}\left(X_1, S_1\right)}{1+r} - \frac{\lambda_1}{1+r} = 0$$

NPV(marginal benefit period 1)	- NPV(marginal cost period 1)	- NPV(user cost period 1)	= 0

(3) $$\frac{dL}{dS_1} = \frac{-C_{S_1}\left(X_1, S_1\right)}{1+r} - \lambda_0 + \frac{\lambda_1}{1+r}\left(\frac{dg}{dS_1} + 1\right) = 0$$

$$\text{rearranging}: \lambda_0 = \frac{-C_{S_1}\left(X_1, S_1\right)}{1+r} + \frac{\lambda_1}{1+r} + \frac{\lambda_1}{1+r}\left(\frac{dg}{dS_1}\right)$$

user cost in period 0	=	NPV(increase in harvesting cost)	+ NPV(user cost associated with having one less unit of stock in period 1)	+ NPV (user cost associated with having less growth in period 1 because you have one less unit of stock in period 1)

(4) $$\frac{dL}{dS_2} = \frac{F_{S_2}(S_2)}{(1+r)^2} - \frac{\lambda_1}{1+r} = 0$$

NPV (marginal salvage value)	-	NPV (user cost in period 1)	= 0

(5) $\frac{dL}{d\lambda_0} = g(S_0) - S_1 + S_0 - X_0 = 0$

The equation of motion between periods 0 and 1 must be satisfied.

(6) $\frac{dL}{d\lambda_1} = g(S_1) - S_2 + S_1 - X_1 = 0$

The equation of motion between periods 1 and 2 must be satisfied.

Notice that keeping a unit of stock unharvested has three effects:

(1) Loss of interest from not harvesting the stock today.

(2) Savings in extraction cost, because $C_s(Xt,S_t) < 0$.

(3) Additional growth of the resource stock. The present value of the additional growth in stock is $\frac{\lambda_1}{1+r}\left(\frac{dg}{dS_1}\right)$.
Recall that S_m is the stock associated with maximum sustainable yield (which occurs where $g_s(S) = 0$, $g_{ss}(S) < 0$).

Notice that additional growth dg/dS can be positive or negative, depending on whether S is less than or greater than the stock associated with maximum sustainable yield, Smsy.
Additional growth is: Positive if $S_t < S_{msy}$.
Negative if $S_t > S_{msy}$.

In a steady state the sum of these effects is zero. Thus,

$\lambda_{t+1} - \lambda_t = 0$ (the shadow value of the resource remains constant) , and

$$r\lambda_t = X_t c_s(S_t) + g_s(S_t)\,\lambda_t$$

Marginal cost of delayed harvest (lost interest)	=	Marginal benefit of delayed harvest (growth + cost savings)

The optimal price of the resource does not change over time if marginal benefit of delaying the harvest of an incremental unit is equal to the marginal cost of delaying the harvest. The marginal cost of delaying the harvesting is the cost of foregone interest.

The marginal benefit of delaying the harvest is the sum of reduced harvesting cost ($X_t c_s(S_t)$) plus the value of the added growth of the stock, $g_s(S_t)\,\lambda_t$.

Thus, at steady state,

$$\left[r - g_s(S_t)\right]\lambda_t = X_t \; c_s(S_t). \qquad \textbf{(7)}$$

Let P_t be optimal price, i.e., $P_t = B_X(X_t)$:

At an optimal resource allocation, from FOC (2):

$$B_X(X_t) = P_t = \underbrace{\lambda_t}_{\text{user cost}} + \underbrace{c(S_t)}_{\text{extraction cost}}. \qquad \textbf{(8)}$$

Since at steady state,

$$S_{t+1} - S_t = 0 \quad \text{and} \quad \lambda_{t+1} - \lambda_t = 0, \quad \text{then } P_{t+1} - P_t = 0.$$

Furthermore, with optimal prices, the steady state equation for $\lambda_{t+1} - \lambda_t = 0$ can be expressed as a function of S and X by re-arranging equations (7) and (8).

$$[r - g_s(S)]\ [B(X) - c(S)] + Xc_s(S) = 0.$$

Yet, a steady-state may not always occur. We need to study the dynamics of a system to be assured that this can happen. A phase diagram is a tool that can be used to implement this (see next page).

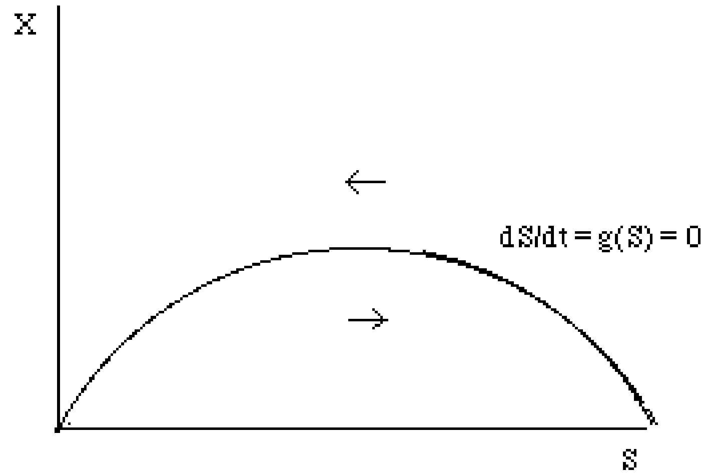
Phase Plane Analysis

Phase plane analysis is a graphical method of analyzing the dynamic behavior of a bioeconomic system. It is useful for determining whether a bioeconomic system will be in steady-state, will drive the resource to extinction, or will cycle.

A phase plane has harvest X on the vertical axis and stock S on the horizontal axis. Two curves are drawn on the graph. We will first talk about each curve separately, then we will put both curves together on the phase plane.

One curve gives all the points at which "the biology is in steady state," i.e., where the stock will not rise or fall, or where $S_0 = S_1 = S_2 = \ldots$. This curve is simply the growth function g(S) that we have seen before (Figure 11.1). On the phase plane it is labeled "dS/dt = 0", i.e., the change in stock over time is zero.

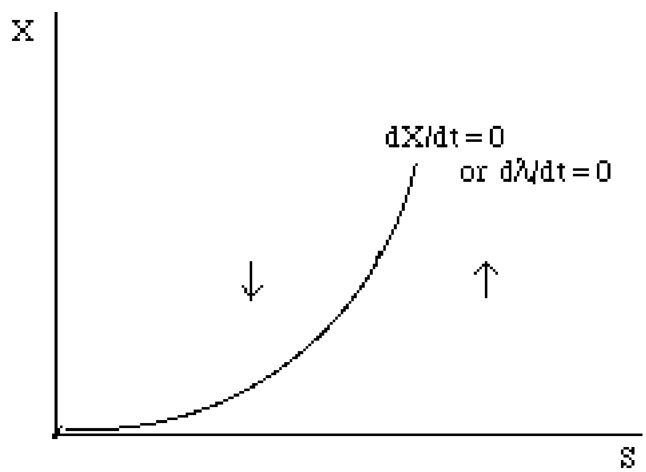
Figure ll.1



The arrows in Figure 11.1 show that, if harvest is above the dS/dt = 0 line, i.e., if harvest is above steady-state harvest, then stock will fall, and if harvest is below the dS/dt = 0 line, stock will rise. **The line, dS/dt = 0, is commonly referred to as an iso-cline**.

The second curve gives all the points at which "the economics is in steady state," i.e., where the economic agent has no incentive to either increase or decrease harvest X, or where $X_0 = X_1 = X_2 = \ldots$, or where $\lambda_0 = \lambda_1 = \lambda_2 = \ldots$. The second curve is derived from the first order conditions when assuming the system is in economic steady state. On the phase plane the second curve is either "dX/dt = 0," i.e., the change in harvest over time is zero, or "dλ/dt = 0," the change in (undiscounted) user cost over time is zero. For example, a curve shaped like the one labeled "dX/dt = 0" in Figure 11.2 can be derived.

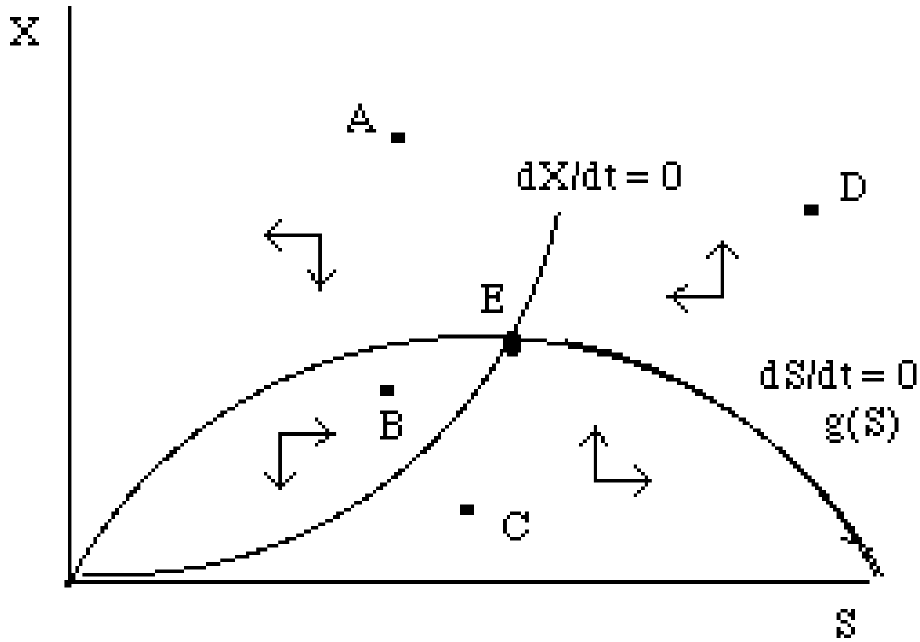
Figure 11.2

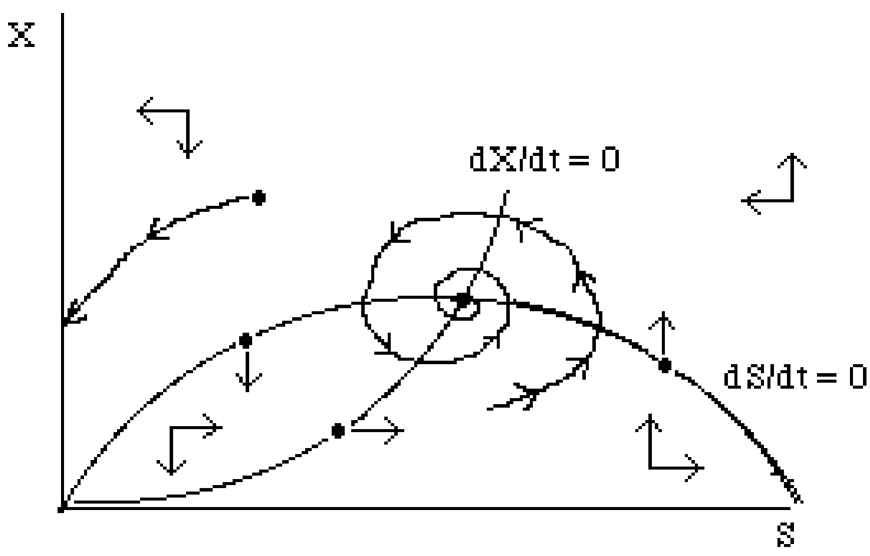


The arrows in Figure 11.2 show that, if stock is to the right of the dX/dt = 0 line, i.e., if stock is high, fish are easy to find in the ocean and harvesting costs are therefore low, then harvest will rise, and if stock is to the left of the dX/dt = 0 line, harvest will fall in order to equate MB = MC of fishing.

Putting the two steady-state curves together, we get Figure 11.3.

Figure 11.3: The Phase Plane





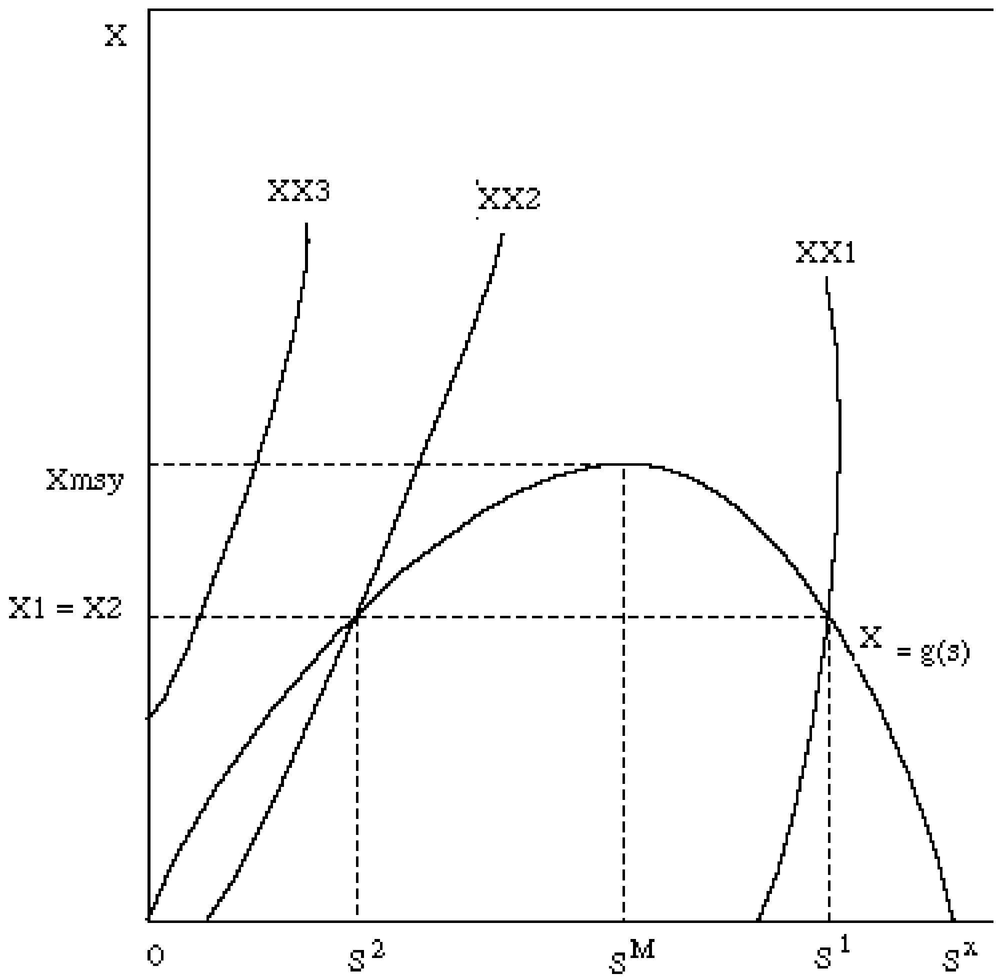


Figure 1: The possible scenarios of optimal management of renewable resources

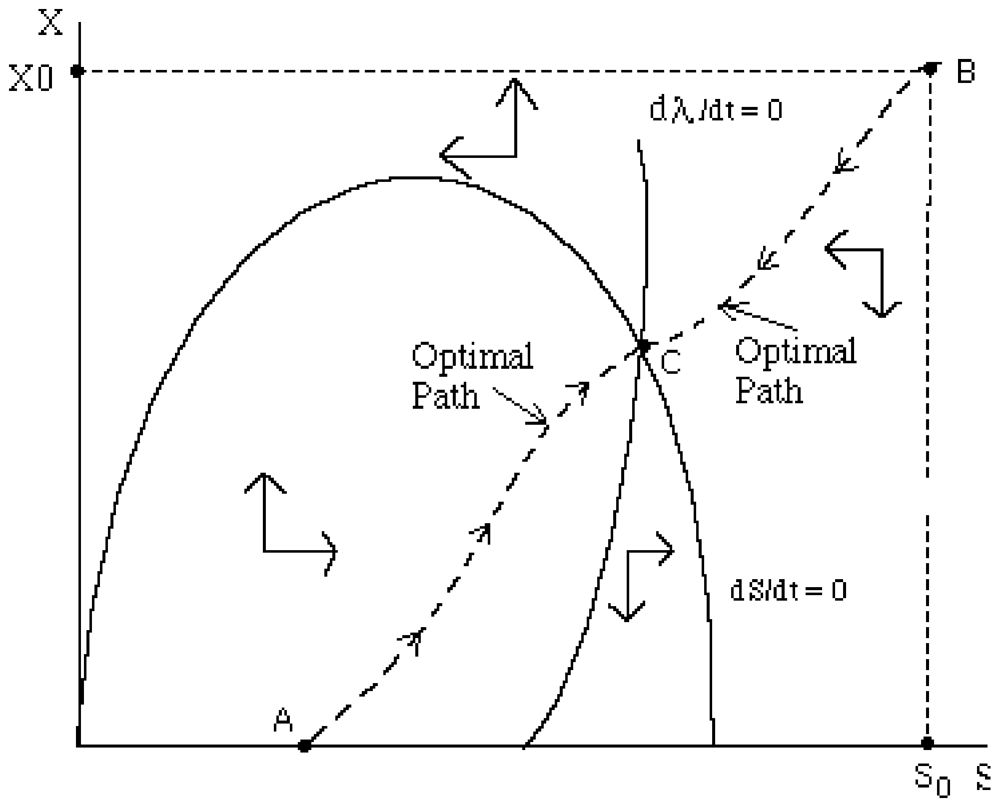
The curve, X = g(s), denotes all X, S combinations that lead to steady state of stock. The curves—XX^1, XX^2, and XX^3—denote all X, S combinations leading to steady state of stock *prices*.

- For each of these curves, $\left[r - g_S(S)\right]\left[B_X - C(S)\right] + XC_S(S) = 0$.

Each curve is drawn to represent a different interest rate; that is we can move between the curves by varying r.

- Curve XX^3 corresponds to the highest r,
- Curve XX^1 corresponds to the lowest r.

The Optimal Path



Phase planes are often drawn with respect to shadow prices. In the figure above, we can see that the implications on the harvest level, X, can be just the opposite when we have drawn the iso-cline dλ/dt = 0 but denote harvest level, X, on the y-axis.

To derive North-South Arrows:

- Say the shadow price of fish jumps up in a given period to a point above the iso-cline dλ/dt = 0
  - the harvest level will increase, because fish are worth relatively more.

- Say the shadow price of fish deviates downward in a given period to a point below the iso-cline dλ/dt = 0
  - the harvest level will decrease, because fish are relatively undervalued.

In this case, we do not get a solution in which the optimal path is a spiral. Here the optimal path is nearly a straight line. The optimal path gives the harvest level associated with any level of stock that will cause the system to converge to a steady-state optimal solution, at the equilibrium level C. For example, if the current stock is at point A, the harvest should initially be zero, then gradually increase along the path as the stock increases towards S*.

Transition from Hunting and Gathering to Farming
(Note: this lecture is on Technology Adoption, Section IX)

A recurring phenomenon in human history is the transition from hunting and gathering the fruit of biological resource systems into more intensive forms of production, such as farming.

Farming can be viewed as the cultivation of the products of biological processes in a (relatively) artificial environment. A similar phenomena is agroforestry, i.e., "tree farming" to produce forest products, which is an implicit assumption in the infinite rotation problem of Forest Economics.

Today, we are seeing a transformation from fishing to fish farming.

- Fish farming is usually called **aquaculture**, or
- **mariculture** if salt water species are grown.

Aquaculture activities include the production of fish, seafood, and algae.

The movement from fishing to aquaculture is similar to the process of transition from hunting and gathering to farming.

In the long run, aquaculture may become the dominant form of fish production, just as

- farming replaced gathering for the provision of fruits and vegetables, and
- ranching replaced hunting for the provision of Beef, Pork, and Chicken.

In principle, **biological production processes have several stages**:

Harvest
Breeding and nurturing
Feeding
Actively managing the environment (land preparation)

The degree of human involvement in the different stages of biological production varies among different systems.

- The only stage performed in **traditional hunting-gathering** activities is **harvest**.

- In cases when **nomadic herdsmen** move their animals, they **provide harvest, breeding, and protection (nurturing)**. They do not provide feeding.

- Going further along the path of transition, **beef ranchers** often **feed their cattle as well as actively manage pastures to supply abundant feed**. In modern **farming operations, humans are in control and manage all stages of production**.

Aquaculture involves the **farming of fish and other freshwater creatures, often in artificial ponds**. Ponds represent a **relatively intensively-managed, controlled environment**, similar to land preparation prior to planting in modern agriculture.

Managers breed salmon in fish hatcheries, and there are other breeding operations which produce oysters and other seafood.

- Artificial pearls are also products of aquacultural breeding activities.

Aquaculture producers often actively feed their animals or plants. In some areas, the government may subsidize or even provide food for aquaculture operations.

- The stocking of fish in a lake for recreational use is one example of subsidized aquaculture.

Aquaculture may support higher sustainable harvest levels of commercial or recreational fish by shifting the growth function of the species upward through providing food, habitat, or improved water quality.

A Model of Economic Transition

The extent of human involvement in the production process is an economic choice problem. When there are small numbers of humans (and many animals), the harvesting cost is low and harvesting may be economically preferable to farming. When the human population is large and the naturally grown food resource base is relatively small, harvesting becomes more expensive relative to farming.

The following is an example of a steady-state model for determining the optimal mix of aquaculture and fishing. It assumes a zero interest rate.

S = stock of fish in a lake

g(S) = growth function for the stock of fish in a lake

X = amount of fish harvested from the lake

Y = amount of fish farmed using aquaculture

Q = consumption of fish

$B(Q)$ = total benefit of consumption.

C(X,S) = total cost of harvesting fish from ocean

F(Y) = total cost of farming fish

In steady state, $X = g(S)$. The social optimization problem is:

$$\max_{X,S,Y,Q} B(Q) - C(X,S) - F(Y)$$

subject to:

$$X + Y = Q, \quad \text{and}$$

$$g(S) = X$$

The Lagrangian expression for this problem is:

$$\max_{X,S,Y,Q,\lambda,\theta} L = B(Q) - C(X,S) - F(Y) + \theta[X + Y - Q] + \lambda[g(S) - X].$$

The FOC's are:

1. $dL/dQ = B_Q - \theta. = 0$

The marginal benefit of consumption equals the shadow price of the consumption constraint. Note that price = marginal benefit, so $P = B_Q = \theta$.

2. $dL/dX = - C_X + \theta - \lambda = 0 \Rightarrow \quad P = \theta = C_X + \lambda$.

The optimal level of fish harvesting occurs where the sum of marginal cost of fishing and the shadow price of the stock is equal to the price of fish.

3. $dL/dS = \lambda g_S - C_S = 0$

The marginal value product of fish growth associated with a change in the fish stock is equal to the marginal change in harvest cost with respect to a change in fish stock. Or, the shadow price of stock = $\lambda = C_S/g_S$.

4. $dL/dY = \theta - F_y = 0 \Rightarrow \quad P = \theta = F_y$

The socially optimal level of fish farming occurs where marginal cost of fish farming is equal to the marginal benefit of producing a fish, or the price of fish.

5. $dL/d\theta = X + Y - Q = 0$ re-states the definition of Q.

6. $dL/d\lambda = g(S) - X = 0$ re-states the steady-state condition.

From FOC's (1)-(4), we can derive the result that have an equilibrium with respect to fishing and farming when:

$$P = B_Q = F_y = C_X + C_S/g_S,$$

i.e., when price equals the marginal benefit of consumption equals the marginal cost of fish farming equals the marginal harvest costs plus the shadow price of the stock of fish.

Example:

Let:

- $B(Q) = 10Q\text{-}0.5Q^2$
- $C(X, S) = \dfrac{3X}{S}$
- $F(Y) = 0.75Y + H$, where H = Fixed Costs (Setup Costs)
- $g(S) = 0.4S - 0.005S^2$

The **Optimization Problem**:

$$\mathsf{L} = 10Q - 05Q^2 - \frac{3X}{S} - 075Y + \theta\big[Q - X - Y\big] + l\big[04S - 0005S^2 - X\big],$$

can also be solved with considerably less algebra by substituting in for the constraints:

- Q = X + Y and g(S) = X imply that Q = g(S) +Y, or

$Q = 0.4S - 0.005S^2 + Y$

$$\mathsf{L} = 10[04S - 0005S^2 + Y] - 05[04S - 0005S^2 + Y]^2 - \frac{3[04S - 0005S^2]}{S} - 075Y$$

with the two FOCs:

(1) $$\frac{dL}{dY} = 10 - \big[04S - 0005S^2 + Y\big] - 075 = 0$$

(2) $$\frac{dL}{dS} = 4 - 01S - \big[04S - 0005S^2 + Y\big]\big[04 - 001S\big] - 0015 = 0$$

Manipulating (1), we get: $Y = 9.25 - 0.4S - 0.005S^2$

Plugging (1) into (2):

$$4.015 - 0.1S - [0.4S - 0.005S^2 + 9.25 - 0.4S - 0.005S^2][0.4 - 0.01S] = 0$$

or,

$$4.15 - 0.1S - [9.25][0.4 - 0.01S] = 0$$

which implies that: **S* = 42**

Plugging S* into the sustainability constraint on harvest, X, we get:

X* = 7.98

Plugging S* into FOC (1) we get: **Y* = 1.27**

Adding X and Y to meet the Quantity constraint: **Q* = 9.25**

And, Using the demand function, P = B'(Q) = 10-Q:

P* = $0.75 **(i.e., Dogfish)**

General Implications:

Technological change improves breeding productivity and reduces feeding costs, which makes aquaculture more attractive. Over time, as knowledge is accumulated, we would expect that the overall marginal cost of fish farming (F_Y) would decrease and that the relative importance of fish farming in fish production would therefore increase.

Farming: Involves Breeding, Feeding, and Nurturing Costs

- **Very Little Harvesting Costs**
- Degree of aquaculture determined by intersection of MC and P.
  - As technology increases breeding productivity and reduces feeding costs, farming becomes an attractive alternative to hunting.

Some Final Points for Consideration

There are considerable differences between farming and fishing. In farming, there are substantial breeding, feeding, and nurturing costs but small harvesting costs. In fishing, harvesting costs are critical.

When people put more effort into pre-harvest activities, they become concerned about the quality of final products.

As one moves hunting and gathering to farming systems, or from fishing to aquaculture systems, property rights systems evolve. We will consider the evolution of property rights systems in greater detail when we discuss water rights.