Lecture 3

Review of Production Economics

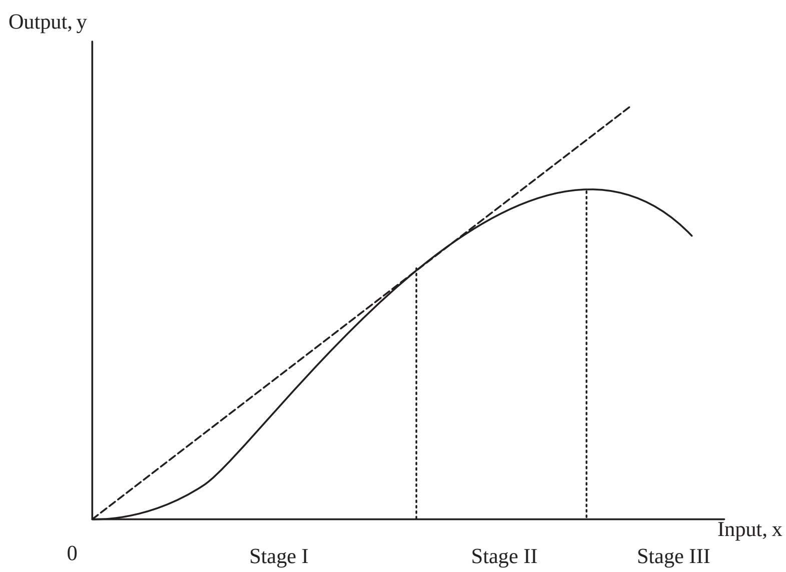
3.1 Production function and its parameters

Production function is defined by the maximum output that can be produced with a given input combination. The basic element is technology. The detail and accuracy of production function depend on its use. It is presented in more generic terms in a general theoretical context than in specific empirical applications. The basic assumptions of production function are presented in two graphs. The first represents output as a function of input and introduces the three stages of production Let $y =$ output and $x =$ input. The production function is $y = f(x)$; marginal productivity (MP) is $f_x = \partial f / \partial x$; and average product (AP) is y/x. Recall that

$MP > AP > 0$	at Stage I of production function
$AP > MP \geq 0$	at Stage II of Production function
$MP < 0$	at Stage III of production function

The second stage is the economic region. This is the stage with positive but decreasing marginal productivity or concave production function. A competitive profit-maximizing firm is likely to operate at this stage of the production function. Many mathematical specifications of production functions, like for example the Cobb-Douglas: $y = Ax^{\alpha}$, with $0 < \alpha < 1$, only represent situations when all outcomes are at the economic regions. Their use precludes identifying situations in which producers operate at the third stage of production and have negative marginal productivity. Quadratic production functions, $y = a + bx - cx^2$, allow outcomes at the second and third regions of production functions, but not at the first. A simple and elegant production function which allows three regions of production is not easy to

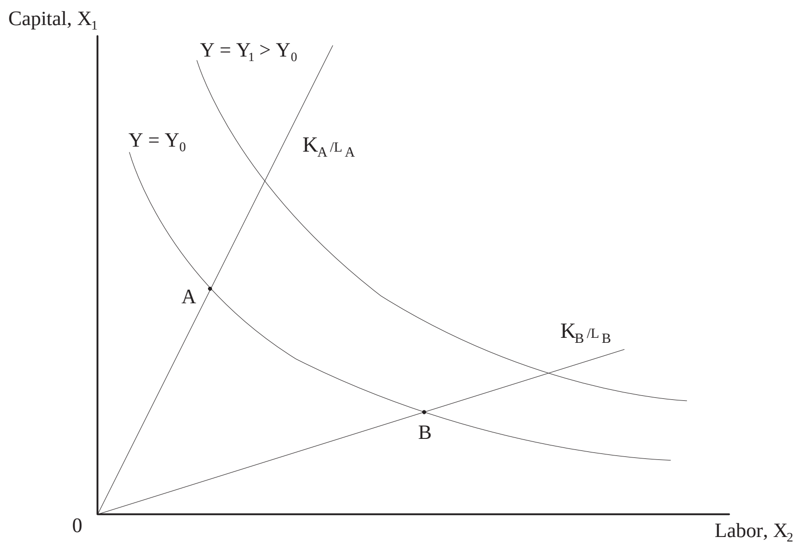
Figure 3.1: Three stages of Production



construct, so we use simple, elegant but flawed production function specifications in many analyses.

Figure 3.1 addresses the relationships between output and input in the production process. A basic issue it raises is that of *economies of scale*. It presents the assumption that below a certain level of output (Stage I), there is increasing returns to scale. However, at the economic region, there is constant, or more likely decreasing returns to scale. Figure 3.2 presents the relationships between inputs in

Figure 3.2: Isoquants and factor intensity



the production process. The *isoquants* depicted represents the different input level combinations producing the same level of output. Isoquants are useful to address issues such as input intensity and input substitutability. If x_1 is capital and x_2 is labor, $\frac{x_1}{x_2}$ measures capital intensity (relative to labor). Production at point A is capital intensive and at B is labor intensive.

Economists are also interested in assessing the ease of replacing one input for another while maintaining output fixed. When production function is of the Leontief type (i.e., with fixed proportion):

$$y = \min \left\{ \frac{x_1}{a_1}, \frac{x_2}{a_2} \right\}$$

the isoquant for each production level is essentially one point

$$\left(x_1 = \frac{y}{a_1}, x_2 = \frac{y}{a_2}\right)$$

and input intensity is constant at $x_1/x_2 = a_1/a_2$. If production function is linear, $y = a_1x_1 + a_2x_2$, the isoquant is a straight line, $x_1 = y/a_1 - a_2x_2/a_1$, and there is infinite substitution possibilities.

To allow quantification and comparison of production technologies, key parameters of the production function were defined. These dimension-free numbers are: the input elasticity, η_i; the scale elasticity, ε; and the elasticity of substitution, σ_{ij}. Let $y = f(x_1, x_2, \ldots, x_n)$ be a production function; y = output; and x_i = inputs, then the following quantities can be defined:

- $f_i = \frac{\partial f}{\partial x_i}$ = marginal productivity.
- $\eta_i = \frac{\partial f}{\partial x_i}\frac{x_i}{y} = f_i\frac{x_i}{y}$= input elasticity
- $\varepsilon = \sum_{i=1}^{n} \eta_i$ = scale elasticity

The elasticity of substitution between input i and j is denoted as σ_{ij}. In case of two inputs, x_1 and x_2, the elasticity of substitution between x_1 and x_2 is:

$$\sigma_{12} = -\frac{\partial(x_1/x_2)}{\partial(f_1/f_2)}\Big/\frac{(x_1/x_2)}{(f_1/f_2)} = -\frac{d\ln(x_1/x_2)}{d\ln(f_1/f_2)}$$

It is a measure of the ease of change in input intensity. $\sigma = 0$ implies fixed proportion production function and input intensity does not change. At the opposite extreme, there is the linear production function ($y = ax_1 + a_2x_2$), in which case $\sigma = \infty$ and input intensity easily changes.

The value of these production function parameters is not necessarily fixed. For example, in the case of one input, $\varepsilon = \eta$, that is, input elasticity is equal to scale elasticity. Thus, at the first stage of the production function it is $\varepsilon > 1$; at the economic stage $1 > \varepsilon > 0$ and at the third stage, $\varepsilon < 0$.

3.1.1 Production Function under Perfect Competition

The parameters of the production function have special interpretation under profit maximization and price taking. Under such conditions, the firm's optimization problem is

$$\max_{x_1,\ldots,x_n} Pf(x_1,\ldots,x_n) - \sum_{i=1}^{n} x_iW_i$$

where W_i is price of input i. The first-order condition for i-th input is

$$Pf_i - W_i = 0$$

and it can be interpreted as stating that the value of marginal product of input i must equal its price. This condition can be expressed in terms of input elasticity: it becomes

$$Py\frac{f_i x_i}{f} - W_i x_i = 0$$
$$\Rightarrow Py\eta_i = W_i x_i$$
$$\Rightarrow \eta_i = \frac{W_i x_i}{Py}$$

that is, the input elasticity equals the share of revenue spent on input i. In the case of constant returns to scale, $\sum \eta_i = 1$ and η_i = share of input i in all expenditures. Under competition,

$$\frac{f_1}{f_2} = \frac{W_1}{W_2}$$

Example: the Cobb-Douglas production function

functional form	$y = Ax_1^{\alpha_1} x_2^{\alpha_2}$
marginal product	$f_i = \frac{\alpha_i y}{x_i}$
input elasticity	$\eta_i = \alpha_i$
scale elasticity	$\varepsilon = \alpha_1 + \alpha_2$
elasticity of substitution	$\sigma = 1$

The Cobb-Douglas production function is limited because it has constant elasticity of substitution; i.e., it does not allow for regions of increasing marginal productivity nor for negative marginal productivity.

3.1.2 Duality and its implications

The cost and profit functions are key relationships for deriving quantities demanded and supplied. The profit function is defined as:

$$\Pi(P, W_1, \ldots, W_n) = \max_{y, x_1, \ldots, x_n} Py - \sum_{i=1}^{n} x_i W_i$$

subject to

$$y = f(x_1, \ldots, x_n)$$

and

$$\frac{\partial \Pi}{\partial P} = y(P, W_1, \ldots, W_n) \quad \text{output supply}$$

$$-\frac{\partial \Pi}{\partial W_i} = x_i(P, W_1, \dots, W_n) \quad \text{input } i \text{ demand}$$

We can use dual relationships to present relationships between quantities as functions of monetary relationships. Dual relationships can be used to estimate supply and demand for agricultural commodities and agricultural input. Furthermore, when data about output levels or input mix are not available, dual relationships can be used to estimate them. For example, one may have accounting data on output of different firms and one may have price. Using duality one can estimate output and input. In principle, one call use duality–based relationships to estimate even production function parameters from monetary data. Indeed, Chambers and Pope (1994) do that in a recent article[1].

One of the key challenges of applied economists is to estimate production parameters. Sometimes it is easier to obtain prices, expenditures or revenue data than quantity data. Duality relationships and other relationships derived under profit maximization provide the base for estimation of technology relationships without data on quantities. For example, if

- revenues = 100,
- expenditures on input 1 (labor) = 20,
- expenditure on input 2 (fertilizers) = 30

the suggested labor elasticity is 0.2 and fertilizer elasticity is 0.3. If relative prices of labor increase by 10 percent and labor expenditures become 21 and fertilizer expenditures become 31, the implied elasticity of substitution under competition can be derived as follows. First, under the initial condition,

$$\frac{W_1^0 x_1^0}{W_2^0 x_2^0} = \frac{20}{30}$$

(x_i^0, W_i^0 = price and quantities under initial outcome). Under new outcome expenditures, the ratio is $W_1^1 x_1^1 / W_2^1 x_2^1$ which is

$$\frac{W_1^1 x_1^1}{W_2^1 x_2^1} = \frac{W_1^0 x_1^0}{W_2^0 x_2^0} 1.1(1+z) = \frac{2}{3} 1.1(1+z) = \frac{21}{31}$$

where z = rate of change in x_1/x_2 or

$$\frac{x_1^1}{x_2^1} = (1+z)\frac{x_1^0}{x_2^0}$$

[1]Robert G. Chambers and Rulon D. Pope, "A Virtually Ideal Production System; Specifying And Estimating the VIPS Model", *American Journal of Agricultural Economics*, Vol.76, No I (February, 1994), pp. 105-113.

$$z = \frac{63}{62 \cdot 1.1} - 1 = 0.0762 \Rightarrow \text{ elast. of subst. } \sigma = -\left(-\frac{0.0762}{0.1}\right) = 0.762$$

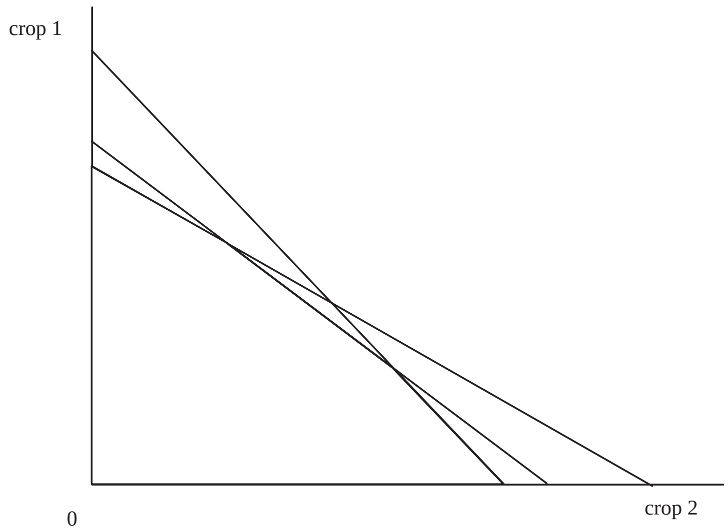
There is a growing literature on the estimation of technological parameters from monetary data assuming profit maximization. However, it is clear that the estimated relationships are not necessarily the true technical parameters. Milton Friedman said that it is not clear that decision makers are profit maximizers, but the data are such that it seems "as if" decision makers are profit maximizers. Production function parameters that are estimated under the profit maximization assumptions are in essence "as if" production function parameters. One can separate between technological relationships and behavioral relationships. Behavioral relationships are relationships that incorporate technological assumptions and behavioral assumptions. Production function parameters that are estimated under duality have a strong behavioral component. Even production theory recognizes that strict profit maximization is unrealistic; behavior has to adjust to uncertainty and risk, and there are new models of production behavior under uncertainty. Simon introduces a notion of bounded rationality. He suggested that the ability of humans to process and analyze data is limited. Therefore, choices are not perfect and reflect this limited ability. One of the challenges is to decipher the factor behind production decisions and to understand what leads producers to make choices. Choices under strict profit maximization may be different than under risk aversion and limited analytic capacities. The same technological relationships may result in different outcomes under different behavioral assumptions. However, it is very difficult to untangle the behavioral and technological contributions to observed outcome.

The production functions that were previously discussed are very stylized. They represent the production function of a salad. All the components are put together and mixed instantaneously. Actual production relationships are more complex. Time plays an important role in production and production includes several stages. One of the challenges of production theory is to introduce the dimensions of time in the production process. Antle developed one model of sequential production process, and there are several other attempts to look at the different stages of production in order to represent more realistic models of a production process.

However, modeling is an act of abstractism. For some purposes, we need a very simple representation of reality and in other uses we need a more realistic representation. In many aggregate analysis, a simple presentation of the production process associated with the traditional production function, is sufficient. For micro analysis, one may need more detailed modeling that takes into account specific biological and physical phenomena.

Before we proceed with explicit production modeling issues, we will discuss an-

Figure 3.3: Production Possibility Frontier



other issue, the measurements and nature of inputs used in the production process.

3.2 Multiproduct Production Function

With two crops and n constraints, it is possible to construct a production possibility frontier. The production possibility frontier denotes the tradeoff between output and every combination of inputs. Its slope is the marginal rate of technical substitution whose formula is given as:

$$MRTS = -\frac{dy_1}{dy_2} = -\frac{f_2}{f_1}.$$

The production possibility curve is a concave function of y_2:

$$\frac{d\left(-\frac{dy_1}{dy_2}\right)}{dy_2} < 0$$

Multiproduct production functions embody behavioral relationships as well as technical relationships. They describe a relationship where there is jointness in production. Define

$$f(y_1, y_2) = g(\mathbf{x})$$

where $\mathbf{x}$ is a vector of input, then $f(y_1, y_2)$ is a multiproduct production function. The function f is concave in y_1 and y_2, e.g.,

$$f(y_1, y_2) = y_1^{\alpha} y_2^{\beta}, \qquad \alpha + \beta < 1.$$

$g(\mathbf{x})$ may be concave in $\mathbf{x}$. One can use the implicit function theorem and write:

$$y_1 = h(\mathbf{x}, y_2).$$

Let $\mathbf{y} = n \times 1$ vector of outputs, $\mathbf{x} = k \times 1$ vector of inputs, $G(\mathbf{y}, \mathbf{x}) = 0$ is an $mx1$ set of relations. Let's divide $\mathbf{y} = (\mathbf{y}_1, \mathbf{y}_2)$, with $\mathbf{y}_1 = m \times 1$, and $\mathbf{y}_2 = (n - m) \times 1$. One can write (assuming an invertible function)

$$\mathbf{y}_1 = h(\mathbf{y}_2, \mathbf{x})$$

where $d\mathbf{y}_1 = -\left\{[G_{y_1}^{-1}]G_{y_2} d\mathbf{y}_2 + [G_{y_1}^{-1}]G_x d\mathbf{x}\right\}$. The key element is $[G_{y_1}^{-1}]$ which is a regular mxm matrix.

Inputs are allocatable and not general. These production functions mentioned above reflect feasibility constraints and not the general technical relationships. Therefore, when we have a multiproduct relationship, we have to define the source of the jointness in production. We need alternative models that determine the source of jointness.

Thus, we have *activity* —an interaction of inputs and output defined by space and time. An activity can be represented by equation or equations.

Let us represent allocable inputs as x, joint inputs as z and outputs as y and technology as a collection of activities. We can then look at the various sources of jointness in production including:

1. *Technical jointness* which includes

    input jointness

    output jointness

2. *Physical jointness*

3. *Behavioral jointness.*

Each type of jointness may lead to a *joint*, a multiproduct production function. Let
j be an index of allocatable inputs;
k an index of nonallocatable inputs;
i an index of output, and
n an index of activity.

3.2.1 Nonjoint Activity

For simplicity, the activity and output can be merged; thus, we have:

- One output.
- Many allocatable inputs:

$$y_i = f_i(x_{i1}, \dots, x_{iJ}).$$

3.2.2 Nonjoint Technology

Consists of nonjoint activities. For agriculture, that may be the case.

3.2.3 Input Joint Technology

Each activity has allocatable inputs and at least one joint input. Again, dropping n,

$$\text{for } i = 1: \quad y_1 = f_1(x_{11}, \dots, x_{1J}, z_1, \dots, Z_k).$$

$$\text{for } i = 2: \quad y_2 = f_2(x_{21}, \dots, x_{2J}, z_1, \dots, Z_k).$$

One can think of a multiproduct production function with input jointness when the inputs (human capital) are unobservable.

Output-Joint Activity

$$(y_{1n}, \dots, y_{In}) = f_n(x_{n1}, \dots, x_{nJ})$$

This is a technically justified joint production. A process has several outputs, e. g., pollution and output, or lambs, wool, and milk.

The same input mix can generate all of them, and no costly decision determines when it will be.

The first figure describes a nonjoint process with J inputs and I outputs, where the allocation of inputs among outputs is not specified. Let $\bar{x}_j$ be total use of the j-th input. The system is defined by

$$q_i = f_i(x_{i1}, \dots, x_{iJ}), \quad i = 1, \dots, I \tag{3.1}$$

$$\bar{x}_i = \sum_{j=1}^{J} x_{ij} \tag{3.2}$$

In this case we have $I + J$ equations with $I + IJ + J$ variables. Using the implicit function theorem, we can present n variables as a function of the other $IJ + I + Jn$ variables.

Figure 3.4: Non joint process

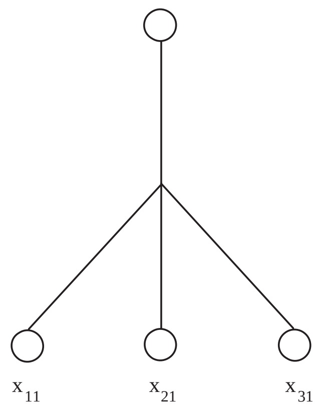


Figure 3.5: Input joint technology

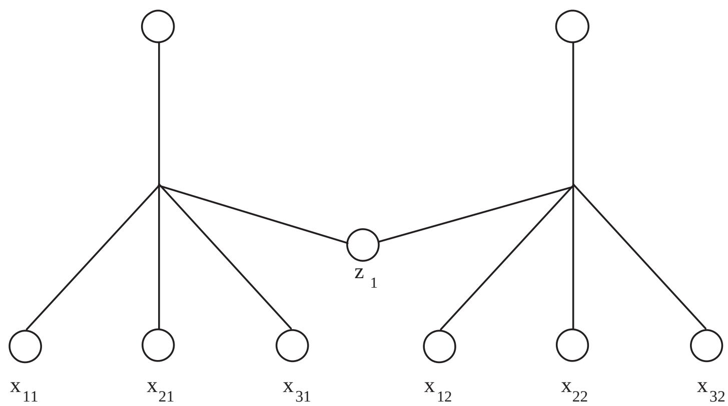


Figure 3.6: Output joint processes

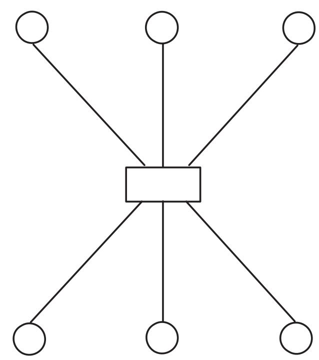
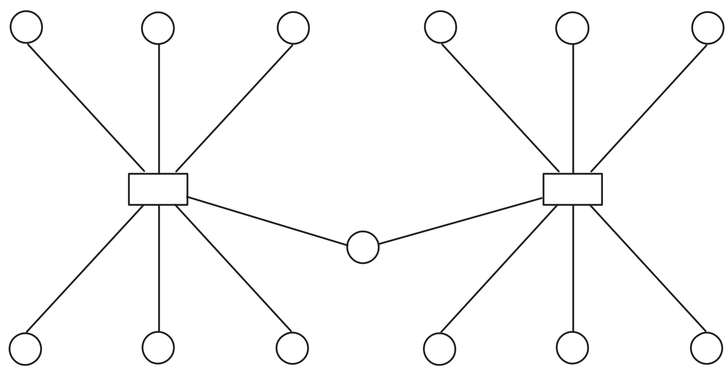


Figure 3.7: Input–output joint processes



In particular, to derive the joint production functions,

$$q_i = h(q_2, \dots, q_I, \bar{x}_1, \dots, \bar{x}_J) \tag{3.3}$$

that reflect technology and resource availability from the original system of equations (3.1) and (3.2), we eliminate $I + J - 1$ equations and $I + J - 1$ variables. In the new system described by equation (3.3), there are $I + J$ variables. Thus, if one can move from (3.1) and (3.2) to (3.3), $IJ + I + J - I - J + 1 = I + J$, or $IJ = I + J - 1$. Thus, if either $I = 1$ or $J = 1$, joint production function (3.3) represents technology and resource availability. For example, with $J = 1$, $I = 2$, suppose

$$q_1 = f_1(x_{11}), \tag{3.4}$$
$$q_2 = f_2(x_{21}), \tag{3.5}$$
$$\bar{x}_1 = x_{11} + x_{21}. \tag{3.6}$$

Then

$$x_{21} = f_2^{-1}(q_2), \tag{3.7}$$
$$x_{11} = \bar{x}_1 x_{21} = \bar{x}_1 - f_2^{-1}(q_2), \text{ and} \tag{3.8}$$
$$q_1 = f_1\left[\bar{x}_1 - f_2^{-1}(q_2)\right] = h(q_2, \bar{x}_1) \tag{3.9}$$

where

$$q_1 = f_1(x_{11}, \bar{x}_2)$$
$$q_2 = f_1(x_{21})$$
$$\bar{x}_1 = x_{11} + x_{21}.$$

We can use a similar process to obtain

$$q_1 = h(q_2, \bar{x}_1, \bar{x}_2).$$

But when

$$q_1 = f_1(x_{11}, x_{12}),$$
$$q_2 = f_2(x_{21}, x_{22}),$$
$$\bar{x}_1 = x_{11} + x_{12},$$
$$\bar{x}_2 = x_{21} + x_{22},$$

one can eliminate three variables out of x_{11}, x_{12}, x_{21}, and x_{22} and present $q_1 = h(q_2, \bar{x}_1 \bar{x}_2, \tilde{x}_{ij})$, where $\tilde{x}_{ij}$ is the one variable out of $x_{ij}, (i = 1, 2; j = 1, 2)$ that was not eliminated.

To have a transformation from a technology representation such as in (3.1-3.2) to one like that in (3.3) is not feasible in the general case, where it involves decreasing the number of variables by IJ and the number of equations by $I + J1$. A transformation to a multiproduct equation may necessitate adding $IJ - I - J + 1$ relationships between the original variables.

Profit maximization can provide such relationships. Consider the problem

$$\max_{\{x_{ij}\}} \sum_{i=1}^{I} \left[p_i f_i(x_{i1}, \dots, x_{iJ}) - \sum_{j=1}^{J} w_j x_{ij} \right]. \tag{3.10}$$

In this case, the first-order conditions are

$$p_i f_{ij} = w_j, \quad i = 1, \dots, I, \quad j = 1, \dots, J \tag{3.11}$$

where $f_{ij} = \partial f_i / \partial x_{ij}$.

For the IJ first-order conditions, add $I + J1$ new variables (prices). Thus, they actually provide $IJ - (I + J - 1)$ independent new relationships of the x_{ij}'s and, by adding relationships such as

$$\frac{f_{ij}}{f_{i1}} = \frac{f_{1j}}{f_{11}}, \quad \text{for } i = 2, \dots, I, \quad j = 2, \dots, J \tag{3.12}$$

to (3.1) and (3.2), a transformation to (3.3) is feasible. Thus, the joint production function (3.3) in most cases reflect both *technological* and *behavioral* considerations.

3.3 Inputs in the Production Process

Capital

Capital consists of all the equipment, structure, and machinery used for production. Capital represents outcomes of previous production activities that are embodied in some assets relating to present production activities. Generally, capital is utilized with viable inputs –labor, energy and fertilizers– that are consumed by the production process. Producers may purchase services of capital goods or they may own capital assets that would be reflected differently in their accounting documents. Capital is measured by the value of the assets that are used as capital goods. In each period there is a cost associated with the use of capital goods. First, it includes the cost of physical depreciation as well as the periodical costs for the resources that were used in the capital investment (interest costs).

Labor

One difficulty in measuring labor comes from the differences in quality between different individuals. Generally, there can be different wage rates according to the quality of labor services provided. An important concept is human capital. Knowledge acquired through training and education in the past is a determinant of productivity in the present. Compensation for workers combines payment for the raw labor services as well as a return for their human capital.

Land

As labor, land is not a homogeneous input. Land quality varies depending on location, physical characteristics, etc. There are different mechanisms for payment of land services including rental fee, sharecropping, etc. Moreover, quality of land may affect the effectiveness of new technologies.

Pesticides

These are damage control agents. Their productivity depends on the environment, the pest situation, and the product.

Water

The value of water depends on its use, quality, and location.

Each input has unique features that may be essential in modeling behavior at the firm level. As the analysis become more aggregated, generic modeling is more relevant.

The next two sections in this lecture will develop models to analyze problems of water and pesticides. The modeling will demonstrate how some of the basic biological or physical properties of water and pest control affect the specifics of the modeling of the production process, the nature of choices that are applied, and the type of outcome that we will observe.

3.4 The economics of land-quality augmenting input application technology

symbols

y = output per acre

e = effective input per acre

a = applied input per acre

i = application technology indicator: $\begin{cases} i = 0 \text{ for traditional technology} \\ i = 1 \text{ for modern technology} \end{cases}$

α = land quality
$0 < \alpha < l$
α measures input use efficiency of traditional technology on soil

$y = f(e)$ is production function, with $f' > 0$ and $f'' < 0$.

$h_i(\alpha)$ = input efficiency function = fraction of input consumed by crop with technology i and land quality α.

Let us consider only one alternative technology, $h_1(\alpha)$, and let us assume

$$0 \leq \alpha \leq h_1(\alpha) \leq 1$$

$$h_i' > 0, \qquad h_i'' < 0$$

P = output price

W = water price

k_i = per acre cost of technology i with $k_1 > k_0$

$\delta_i = \begin{cases} 1 & \text{if technology } i \text{ is chosen} \\ 0 & \text{otherwise} \end{cases}$

The optimization problem faced by a farmer when chosing the technology, is:

$$\max_{\delta_i, a_i} \sum_{i=0}^{1} \delta_i \left(Pf(h_i(\alpha)a_i) - Wa_i - k_i \right) \tag{3.13}$$

with:

$$\delta_i \in \{0, 1\}$$

$$0 \leq \sum \delta_i \leq 1$$

The search for an optimal solution is conducted in two stages. First, the optimal continuous choice is analyzed for each of the alternative technologies:

$$\Pi_i = \max_{a_i} Pf(h_i(\alpha)a_i) - Wa_i - k_i \tag{3.14}$$

With F.O.C.

$$
\begin{aligned}
Pf'h_i &= W \qquad (3.15)\\
\left\{ \begin{array}{r} \text{value of} \\ \text{marginal product input} \end{array} \right\} &= \text{price of input}\\
\Rightarrow Pf' &= \frac{W}{h_i(\alpha)}\\
\left\{ \begin{array}{r} \text{value of marginal} \\ \text{product of effective input} \end{array} \right\} &= \text{price of effective input}
\end{aligned}
$$

Once the optimal quantity of input to be used under each technology, a_i, is found, the *discrete choice* problem is solved, choosing

$$\delta_1 = 1 \text{ if } \Pi_1 > \Pi_0, \Pi_1 > 0$$

$$\delta_0 = 1 \text{ if } \Pi_0 > \Pi_1, \Pi_0 > 0$$

$$\delta_1 = \delta_0 = 0 \text{ if } \Pi_1, \Pi_0, < 0$$

The second-order condition of (3.14) is

$$Pf_i''h_i^2 < 0 \qquad (3.16)$$

Total differentiation of (3.15) yields

$$Pf''h_i^2 da + f'h_i dP + [Pf''h_i h_1' a + Pf'h_1']d\alpha - dW = 0$$

Let
$\Psi(e) = f'(e)e/f(e)$ be the *output elasticity of effective water*;
$\phi(e) = -f''(e)e/f'(e)$ be the *elasticity of marginal productivity* of e, EMP; and
$\eta(\alpha) = \frac{h_i'(\alpha)\alpha}{h_i(\alpha)}$ the *elasticity of input use efficiency*

From (3.16)

$$
\begin{aligned}
\frac{da_i}{dP} = \frac{-f'h_i}{Pf''h_i^2} &= -\frac{a_i f'}{Pf''e} = \frac{a_i}{P\phi} > 0 \qquad (3.17)\\
\frac{da_i}{dW} = \frac{1}{Pf''h_i^2} &= \frac{f'}{Pf'h_i f''h_i} = -\frac{a_i}{W\phi} > 0 \qquad (3.18)\\
\frac{da_i}{d\alpha} = -\frac{[Pf''h_i h_i' a + Pf'h_i']}{Pf''h_i^2} &= -\frac{a_i h_i'}{h_i} - \frac{f'h_i'}{f''h_i^2} \qquad (3.19)\\
= -\frac{a_i \eta_i(\alpha)}{\alpha}\left[1 - \frac{1}{\phi_i(a)}\right] &\gtreqless 0 \quad \text{if } \phi_i(a) \gtreqless 1 \qquad (3.20)\\
\frac{dy_i}{dP} = f'h_i \frac{da_i}{dP} &= \frac{f'h_i a_i}{P\phi} = \frac{y_i f'e}{f \cdot P\phi} = \frac{y_i \Psi}{P\phi} > 0 \qquad (3.21)
\end{aligned}
$$

Similarly,

$$\frac{dy_i}{dW} = -\frac{y_i\Psi}{W\phi} \tag{3.22}$$

$$\begin{aligned}\frac{dy}{d\alpha} = -f'h_i(\alpha)\frac{da_i}{d\alpha} + f'a_ih_i'(\alpha) &= f'h_i(\alpha)\left[\frac{da_i}{d\alpha} + \frac{a_i\eta_i}{\alpha}\right] \\ = \frac{f'h_i\alpha\eta}{\phi\alpha} &= \frac{y_i\Psi\eta}{\phi\alpha} > 0 \end{aligned} \tag{3.23}$$

3.4.1 Comparison of Input Use and Output Under the Two Technologies

Technology switch from $i = 0$ to $i = 1$ is equivalent to land quality improvement from α to $h_1(\alpha)$. Therefore,

$$\begin{aligned} a_1 &\approx a_0 + \frac{\partial a_0}{\partial \alpha}(h_1(\alpha) - \alpha) \\ \Delta a \equiv a_1 - \frac{a_0}{\alpha} &\approx \frac{\partial a_0}{\partial \alpha}(h_1(\alpha) - \alpha) \\ &= -a_0\left[1 - \frac{1}{\phi}\right](h_1(\alpha) - \alpha) \\ &\Rightarrow \Delta a \gtreqless 0 \text{ where } \phi \gtreqless 1 \end{aligned} \tag{3.24}$$

Similarly,

$$\Delta y \equiv y_1 - y_0 \approx \frac{\partial y_0}{\partial \alpha}(h_1(\alpha) - \alpha) = \frac{y_0\Psi}{\alpha\phi}(h_1(\alpha) - \alpha) > 0 \tag{3.25}$$

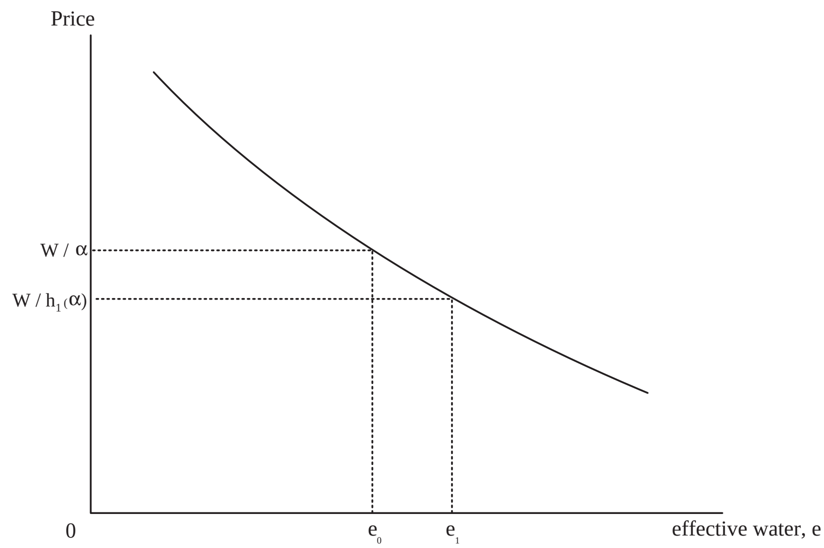
Thus, adoption of modern technology always increases yield and saves water only when $\phi > 1$.

$\Delta a_1 < 0$ when yield effect is small $\Delta a_2 > 0$ when yield effect is big.

What do we know about the EMP, ϕ? Assuming that $f(\cdot)$ has three regions of production, its marginal and average productivity (MP and AP) are depicted in Figure 3.

The economic region ($f'' < 0$, $MP < AP$) is between C and D in Figure 3, and MP is negative to the right of G. The MP reaches its peak at B, where $f''(e_b) = 0$ and, hence, the EMP $= \phi(e_G) = -f''(e_G)(e_G)/f'(e_G) = \infty$. Thus, the EMP increases from 0 to ∞ between B and G, the EMP increases, and assuming continuity there is a point D with $\phi(e) \gtreqless 1$ if $e \gtreqless e_D$, $e < e_G$.

Figure 3.8: Value Marginal Product of Effective Water



$$e_0 < e_1 \Rightarrow y_0 < y_1$$

Figure 3.9: Technology adoption and input use

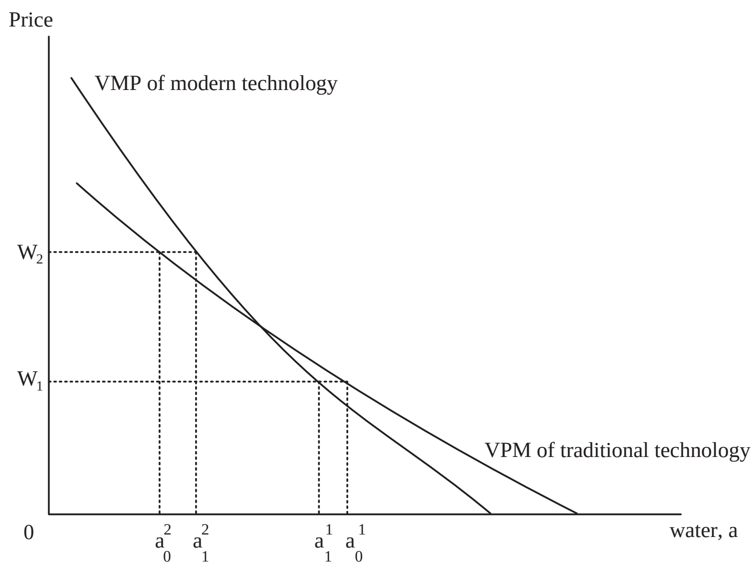
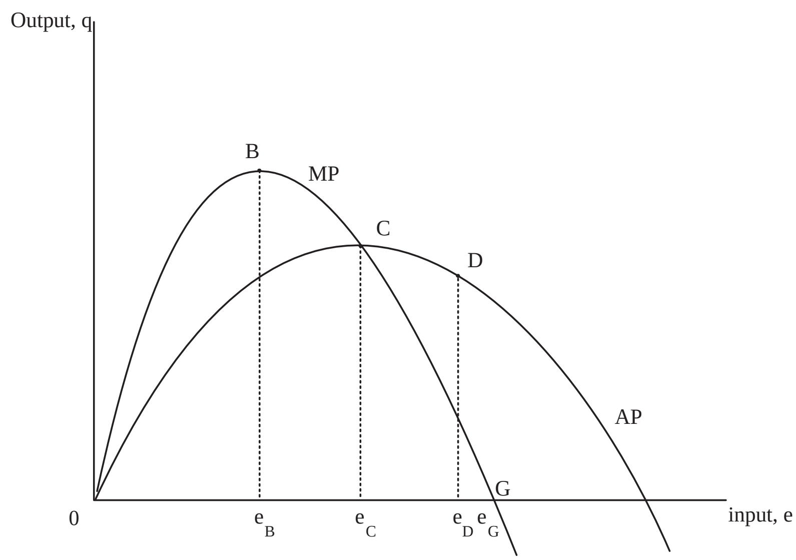


Figure 3.10: Average and Marginal Product



3.4.2 Implication for Cobb-Douglas

The Cobb-Douglas production function is quite popular because of its ease of use. We will argue here that it is not very realistic to apply it to microlevel studies. Suppose $y = Ae^{\psi_0}$, with $(1-\psi_0) < 1$. In this case, the two elasticities of interest are constant, $\psi(e) = \psi_0$, the EMP, $\phi(e) = 1-\psi_0 < 1$. However, Cobb-Douglas does not allow a region with negative marginal productivity. Furthermore, consider the case with $i = 0$, the F.O.C. is $\psi_0 PAE^{\psi_0 - 1} = W/\alpha$. Hence,

$$e = \left(\frac{\Psi_0 W}{PA\alpha}\right)^{1-\Psi_0}, \quad a = \left(\frac{\Psi_0 W}{PA}\right)^{1-\Psi_0}\frac{1}{\alpha}, \quad aW = (1-\alpha)Py$$

The share of (effective) water cost per acre is rarely constant. It is likely to increase with W. Furthermore, water prices vary radically in California and water use per acre does not respond as drastically as predicted by Cobb-Douglas. Suppose $\psi_0 = .5$. If $W_A = 10W_B$, we are unlikely to observe that $a_B = \sqrt{10W_B}$. Therefore, production functions like the quadratic, may be more realistic for depicting microlevel behavior.

3.4.3 Quality and Technology Choices

We will argue that there are segments of lower quality lands that adopt the new technology. To show that note first that

$$\frac{d\pi_i}{d\alpha} = \frac{\partial[Pf(h_i(\alpha)a_i) - wa_i - k_i]}{\partial\alpha} =$$

$$P(f'h_i - W)\frac{\partial a_i^*}{\partial\alpha} + Pf'h_i'a_1^* =$$

$$Pf'h_i a_i \frac{\eta_i}{\alpha} = \frac{Wa_i\eta_i}{\alpha} > 0$$

Thus, profits increase with land quality

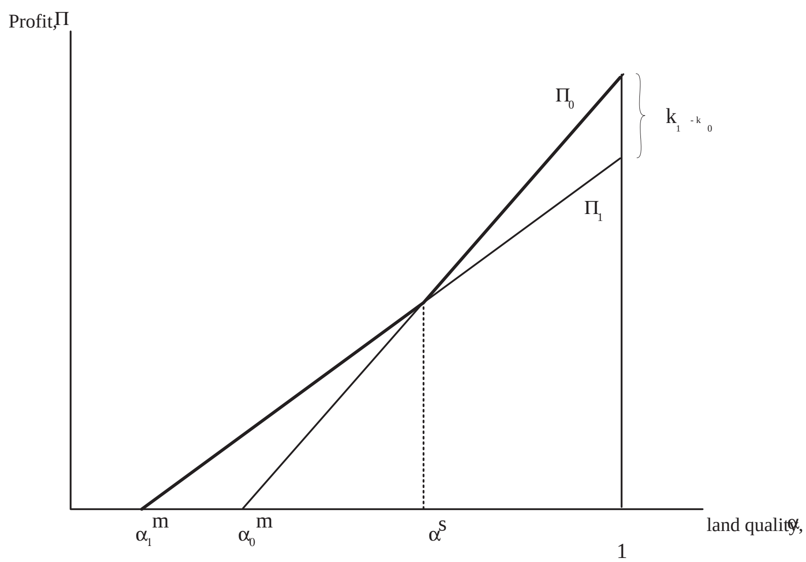
$$\frac{d\Delta\pi}{d\alpha} = \frac{d(\pi_1 - \pi_0)}{d\alpha} = W\left[\frac{a_1^*\eta_1 - a_0}{\alpha}\right](\eta_0(\alpha) - 1)$$

For $\alpha = 1$, $a_1^*(1) = a_0^*(1)$, $\pi_o(1) = \pi_1(1) + k_1 - k_0$;

$$\frac{d\Delta\pi}{d\alpha}(1) = Wa_1^*(\eta_1(1) - 1) < 0$$

The modern technology is less profitable for $\alpha = 1$, but the profitability gaps decline as α becomes smaller, and at $\alpha = \alpha_1^s$ their profits per acre are equal.

Figure 3.11: Switching Land Quality



There may be many feasible patterns of technology adoption as functions of quality, but the highest quality land never adopts. The pattern we analyze is depicted in Figure 3.11.

$$\delta_0 = 1 \qquad \text{for } \alpha > \alpha^s, \qquad \text{where } \pi_0 > \pi_1 > 0$$

$$\delta_1 = 1 \qquad \text{for } \alpha_1^m < \alpha < \alpha^s, \qquad \text{where } \pi_q > \pi_0 > 0$$

The quality α^s is switching quality land. At this quality,

$$\pi_1(\alpha^S) = \pi_0(\alpha^S)$$

$$\Rightarrow Pf[h_1(\alpha^S)a_1^*(\alpha^S)] - Wa_1^*(\alpha^S) - k_1 + k_0 = Pf[\alpha^S a_0^*(\alpha^S)] - Wa_0^*(\alpha^S) \tag{3.26}$$

$$\pi_i(\alpha_i^m) = 0$$

$$Pf[h_i(\alpha_i^m)a_i^*(\alpha_i^m)) - Wa_i^*(\alpha_i^m) - k_i = 0 \tag{3.27}$$

The introduction of the new technology will lead to adoption at the extensive margin when $\alpha_1^m < \alpha < \alpha_0^m$ and switching from traditional to modern technology for $\alpha_0^m \leq \alpha \leq \alpha^s$

The rental rate function of the industry is

$$r = \begin{cases} \pi_1(\alpha) & \alpha_1^m < \alpha < \alpha^s \\ \pi_0(\alpha) & \alpha_1^S < \alpha < 1 \end{cases}$$

The switching quality α^S and marginal qualities are functions of prices. Total differentiation of (11) yields

$$[Pf_1'h_1 - W]da_1^* + Pf_1'a_1^*d\alpha^S + f_1dP - a_1^*dW - dk_1$$
$$= [Pf_0h_1 - W]da_0^* + Pf_1'a_0^*d\alpha^S + f_0dP - a_0^*dW - dk_0$$
$$\Rightarrow \frac{W[a_1^*\eta_1 - a_0^*]}{\alpha^S}d\alpha^S + (y_1 - y_0)dP - (a_1^* - a_0^*)dW - dk_1 - dk_0 = 0$$
$$\Rightarrow \left\{ \frac{d\alpha^S}{dP} = -\frac{(y_1-y_0)\alpha^S}{W[a_1^*\eta_1 - a_0^*]} > 0 \frac{d\alpha^S}{dW} = -\frac{(a_1^*-a_0^*)\alpha^S}{W[a_1^*\eta_1-a_0^*]} > 0 \quad \text{if EMP } > 1 \right.$$

Total differentiation of (12) yields

$$[Pf_1'h_1 - W]da_1^* + Pf_1'h_1'a_1^*d\alpha_1^m + y_1dP - a_1^*dW - dk_1 - 0$$
$$\Rightarrow \begin{cases} \frac{d\alpha_1^m}{dP} = -\frac{y_1}{Pf_1'h_1'a_1^*} < 0 \\ \frac{d\alpha_1^m}{dW} = -\frac{a_1^*}{Pf_1'h_1'a_1^*} > 0 \end{cases}$$

A higher input price will trigger the existence of the rent-efficient firms. When $\phi > 1$ it will trigger adoption of modern technologies by firms around α^s. Higher input prices will trigger technology switching toward the modern one at $\alpha = \alpha^s$ and entry of producers with marginal quality which will adopt the modern technology.

3.4.4 Aggregation

Suppose the distribution of land quality is

$$\int_0^\infty g(\alpha)d\alpha = A$$

$g(\alpha)\Delta\alpha$ denotes the cost of land of quality in $\left(\alpha - \frac{\Delta\alpha}{2}, \alpha + \frac{\Delta\alpha}{2}\right)$. Aggregate supply is

$$Y^S = \int_{\alpha_1^m}^{\alpha^s} y_1 g(\alpha)d\alpha + \int_{\alpha^s}^{1} y_0 g(\alpha)d\alpha$$

The marginal change in supply with respect to price is given by

$$Y_P^S = \int_{\alpha_1^m}^{\alpha^S} \frac{\partial y_1}{\partial P} g(\alpha)d\alpha + \int_{\alpha^S}^{1} \frac{\partial y_0}{\partial P} g(\alpha)d\alpha$$

$$+g(\alpha^S)\left[y_1(\alpha^S) - y_0(\alpha^S)\right]\frac{\partial \alpha^S}{\partial P} - g(\alpha_1^m)\frac{\partial \alpha_1^m}{\partial P} > 0$$

and

$$Y_W^S = \int_{\alpha_1^m}^{\alpha^S} \frac{\partial y_1}{\partial W} g(\alpha)d\alpha + \int_{\alpha^S}^{1} \frac{\partial y_0}{\partial W} g(\alpha)d\alpha$$

$$+\frac{\partial \alpha^S}{\partial W}\left[y_1(\alpha^S) - y_0(\alpha^S)g(\alpha^S)\right] - y(\alpha_1^m)g(\alpha_1^m)\frac{\partial \alpha_1^m}{\partial W} \gtreqless 0$$

The first three items are negative, but the switching effect of higher W on supply may be positive.

3.4.5 Impact of Pollution Tax

Suppose $Z = [1 - h_i(\alpha)]\alpha_i$. If pollution tax is V, the maximization problem for technology i becomes

$$\max_{a_i} Pf(h_i(\alpha)a_i) - Wa_i - [1 - h_i(\alpha)]a_i V - k_i \tag{3.28}$$

with FOC

$$Pf'h_i = W + V(1 - h_i(\alpha))$$
$$Pf' = \frac{W}{h_i} + V\left[\frac{1 - h_i(\alpha)}{h_i(\alpha)}\right]$$

3.5 The economics of pesticides

Pesticides are chemicals used in controlling agricultural pests. There are three major classes of pesticides: *insecticides*, *fungicides*, and *herbicides* The use of pesticides in agriculture presents some interesting aspects to be considered:

- they need to be chemically updated over time as pests build resistance
- there are adverse human and animal health effects associated with pesticide use, as well. The adverse human health effects of different types of pesticides depend on the similarity between human or animal biology and the biology of the target pest; insecticides, for example, are generally worse for human health than fungicides.

3.5.1 A Brief History of Pesticide use

Herbicides: —
From 1965 to 1980, growth in the relative price of labor increased the use of herbicide as a factor of production. This occurred because **herbicide use is a substitute for labor**

During the 1980s, lower agricultural commodity prices and reduced crop acreage led to an overall reduction in herbicide use.

Insecticides:—
During the 1970's, the creation of the EPA and an increase in energy prices led to a reduction in insecticide use.

Fungicides:—
Fungicide use has remained relatively stable over the past 30 years, although recent legislation banning the use of carcinogenic chemicals in the Delaney Clause will soon outlaw many fungicides (and several popular insecticides and herbicides).

3.5.2 Pesticides in a Damage Control framework

Pesticides are damage control agents. In formal terms, production can be modeled as

$$Y = g(Z)[l - D(n)]$$

where:
Y is total output,
$g(Z)$ is potential output, that is the maximum output that can be produced in assence of damage due to the presence of a pest,
Z are all inputs not related to pest control, and

$D(n)$ is the *damage function*, expressed as percent of output lost to pest damage, and assumed to be function of the pest population, n.

The effect of pesticide use is that of controlling the pest population, according to:

$$n = h(n_0, X, A)$$

where
n_0 is the initial level of pest population, before pesticide application,
X is the level of applied pesticide,
A is some alternative pest control method, such as *Integrated Pest Management* (IPM);
and where:

$$h_X < 0, h_A < 0.$$

The Economic Threshold
Obviously, there are costs associated with pesticide application. If the total damage from pests is less than the social cost associated with a single application of a pesticide to a field (including *Marginal External Cost* MEC), then the welfare–maximizing level of pesticide use is zero. Note that this implies toleration of some pests in the field as well as toleration of the associated pest damage, such as less visibly appealing fruits and vegetables.

When the level of pest damage rises above the social cost of one pesticide application, then it is welfare–maximizing to apply the pesticide. Profit maximization in the private market will determine the **economic threshold**, $\bar{n}_0$ as the pest population level at which it becomes profit–maximizing to apply the pesticide. The economic threshold is determined by setting total pest damage equal to the total cost of a single pesticide application and solving for $\bar{n}_0$:

$$Pg(Z)D(\bar{n}_0) = w$$

where P is output price, and w is the cost of applying pesticide.

Given function forms for $g(\cdot)$ and $D(\cdot)$, one could solve the above equation for $\bar{n}_0$.

In the models that follow, we will assume that the pest population is above the economic threshold.

3.5.3 Model of pesticide use with known pest population and pest control alternatives

The optimal level of pesticide use is determined by solving:

$$\max_{X,A} \{Pg(Z)\,[1 - D(h(n_0, X, A))] - VA - wX\} \tag{3.29}$$

where the symbols are as defined above, and V is the unit cost of alternative control methods.

The FOC's are:

$$\frac{dL}{dX} = -Pg(Z)D_n h_X w = 0 \tag{3.30}$$

$$\frac{dL}{dA} = -Pg(Z)D_n h_A V = 0 \tag{3.31}$$

Thus, one would maximize profits by applying pesticides until the value of marginal product (marginal benefit) of pesticide application equals the marginal cost of pesticide application. The model predicts that the use of pesticides will increase following:

- an increase in initial pest population (n_0),
- an increase in the output price (P),
- an increase in potential output ($g(Z)$),
- an increase in the price of alternative controls, or
- a decrease in the price of pesticides.

Analogous results hold for the alternative pest control method.

A model with a secondary pest

Sometime, different pests may be present at a given time. Pests are classified as primary or secondary according to the severity of damage. The primary pest is usually the one that is targeted by the pesticide use.

In this model we will consider the presence of a secondary pest. For simplicity we will not consider any direct alternative controls besides the use of pesticides, but we will assume a specific biological relationship between the two pest populations: in particular, let us assume that both pests cause damage, and both populations levels are known, but pest 1 is also a *predator* of pest 2.

The damage function, $D(\cdot$ will be function of both pests' population levels:

$$D(n_1, n_2)$$

Pest 1 population is directly affected by the pesticide, so that

$$n_1 = n(n_0, X)$$

while pest 2 population level is a decreasing function of pest 1 population level:

$$n_2 = \Psi(n_1), \quad \text{with } \Psi_{n_1} < 0.$$

In other words, using pesticide to control pest 1 may lead to an increase in the population of pest 2, since pest 1 is a predator of pest 2.

The problem of determining the optimal level of pesticide to use become, thus,

$$\max_X \{Pg(Z)\,[1 - D(h(n_0, X, \Psi(n_1)))] - wX\} \tag{3.32}$$

with first order condition:

$$-Pg(z)[D_{n_1}h_X + D_{n_1}h_{n_2}\Psi_{n_1}] - w = 0$$

Both the direct impact on n_1 and indirect impact on n_2 on crop damage have to be considered on determining the optimal level of X. Lack of recognition of the biological predator–prey relationships may lead to economically inefficient over–application of pesticides, since the beneficial effect of the predator pest on reducing pest 1 is ignored.

Pesticide resistance

Through the biological process of natural selection, pests exposed to pesticides gradually develop genetic resistance to pesticides. Higher levels of pesticide application may accelerate buildup of resistance due to genetic selection of resistant genes. Short run pesticide control problems in a given season will be inefficient if long term resistance effects are not considered. Therefore, the calculation of optimal dosage of pesticide should take into account:

- resistance buildup (pesticide effectiveness is an "exhaustible resource" and should be modeled as such), and
- use of alternative chemicals or alternative pest control methods (such as the use of alternative cropping methods, crop rotation, natural diseases and predator–prey relationships) should be considered in order to reduce resistance buildup.

Unknown pest population and pest population monitoring

When pest populations are unknown, as is usually the case, one can distinguish between *preventive* and *reactive* pesticide application. Contrary to what is considered common wisdom in human medicine, for agricultural pest control, preventing may be worse (less efficient) than reacting.

With **preventive application**, pesticides are applied without an attempt to determine potential pest populations. Instead, based on experience or historical data, the farmer makes educated guesses about the probabilities of various pest population levels occurring. The farmer then chooses a level of pesticide use to maximize expected profit. For example, and to keep the analysis simple, suppose there are two possible initial pest population levels, low (n_1) and high (n_2).

Assuming preventive application, the problem of deciding how much pesticide to apply is, then,

$$\max_X E(\pi) = p\{Pg(Z)[1-D(h(n_1,X))]-wX\}+(1-p)\{Pg(Z)[1-D(h(n_2,X))]-wX\}$$

where p is the probability of initial pest population n_1 occurring, $(1-p)$ is the probability of initial pest population n_2 occurring. The FOC is:

$$p\{-Pg(Z)D_h h_X(n_1) - w\} + (1-p)\{-Pg(Z)D_h h_X(n_2) \ - w\} = 0.$$

Given specific g, D, and h functions, one could solve this FOC for X. Plugging the optimal value, X, back into the objective function would then give the level of expected profit associated with preventive pesticide application. Note that because the pest population is uncertain, X will be the same regardless of which pest population level, n_1 or n_2, actually occurs. This is inefficient because we would like to use less pesticide if n_1 occurs and more if n_2 occurs.

To decide between preventive and reactive application methods, we need to compare the level of expected profits under preventive pesticide application with the level of expected profits under the following model of reactive application.

With **reactive application**, a fixed monitoring cost is paid to determine the pest population level, and then the optimal X is chosen for the specific pest level. This enables more precise pesticide use. The problem is then:

$$\max_{X_1,X_2} E(\pi) = p\{Pg(Z)[1-D(h(n_1,X_1))]-wX_1\} + \\ (1-p)\{Pg(Z)[1-D(h(n_2,X_2))]-wX_2\} - m$$

where m is the fixed cost of monitoring

the FOC's are:

$$\frac{\partial E(\pi)}{\partial X_1} = p\{-Pg(Z)D_h h_{X_1}(n_1) - w\} = 0$$

$$\frac{\partial E(\pi)}{\partial X_2} = (1-p)\{-Pg(z)D_h h_{X_2}(n_2) - w\} = 0$$

Given specific g,D and h functions, one could solve the FOC's for the optimal X_1 and X_2. Plugging X_1 and X_2 back into the objective function gives the level of expected profits under reactive pesticide application. Note that the resulting equation for expected profits will contain monitoring costs, m. With reactive application, there is a tradeoff between monitoring costs, m, and the savings in pesticide costs made possible by monitoring.

Two points need to underlined:

- If the difference between X_1 and X_2 is large, and m is relatively small, then reactive application will give a higher level of expected profits than would preventive application. Monitoring and reactive application are key components of modern Integrated Pest Management (IPM) programs.
- Even if the difference between X_1 and X_2 is large, farmers may still prefer preventive application whenever the the price of pesticides is very low or monitoring cost is very high, so to ensure higher expected profits.

In order to get combine the profit-maximizing decision with social-welfare maximization, an appropriate tax would be imposed on pesticides so that effective prices would reflect MEC.

Regional cooperation in pest control activities

Pests do not recognize property rights. When it comes to resistance, either no chemical treatment or chemical treatment might lead to externality problems. Pest control districts are introduced to overcome these problems (e.g., mosquito control districts). The activities of such districts encompass joint effort in monitoring activities, coordinate crop management and rotation, and coordinate pesticide spraying.

Health-risk and environmental effects of pesticide use

Health risk is the probability that an individual selected randomly from a population contracts adverse health effects (mortality or morbidity) from a substance. The health risk-generating process contains three stages: *contamination*, *exposure* and *dose response*.

Contamination is the presence of toxic pesticide or its derivates on the agricultural product. It is direct result of pesticide application: the chemicals are spread through the air and water and become absorbed by the product.

Exposure is the contact of toxic substances with human or animal organisms. It may result from eating (consumers), breathing or touching (agricultural or chemical workers), drinking water that is contaminated.

The dose-response relationship translates exposure to probability of contracting certain diseases. We usually distinguish between acute and chronic risks.

- Acute risks are the immediate risks of poisoning.
- Chronic risks are risk that may depend on accumulated exposure and which may take time to manifest themselves, like for example the higher incidence of certain type of cancer in populations that are exposed for a long time.

Risk assessment models

The processes that determine contamination, exposure and the dose/response relationship are often characterized by heterogeneity, uncertainty and random phenomena (e.g., weather). Thus, contamination, exposure and the dose/response relationship need to be analyzed with models that accounts for the inherent uncertainty. *Risk assessment models* estimate health risks associated with pesticide application by making use of estimated probabilities.

Let $r =$ the represent individual health Risk. It can be expressed as:

$$r = f_3(B_3)f_2(B_2)f_1(B_1, X)$$

where:
X is the level of pollution on site (i.e., the level of pesticide use)
B_1 is the damage control activity at the site (i.e. protective clothing, re-entry rules, etc.)
B_2 is the averting behavior by potentially exposed individuals (i.e., washing fruits and vegetables)
B_3 is the dosage of pollution (i.e., the type of pesticide residual consumed).

The health risk of an average individual is thus the product of three functions:

- $f_1(B_1, X)$ is the contamination function. The function relates contamination of an environmental medium to activities of an economic agent (i.e., relates pesticide residues on apples to pesticides applied by the grower)
- $f_2(B_2)$ is the human exposure coefficient, which depends on an individual's actions to control exposure (i.e., relates ingested pesticide residues to the level of rinsing and degree of food processing an individual engages in)
- $f_3(B_3)$ is the dose-response function which relates health risk to the level of exposure of a given substance (i.e., relates the proclivity of contracting cancer to the ingestion of particular levels of a certain pesticide), based on available medical treatment methods, B_3. Dose-Response functions are usually estimated in epidemiological and toxicological studies of human biology

The product $f_2(B_2)f_1(B_1, X)$ is the overall exposure level of an individual to a toxic material (e.g., the amount of pesticide present on an apple times the percentage *not* removed by rinsing the apple). The degree of overall exposure can be effected by improved technology and by a greater dissemination of information.

Estimating these functions involves much uncertainty.

1. Scientific knowledge of dose-response relationships of pesticides is generally incomplete, especially for pesticides ingested in small doses over long periods of time.

2. Contamination function depends partly on assimilation of pollution by natural systems, which can differ regionally (i.e., wind distributed residues).

3. Exposure coefficient depends on education of population (i.e., are consumers aware of pesticide residue averting techniques, such as washing?)

Uncertainty is included in the economic model by using a **safety-rule approach**.

The **policy goal** of pesticide use regulation should be that of maximize welfare subject to the constraint that the probability of health risk remains below a certain threshold level, R, an acceptable percent of the time, α.

α, the safety level, measures the degree of social risk aversion. It might represent the degree of confidence we have in our target risk.

For any target level of risk and any degree of significance, the model can be solved for the optimal levels of pesticide use, damage control activities, averting behavior by consumers, and preventative medical treatments.

General implications of this way of approaching the modeling include:

1. the optimal solution involves some combination of pollution control, exposure avoidance, and medical treatment;

2. the cost of reaching the target risk level increases with the safety level *alpha*;

3. the shadow price of meeting the risk target depends oil the degree of significance we have that the target is being met. The higher α, or the greater the uncertainty we have in our estimate of risk, the higher the shadow value of meeting the constraint.

Example:–

Say there is no uncertainty regarding the health effects of pesticide use, that is, toxicologists know with certainty a point estimate of the dose-response function.

Let:

X = the level of pesticides used on a field
A = the level of alternate pest control activities
P = the value of farm output(i.e., the price of a basket of produce)
Y = the level of farm output

W = the price of pesticide
V = the price of alternative controls ($V > W$)
R = the level of health risk in society
B_1 = damage control activities by the farm (i.e., pesticide reentry rules)
B_2 = aversion activities by members of the population (i.e., washing residues off)
B_3 = available level of medical treatment
$Y = f(X, A)$ is the farm production function (i.e., a pesticide damage function)
$r = f_3(B_3)f_2(B_2)f_1(B_1, X)$ is the Health Risk of pesticide use
$C(R)$ is the cost to society of health risk R.

Then, the objective of the society is:

$$\max_{X,A,r,B_1,B_2,B_3} Pf(X, A) - C(R) - C(B_1, B_2, B_3) - WX - VA$$

subject to:

$$R = f_3(B_3)f_2(B_2)f_1(B_1, X)$$

which can be written in Lagrangian form as:

$$\max_{X,A,R,B_1,B_2,B_3} L = Pf(X, A) - C(R) - C(B_1, B_2, B_3) - WX - VA + \lambda[R - f_3(B_3)f_2(B_2)f_1(B_1, X)]$$

with the FOCs:

$$\frac{dL}{dA} = Pf_A - V = 0 \tag{3.33}$$

the MRP of the alternative control equal the MC of the alternative control

$$\frac{dL}{dr} = -C'(R) + \lambda = 0 \tag{3.34}$$

the MSC of health Risk = shadow value of risk (The MC of risk in terms of social damages is equal to the shadow price of reducing societal risk.)

$$\frac{dL}{dX} = Pf_X - W - \lambda\left[f_3 f_2 \frac{df_1}{dX}\right] = 0 \tag{3.35}$$

$$\frac{dL}{dB_1} = -C_{B_1} - \lambda\left[f_3 f_2 \frac{df_1}{dB_1}\right] = 0 \tag{3.36}$$

$$\frac{dL}{dB_2} = -C_{B_2} - \lambda\left[f_3 f_1 \frac{df_2}{dB_2}\right] = 0 \tag{3.37}$$

$$\frac{dL}{dB_3} = -C_{B_3} - \lambda \left[f_2 f_1 \frac{df_3}{dB_3} \right] = 0 \tag{3.38}$$

We can re-write equations (3.35)–(3.38) using equation (3.34) as:

$$Pf_X = W + C'(R) \left[f_3 f_2 \frac{df_1}{dX} \right]$$

The MPR of pesticides to the farm is equal to the MPC of pesticides plus the (MC of risk) × (marginal contribution of pesticides to Health Risk)

$$C_{B_1} = -C'(R) \left[f_3 f_2 \frac{df_1}{dB_1} \right]$$

The MC of damage control equals (avoided MC of risk) × (marginal improvement in risk from engaging in damage control activities)

$$C_{B_2} = -C'(R) \left[f_3 f_1 \frac{df_2}{dB_2} \right]$$

The MC of averting behavior equals (avoided MC of risk) × (marginal improvement in risk from engaging in averting behavior)

$$C_{B_3} = -C'(R) \left[f_2 f_1 \frac{df_3}{dB_3} \right]$$

The MC of medical treatment equals (avoided MC of risk) × (marginal improvement in risk from engaging in medical treatment).

The optimal solution involves equating all six FOCs. Equations (3.34)–(3.38) can be expressed as:

$$\lambda = C'(R) = \frac{Pf_X}{\left(f_3 f_2 \frac{df_1}{dX} \right)} = \frac{-C_{B_1}}{\left(f_3 f_2 \frac{df_1}{dB_1} \right)} = \frac{-C_{B_2}}{\left(f_3 f_1 \frac{df_2}{dB_2} \right)} = \frac{-C_{B_3}}{\left(f_2 f_1 \frac{df_3}{dB_3} \right)}$$

which says that the optimal solution involves equating the shadow price of risk with a series of ratios.

The denominator of each expression transforms marginal benefits and marginal costs of health-related activities into changes in health risk.

When parameters are known, the model can be solved for the optimal levels.

Some general implications:

1. If there is no tax on pesticide use, $t^* = \lambda$ and no subsidy on farm-level damage control, $s^* = \lambda$, then the farm will not recognize the effect of pesticide use on societal health, and operate as if $\lambda = 0$. As a cosequence:

- an inefficiently high level of pesticides will be used,
- an inefficiently low level of damage control will be applied.

2. The optimal solution may involve a large level of pesticide use, little damage control, little medical treatment, and a high degree of averting behavior.

    - Rinsing and washing produce may be the least expensive method of reducing health risk in society.

Example 2 (a model with uncertainty): —
Let r be the probability of an individual contracting a disease. $r = c \cdot e \cdot d \cdot x$ where:
c = contamination probability
e = exposure probability
d = dose/response probability
x = amount of pesticide applied.

Let

$$c = \begin{cases} 1 \text{ with probability } .5 \\ 2 \text{ with probability } .5 \end{cases}$$

$$e = \begin{cases} 1 \text{ with probability } .5 \\ 3 \text{ with probability } .5 \end{cases}$$

$$d = \begin{cases} 10^{-5} \text{ with probability } .5 \\ 10^{-6} \text{ with probability } .5 \end{cases}$$

For $x = 1$,

$$r = \begin{cases} 10^{-6} \text{ with probability } 1/8 \\ 2 \cdot 10^{-6} \text{ with probability } 1/8 \\ 3 \cdot 10^{-6} \text{ with probability } 1/8 \\ 6 \cdot 10^{-6} \text{ with probability } 1/8 \\ 1 \cdot 10^{-5} \text{ with probability } 1/8 \\ 2 \cdot 10^{-5} \text{ with probability } 1/8 \\ 3 \cdot 10^{-5} \text{ with probability } 1/8 \\ 6 \cdot 10^{-5} \text{ with probability } 1/8 \end{cases}$$

(Note: 10^{-6} means "one person per million people" contracts the disease. 10^{-5} means "one person per hundred thousand people" contracts the disease.)

Then, expected risk is:

$$\frac{13.2}{8} \cdot 10^{-5} = 1.65 \cdot 10^{-5}$$

or one person in 165,000, on average, contracts the disease. Yet the variability of this estimate is substantial, which implies that α is large.

In many cases, the highest value (worst case estimator) of each probability is used when the risk generation processes are broken down to many sub-processes. This creates a "creeping safety" problem, in that the multiplication of many "worst case" estimates may lead to wildly unrealistic risk estimates. Of course, the variability and uncertainty associated with risk estimates can be reduced by expenditures on research and through information–sharing.

Pesticide Policy

Current pesticide policy separates pesticide economics from health considerations. New policy is triggered solely by health considerations - when a chemical is found to be carcinogenic or damaging to the environment, it is banned, or "canceled".

The impacts of pesticide cancellation depend on the available alternatives. If there are no alternatives, then cancellation causes losses in crop yields due to higher pest damages and to increases in costs, since alternative methods of control are generally more expensive. If chemicals have alternatives, the impact is mostly on cost

To estimate overall short–term impacts, the impacts on yield per acre and cost per acre are evaluated using one of the following methods:

Delphi method: –
The Delphi method uses "guesstimates by experts", which are easy to obtain but are arbitrary and sometimes baseless (named after the famous "Oracle at Delphi" in ancient Greece).

Experimental studies: –
These studies are based on data from agronomical experiments, but experimental plots often do not reflect real farming situations.

Econometric studies: –
Statistical methods based (ideally) on data gathered from real farming operations. However, these studies are often not feasible because of data limitations and the difficulty of isolating the specific effects of pesticides.

Cost Budgeting Method: –
Given:
y_{ij} = output per acre of crop i at region j with pesticide;
P_{jj} = price of crop i in region j;
A_{ij} = acreage of crop i, region j;
Δy_{ij} = yield reduction per acre because of cancellation;
Δc_{ij} = cost increase per acre because of cancellation;

under a partial crop budget, impacts on social welfare are estimated as:

$$\sum_{i=1}^{I}\sum_{j=1}^{J}(P_{ij}\Delta y_{ij} + \Delta c_{ij})A_{ij}$$

or, a pesticide cancellation causes losses in revenue from lower yields per acre and increased costs per acre, which is multiplied the total acreage in all regions and across all types of crop affected by the ban.

The cost budgeting approach has several limitations.

1. It ignores the effect of a change in output on output price; this tends to overestimate producer loss and underestimate consumer loss.
2. It ignores feedback effects from related markets.

In general, this method does not consider the interaction of supply and demand, and does not attempt to find the new market equilibrium after the application of a pesticide ban.

General Equilibrium Method: –
This method is based on analyzing the impact of a pesticide ban on *equilibrium* prices and output, taking into account the interaction of supply and demand and any feedback effects from related markets. In addition, this method offers a better assessment of equity effects by computing welfare changes for various groups.

As a result of a pesticide ban, marginal cost per acre increases, output declines, and output price increases. The magnitude of the change in output price depends on the elasticity of demand and any feedback effects from related markets, such as markets for substitute goods.

General equilibrium analysis recognizes heterogeneity in welfare effects: the welfare of non-pesticide-using farmers increases due to the increase in output price, but the welfare of pesticide-using farmers decreases if demand is elastic (but may increase if demand is inelastic); consumer welfare decreases due to price increases.

Example 1: –
Say there are two agricultural regions, 1 and 2, and the pesticide is banned only in region 2.

(figure 7.3 here)

Let:

S^1 = supply of region 1

S_0^2 = supply of region 2 before pesticide ban
S_1^2 = supply of region 2 after pesticide ban
$S^1 + S_0^2$ = total supply before regulation, regions 1 and 2
$S^1 + S_1^2$ = total supply after regulation, regions 1 and 2
Y_0 = total output before regulation
Y_1 = total output after regulation
y_0^1 = output of region 1 before regulation
y_1^1= output of region 1 after regulation
$y_0^2 = Y_0 - y_0^1$ = output before regulation, region 2
$y_1^2 = Y_1 - y_1^1$ = output after regulation, region 2
P_1abP_0 = consumer loss from cancellation
P_0cdP_1 = producer gain, region 1
$P_0hn - P_1em$ = welfare loss, region 2.

Results:–

When the ban affects only one of two or more regions, growers in the regions without the ban gain from the pesticide ban. Thus, some farmers may support pesticide bans if they feel the effect of the ban on other producers to a greater degree. For example, say farmers in region 2 grow pesticide-free produce.

Example 2: – Agricultural price support policies may lead to oversupply, so that pesticide regulation may increase welfare by reducing excess supply.

Pesticide Regulation and Agricultural Policy

(Figure 7.4 here)

S_1 = supply before cancellation
S_2 = supply after regulation
P_S = price support
P_c^1, P_c^2 = output price before and after
Q^1, Q^2 = output before and after

- welfare loss because of price support before regulation = area mcf
- welfare loss after ban = area $ubecna = ubna$ (extra cost) + ecn (under–supply).

If price support is very high, pesticide cancellation reduces government expenditure substantially. This results in an increase in taxpayer welfare of area $P_SmfP_c^1 - P_SbeP_c^2$.

Consumer welfare declines by $P_c^1feP_c^2$. Producer surplus declines by $P_Sma - P_Sbu$.

Alternative Policies

Pesticide effects include several related issues:

- Food safety
- Worker safety
- Ground water contamination
- Environmental damage

Pesticide bans and taxes address all these issues. However, a pesticide ban is an inefficient policy. Pesticide uses and impacts vary significantly across regions.

Economic mechanisms (taxes, partial bans) that discriminate across different types of uses may eliminate most of the pesticide damage but retain most pesticide benefits.

Although a pesticide ban may provide the incentive to develop new, less dangerous pest control methods, a pesticide tax may serve the same purpose and also allow a more gradual and efficient transition to the new technology. However, if a pesticide tax is used, policymakers should keep in mind that pesticide use patterns could shift significantly across geographic regions.

Other policy tools can affect different stages of the risk generation process:

1. Pollution controls affect contamination.
2. Protective clothing affect exposure,
3. Medical treatment affect dose/response.

Green markets Tolerance standard	address water safety concerns
Re–entry regulation Protective Clothing	address worker safety concerns
Liability Water disposal regulation	affects ground water contamination

3.6 Economic Analysis of Investments

In this section, we will study the putty-clay framework, which is a general framework to view production choices and their outcomes. Some of the basic points that will be emphasized include:

1. Some decision variables are discrete and others are continuous. Firms have to make simultaneous choices about the nature of technology —whether they will use drip or sprinkler irrigation or biological or chemical control. These choices are dichotomous choices and decision variables can assume values of 0 and 1. The types of choices are also dealt by technology adoption models. Other choices are with respect to the value of a given variable, for example, how much water should be applied. The variables in this case are determined from a continuous set.

2. There is heterogeneity in production. Producers operate under varying sets of circumstances that may result in different outcomes. The causes for variability may be differences in environmental conditions (land quality), human capital, and physical capital.

3. There are differences in long-run and short-run choices. Short-run choices entail much less flexibility than long-run choices. However, the outcome of short-run choices are much easier to predict.

4. Aggregation is a challenge in both short-run and long-run analysis. To obtain meaningful predictions of production choices and market outcomes under heterogeneity, meaningful aggregation procedures are essential.

Modeling production processes is essential for developing realistic policy analysis frameworks. In all of the modeling, one needs to investigate the implications of the approach for policy purposes.

key concepts

- Notions of present value
- Internal rate of retum
- Cost of capital
- Depreciation
- Obsolescence
- The Cambridge controversy
- Putty-clay models
- Ex ante vs. ex post production functions
- Micro vs. macro production functions

3.6.1 The Cambridge Controversy

The notion of production function is applied for different levels of aggregation. We can speak about the production function of an individual process (a production function of wheat in one field), production function of producers (a production function of wheat producers with several fields); production function of an industry producing the same product; production function of a sector that includes several industries; and an economy aggregate production function.

Aggregation may require a redefinition of input and output, especially for conceptual analysis, as one has to reduce the number of variables to a bare minimum to illustrate some concept without having an extremely complicated analysis. Even empirical analysis may require reducing the dimensionality and aggregation. One question is: "Under what condition would aggregation become meaningless and the results not useful?" The biggest controversy has been related to economy–wide production functions. One of the most important areas of research after Word War II were attempts to understand the process of economic growth. Kuznets established a national accounting data on output, capital, and aggregate labor. Many researchers, most notably Robert Solow, developed a neoclassical growth theory to analyze these data. The growth literature that Solow developed was very important during the 1960's and early 1970's, and it spawned another body of literature that attempted to explain the process of innovation. The first critical seminal article in the literature on innovation and growth was an article on learning by doing by Kenneth Arrow. The article was published in 1967. There has been a resurrection in the mid-1980's because of the works of Lucas and, in particular, Paul Romer, who introduced a new concept: endogenous growth. Romer's work has become an important element of microeconomics, but we will return to our discussion of production and, in particular, the Cambridge controversy that led to the putty-clay model which is our subject of interest.

The Cambridge controversy was a debate between economists in Cambridge, Massachusetts, headed by Robert Solow and Paul Samuelson, proponents of the neoclassical production function and neoclassical growth theory, and economists in Cambridge, England, headed by Joan Robinson, Piero Sraffa, and Luigi Pasinetti. Neoclassical growth theory assumes the existence of an aggregate production function where national output is produced by aggregate labor and aggregate capital stock. It also assumes that there is an endogenous process of technological change that increases input productivity overtime. Solow estimated an aggregate model of economic growth of the form:

$$Y_t = AL_t^{\alpha}K_t^{\beta}e^{\eta_t}$$

where:[0.1in] Y_t = aggregate output

L_t = aggregate labor
and
K_t = aggregate capital.

His model has had a good statistical fit and η, the time coefficient, was found to be quite substantial, indicating the importance of technological change. The model assumes that the economy has a stock of capital, K_t, which is augmented by investment I_t, but may decline due to depreciation. This approach suggests measuring capital by dollar units and assumes that capital goods are malleable.

The malleability of capital seems unreasonable to the Cambridge, England, economists. The English economists argued that there is much specialization of capital goods —a tractor cannot print books. Therefore, the notion of aggregate capital is meaningless, and policies based on assumption of smooth substitution between capital and labor may be wrong.

The Cambridge controversy was a debate about the formulation of production and microeconomics. Both groups have valid points. The basic idea of assessing aggregate productivity in the economy —taken by the Cambridge, Massachusetts, scholar— was viable. The effort they started led to important results, and growth theory is a very important area of research. However, the England group was correct in saying that higher capital expenditures do not necessarily mean more flexibility in production since capital goods are limited in theft uses. One of the important elements in Romer's new model is the explicit recognition of the role of specialized capital goods and the limited extent of malleability that capital goods have. The Cambridge controversy can be summarized succinctly as the argument about the magnitude of the elasticity of substitution between capital and labor. The neoclassical economists assume that the elasticity of substitution is quite high and the English economists assume that it is very low and relationships are converging to a fixed proportion production function The compromise was presented in "putty clay" models.

3.6.2 Putty–Clay Models

Putty–clay models were introduced by Johansen and Salter. They separated between micro and macro and *ex ante* and *ex post* production functions. A micro production function is the production function of an individual producer. A macro production function is a production function of an industry. One challenge is to develop aggregation procedures to move from micro to macro relationships. The ex ante choices are the putty stage, before the shape of the final machine is determined. Ex post choices are at the clay stage where the equipment is well formed and limits the flexibility of choices. The ex ante production function is used for long-run choices before investment takes place and where the capital level is flexible. An ex post production function reflects choices when capital outlay is completed and capital is less flexible. Putty–clay models assume that, at the microlevel, ex ante production functions are neoclassical and have positive elasticities between capital and other inputs, but ex post functions have fixed proportions and zero elasticity of substitution. Thus, the putty-clay models separate between

1. micro ex post production function,
2. micro ex ante production function,
3. aggregate ex post production function, and
4. aggregate ex ante production function.

The Salter Model

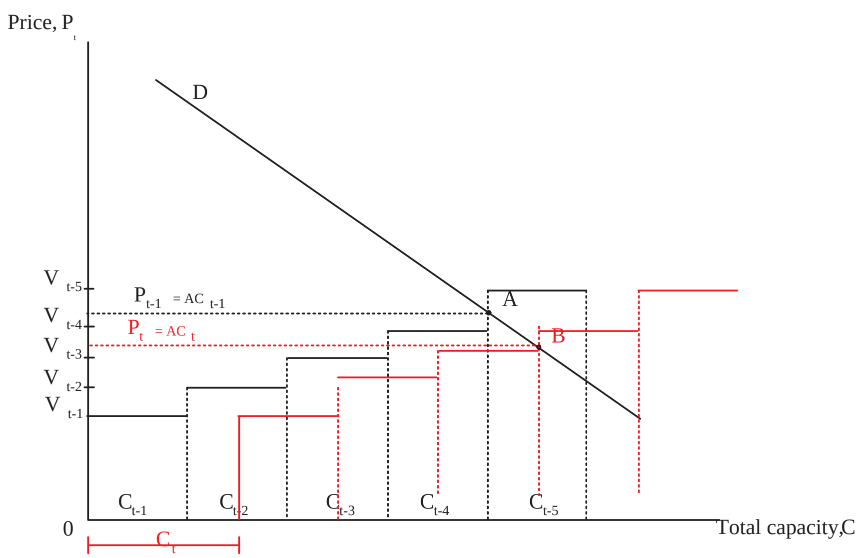
Salter introduced a graphical presentation that is very useful for explaining the putty–clay model. His model is dynamic, and he looks at determination of prices and investment at a given period. At the start of the period, the industry has a distribution of existing production units that were built in previous years. Every year entrepreneurs make ex ante decisions about new capital. In a later lecture, we will study in detail the determination of capital and labor costs of a new technology for a given moment in time; however, here we will make some general assumptions about the trend in capital costs and variable costs over time.

Every year entrepreneurs determine the cost of a new capital good, its production technology, and its production capacity. Salter assumes that technology has constant returns to scale, and the cost of variable inputs such us labor increases over time relative to capital[2]. Technological change and the relative price of labor results in new technology with lower variable costs but may have slightly higher annualized fixed costs.

Suppose we are at the beginning of period t. The industry inherits capital that was built in previous periods. Let C_{t-j} be the productive capacity of facilities that were built j years before t. We can refer to these machines as vintage $t-j$, and C_{t-j} is the productive capacity of vintage $t-j$. Productive capacity is the maximum output that these machines can produce if they are utilized. Let V_{t-j} be the variable input cost per unit of output of machines of vintage $t-j$. Thus, at the beginning of the period, the industry has output supply of an existing plant that is a step function such as the one depicted in Figure 3.12.

[2] The reason that capital becomes cheaper overtime is that technological change results in improved machinery. Labor cost may decline only when population growth is very drastic, but in most developed countries capital cost has declined relative to labor costs.

Figure 3.12: Total capacity and price in the Salter Model



If output price P is smaller than V_{t-j}, then the capacity of vintage $t-j$ would not be utilized. If output price is greater than V_{t-j} then the output capacity of vintage $t-j$ will be utilized. Part of the capacity will be utilized if the price is equal to $Vt-j$. Let AC_t be average cost per period (total cost divided by output) of a machine of vintage t. AC includes both variable cost and annualized fixed cost[3]. New productive capacity is introduced in period t as long as price is greater than average cost. Thus, in equilibrium, output price has to be equal to average cost of the current vintage.

Figure 3.12 will help us to understand the determination of the equilibrium of time t, assuming that the industry is facing negative sloped demand curve D. Suppose that the equilibrium at period $t-1$ was at point A. During period $t-1$, the industry produced q_{t-1} units of output using the productive capacity of vintage $t-1$, $t-2$, $t-3$, and $t-4$. The productive capacity of vintage $t-5$ was idle because the variable cost of this vintage, V_{t-5}, was higher than the price. And the price at period $t-1$ is equal to the average cost of vintage $t-1$ which is AC_{t-1}. Now suppose that the average cost of vintage t is AC_t, which is smaller than AC_{t-1}. Suppose that AC_t is between V_{t-3} and V_{t-4}. The new output price P_t will be equal to AC_t. The capacity of vintage C_{t-4} will not be utilized. The new capacity of vintage t, C_t, introduced at time t will be equal to C_{t-4} plus the increase in quantity demanded because of lower prices. This can be represented by a shift to the right of the supply step function. The new equilibrium is in point B in the figure. Thus, Salter's analysis suggests that old capital equipment continues to operate as long as revenues can cover its variable cost. However, at a certain time, this capital will grow out of production because its variable costs are too high. In his model, capital is not being destroyed, it is only becoming obsolete. At the same time, new capital is introduced reflecting the fact that there is a technological change that reduces average cost below the previous prices. When a firm makes ex ante investment decisions, it has to recognize that the economic life of capital is limited and it has to compute the cost of capital accordingly. Furthermore, when computing the cost of capital, it has to recognize that variable costs may increase over time and may reduce both the economic life of capital and its future earning capacity. The next section addresses the investment choice taking into account changes in prices over rime and final economic life.

The Salter model provides the framework for long-term decisions when investments in new capital is incorporated explicitly into the analysis. In the shorter run, choices are limited to existing equipment so if one wants to know the immediate effect of changes in policies, he may ignore the possibility of developing new equipment but considers the impact given existing vintages.

[3]The assumption of constant returns to scale allows us to present average cost regardless of size.

A Quantitative Analysis of investments

An investment involves an initial outlay of capital and results in a stream of benefits. To analyze an investment, one needs to know the stream of costs and benefits over time. Let x_0, x_1, and x_T denote the net benefit from a project at period $0, \ldots, T$. When $x_t < 0$, it represents a cost. For example, x_0 may be the initial investment, $x_t, \quad t = 1, \ldots, T$ are the returns. In some cases there may be several periods of negative outlay. The interest or discount rate, denoted by r, is a fee for the use of \$1.00 for one period. Whoever provides the money is paid for the use of this money for say, consumption for one period. The interest rate can provide a base for comparing an income stream at different time periods. A dollar earned next year is worth $\left(\frac{1}{1+r}\right)$ today and a dollar carried five years from now is worth $\left(\frac{1}{1+r}\right)^5$ dollars today. The net present value of an investment is $NPV = \sum_{t=0}^{T} \frac{x_t}{(1+r)^t}$. When time is continuous, one can use $NPV = \int_0^T e^{-rt} x_t dt$

A project is worthwhile if $NFV > 0$. One way to compare projects is to compare their *internal rate of return* (IRR). It is defined by z where z solves

$$\sum_{t=0}^{T} \frac{x_t}{(1+z)^t} = 0$$

or

$$\int_0^T e^{-zt} x_t dt = 0$$

Suppose a project requires an investment of K to be paid in T equal payments with an interest rate of r. The annual payment will be Y where:

$$\int_0^T e^{-rt} Y dt = K$$

or

$$Y = \frac{r}{1 - e^{-rT}} K$$

When a project is of infinite length, then it is simply $Y = rK$.

Capital Expenditure – When capital equipment worth K dollars is used in the production process, the capital expenditure during period t includes:

- Interest cost rK (or $rK/(1e^{-rT})$)

- Depreciation. the loss of value because of utilization.

Sometimes depreciation is assumed to be proportional to the value of stock and depreciation is δK, with δ, a fixed depreciation coefficient. Investment choices, purchases, and use of capital goods are the results of choices over time. The production function, however, is a simplified concept used for firm–level static analysis that can lead to much confusion in dynamic and aggregate settings.

Long-Run Micro Model – This involves the choice of capital. The information available to farmers is output prices P_t, and labor prices, W_t, for $t = 0, \ldots, \infty$. The production function is $f(K, L)$ where K is capital and L is labor. The objective function is:

$$\max_{K, L_t} \int_0^\infty e^{-rt} \left[P_t f(K, L_t) - W_t L_t\right] dt - K$$

with constraint:

$$L_t \geq 0$$

The ex post choice problem given K is:

$$\max_{L_t} P_t f(K, L_t) - W_t L_t, \qquad \text{for} t = 1, \ldots, T$$

and the ex post decision rules are to choose L_t such that:

$$P_t \frac{\partial f}{\partial L} - W_t = 0 \qquad \text{for all} t$$

and

$$P_t f(K_t, L_t) - W_t L_t \geq 0$$

For every K, there is a function $L_t(K)$ denoting labor use over tine. Once the optimal ex post decision rules are determined, the ex ante choice is to choose K to:

$$\max_{K} \int_0^\infty \left\{e^{-rt} P_t f(K, L_t(K)) - W_t L_t(K)\right\} dt - K$$

If P_t grows faster than W_t, than L_t will be positive forever. However, when the labor price grows faster than the price of capital, there may be a period T_1 where maximum short-term profit is zero. Beyond this period, no output is produced. T_1 is the economic life of capital. Here capital stops operation because of obsolescence.

- **Issue** –When prices fluctuate a lot, then you may shut down, then operate, shut down, etc.

Consider the simpler case where the ex post technology is a fixed proportion. Then, at time zero, both capital and labor are determined and ratios are followed thereafter. The objective function is

$$\max_{K,L} \int_0^\infty \left\{ e^{-rt} P_t f(K,L) - W_t L \right\} dt - K$$

subject to

$$P_t f(K,L) W w_t L > 0$$

Consider the case when output and labor prices grow exponentially, $P_t = P_0 e^{\gamma t}$ and $W_t = W_0 e^{\theta t}$. The rate of growth of output price is larger than that of labor price, $\gamma > \theta$ but $r > \gamma > \theta$. In this case production occurs between $t = 0$ and $t = T_1$, and the optimization problem is

$$\max_{T_1,K,L} \frac{P_0 f(K,L)}{r-\theta}\left(1 - e^{-(r-\theta)T_1}\right) - \frac{W_0 L}{r-\gamma}\left(1 - e^{-(r-\gamma)T_1}\right) - K$$

This optimization function can be rewritten as:

$$\max_{T_1,K,L} P_0 f(K,L) - W_0 L \frac{r-\theta}{r-\gamma} \frac{\left(1 - e^{-(r-\theta)T_1}\right)}{\left(1 - e^{-(r-\gamma)T_1}\right)} - \frac{r-\theta}{1 - e^{-(r-\theta)T_1}} - \frac{W_0 L}{r-\gamma}\left(1 - e^{-(r-\gamma)T_1}\right) K \tag{3.39}$$

The economic life of capital is determined solving

$$P_0 e^{\theta T_1} f(K,L) = W_0 e^{\gamma T_1} L$$

When $\theta = \gamma$, the dynamic problem becomes the classical static problem. Once T_1 is solved for, the ex ante optimal level of K and L are derived solving equation (3.39). In this case, the dynamic optimization looks like a standard static optimization with labor and capital costs adjusted to deal with different growth rates and economic lives of capital.

3.6.3 Aggregation From Micro to Macro

Short–run derivation in putty–clay models

Following the putty–clay assumption, each production unit has a fixed-proportion, ex post production function. Production coefficients vary among production units to reflect their vintages and other variables that affect the ex ante choices. Suppose we have one input. Let y denote the productivity of a unit expressed as output/input ratio; P is the output price; and W the input price. Production units (machines or plants) with $y > W/P$ will operate with full capacity, while production units with

$y < W/P$ will be idle. Let the density of the distribution of output capacity as a function of output/input ratio be denoted by $f(y)$. That means that the output capacity of firms with $y' - \frac{\Delta y}{2} < y < y' + \frac{\Delta y}{2}$, for very small Δy, is $f(y')\Delta y$, and total output capacity of the industry is $C = \int_0^\infty f(y)dy$. The output supply of industry is given by

$$Y(P, W) = \int_{W/P}^{\infty} f(y)dy \tag{3.40}$$

and the input demand is given by

$$X(P, W) = \int_{W/P}^{\infty} \frac{f(y)}{y} dy \tag{3.41}$$

Suppose y has a Pareto distribution (cf. figure 3.13). This distribution is found to be useful describing income distribution and farm size distribution:

$$f(y) = \begin{cases} Ay^{-(\alpha+1)} & y \geq K \\ 0 & y < K \end{cases}$$

with $\alpha > 0$. What is A? We know that total productive capacity is C.

$$C = \int_K^\infty Ay^{-(\alpha+1)}dy = \left[\frac{A}{-\alpha} y^{-\alpha}\right]_K^\infty = \frac{A}{\alpha K^\alpha} \tag{3.42}$$

From (3.42), $A = \alpha C K^\alpha$ and, hence,

$$f(y) = \frac{\alpha C}{K} \left(\frac{y}{K}\right)^{-(\alpha+1)}$$

K is output/input ratio of the least efficient machine. Under the Pareto distribution, productive capacity declines as efficiency (measured by y) increases. Note that

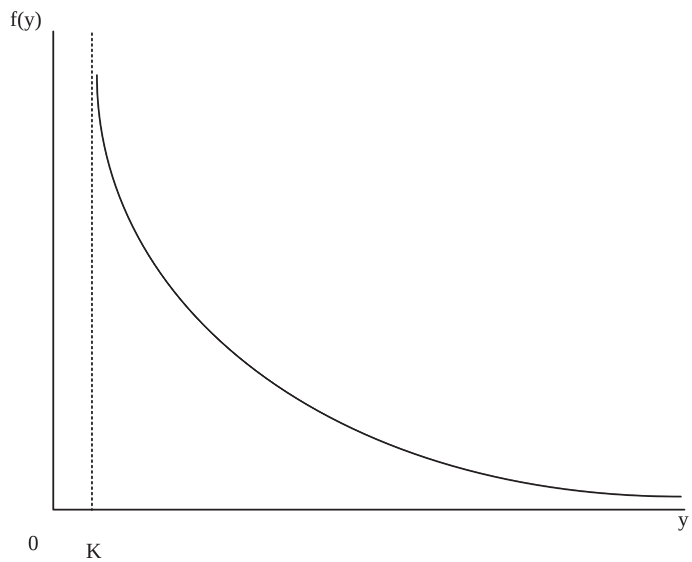
$$(\alpha + 1) = -\frac{\partial f(y)}{\partial y} \frac{y}{f(y)}$$

is the rate of decline in productive capacity associated with increased efficiency. Higher α assures more skewed capacity distribution. The Pareto distribution, as well as the log normal and exponential distribution is approximated with the empirical distribution of income but not necessarily distribution of output capacity.

From (3.40), the supply when capacity is distributed as a Pareto function of output/input ratio is

$$Y(P, W) = \int_{W/P}^{\infty} Ay^{-(\alpha+1)}dy = \left[-\frac{A}{\alpha} y^{-\alpha}\right]_{W/P}^{\infty} = \frac{A}{\alpha}\left(\frac{P}{W}\right)^{\alpha} \tag{3.43}$$

Figure 3.13: The Pareto Distribution



and from (3.41)

$$X(P,W)=\int_{W/P}^{\infty}\frac{Ay^{-(\alpha+1)}}{y}dy=\left[-\frac{A}{(\alpha+1)}y^{-(\alpha+1)}\right]_{W/P}^{\infty}=\frac{A}{\alpha+1}\left(\frac{P}{W}\right)^{\alpha+1} \tag{3.44}$$

From (3.44), one obtains

$$\frac{P}{W}=\left[\frac{\alpha+1}{A}X\right]^{\frac{1}{\alpha+1}}$$

Substituting that to (3.43) yields

$$Y=\frac{A}{\alpha}\left[\frac{\alpha+1}{A}X\right]^{\frac{\alpha}{\alpha+1}}$$

or

$$Y=BX^{\frac{\alpha}{\alpha+1}} \tag{3.45}$$

where

$$B=\frac{A}{\alpha}\left[\frac{\alpha+1}{A}\right]^{\frac{\alpha}{\alpha+1}}$$

Equation (3.45) provides what seems to be an aggregate production function, but this is not a “technological” relationship. It is a hybrid incorporating both technological (micro production function) and behavioral (profit maximization) elements and relating aggregate output and inputs under profit maximization.

The above analysis suggests that the input elasticity of the aggregate production function, $\alpha/(\alpha+1)$, reflects the productive capacity distribution and not a “technical” coefficient. Instead, $\alpha/(\alpha+1)$ is an outcome of investment decisions of the past that have resulted in the distribution of productive capacity. The Pareto distribution used here is not realistic for cases where you have growing industries and where there are relatively more capacity in newer vintages than older ones. It is reasonable in describing declining industries. When the input is land, it describes situations where higher quality lands (with higher y) are growing scarce and the mode of the productivity capacity distribution is at lower land qualities. The reservation we have regarding the Pareto distribution raise doubt on the appropriateness of aggregate Cobb-Douglas production functions.

A simple model of aggregation with some variability

The Putty–Clay framework does not allow short-term flexibility. However, existing plants may change variable inputs to affect output. Thus, let us assume that production function is of constant returns to scale, but plants vary in quality. Let q

be a variable measuring the quality of the capacity unit. Higher q represents, say, higher human capital of plant managers. Let production per capacity unit be

$$y = f(q, x)$$

when x is the variable input per capacity unit. Assume

$$f_x > 0, \qquad f_{xx} < 0, \qquad f_{qx} > 0, \qquad f_{qxx} < 0$$

For example,

$$f(q, x) = q^\gamma x^\beta, \qquad 1 > \gamma > 0, \qquad 1 > \beta > 0$$

Let us also assume when a capacity unit operates, a fixed-cost C is required. For a micro unit, the two choices are:

(i) Whether to operate or not

(ii) How much input per capacity unit to use.

Let P indicate the output price and W the input price. The decision problem is

$$\max_x Pf(q, x) - Wx - C$$

The first-order condition is

$$Pf_x - W = 0$$

The capacity unit operates if

$$Pf(q, x) - Wx - C > 0$$

For the case $f(q, x) = q^\gamma x^\beta$, at the optimal solution

$$P\beta q^\gamma x^{\beta-1} = W \tag{3.46}$$

or microlevel input demand is

$$x(P, W, q) = \left[\frac{P\beta q^\gamma}{W}\right]^{\frac{1}{1-\beta}}$$

and microlevel supply is

$$y(P, W, q) = q^\gamma \left[\frac{P\beta q^\gamma}{W}\right]^{\frac{\beta}{1-\beta}}$$

In this case, x increases with quality and output price and declines with input price. Profit per capacity unit is

$$\pi(P,W,q,C) = Pq^{\gamma}\left[\frac{P\beta q^{\gamma}}{W}\right]^{\frac{\beta}{1-\beta}} - W\left[\frac{P\beta q^{\gamma}}{W}\right]^{\frac{1}{1-\beta}} - C$$

Setting this expression equal to zero and solving for q the marginal producing quality, q^m, can be found; it is a function of P, W, and C.

$$q^m(P,W) = \left(\frac{W}{P}\right)^{\frac{1}{\gamma}}\left[\frac{C\beta}{(1-\beta)W}\right]^{\frac{1-\beta}{\gamma}}$$

Let the distribution of capacity be denoted by $g(q)$ when $g(q)\Delta q$ denotes the number of capacity unit between $q - \frac{\Delta q}{2}$ and $q + \frac{\Delta q}{2}$. Let

$$\int_0^{\infty} g(q)d(q) = M$$

where M is the number of capacity units, then the aggregate supply is

$$Y(P,W,q) = \int_{q^m(p,q)}^{\infty} y(P,W,q)dq$$

To find the supply slope, we need to differentiate $Y(P,W,q)$. The Leibnitz rule is used for differentiating integrals. For a function

$$F(x) = \int_{a(x)}^{b(x)} g(x,z)dz$$

the Leibnitz rule tells that $F_x(x) = b_x g(x,b(x)) a_x g(x,a(x)) + \int_{a(x)}^{b(x)} g_x(x,z)dz$. Using this rule, the slope of the supply curve is

$$Y_P(P,W) = \underbrace{\int_{q^m}^{\infty} y_P(P,W,q)dq}_{(1)} - \underbrace{\frac{\partial q^m}{\partial P}(P,W,q^m)g(q^m)y(q^m)}_{(2)}$$

Marginal change in supply includes change in intensive margin (1) and change in extensive margin (2). A higher price will reduce marginally output of every operating production unit, and it will cause closure of some borderline capacity units.

Determination of aggregate relationship in cases with varying input quality

In this section the irrigation model that was presented in the previous section is used as a base for aggregation. Let us recall that model. Suppose we have a constant returns to scale technology, in which agricultural product is produced using land and E, effective input. Given constant return to scale, output and input use can be expressed on a per–acre basis:

$y = f(e) =$ yield per acre

$e =$ effective input per acre (input used in production process)

$a =$ total applied input

$\alpha =$ quality of land, a continuous variable from 0 to 1.

The amount of effective input is a function of the amount applied, of land quality and of the kind of technology: $e = h(i, \alpha)a$ where i is a technology index, assuming value 2 for modern technology and value 1 for traditional ones. When no technology is used, $i = 0$. $h(i, \alpha) = e/a$ is input-use efficiency. When input is water, it is irrigation efficiency. Let α be an index of land quality with respect to input use (e.g., water-holding capacity). Assume $1 \geq h(2, \alpha) \geq h(1, \alpha) = \alpha > 0$. Let us also assume that production requires fixed cost per acre, C_i with $C_2 > C_1$. Assuming profit maximization for each α, there is an optimal a (applied water) and an optimal i (technology) determined solving

$$\max_i \begin{cases} \Pi(i) = \max_a Pf(h(i,\alpha)a) - Wa - C_i, & i = 1,2 \\ \Pi(0) = 0 \end{cases}$$

when P is output price, W is input price and C_i is the cost of adopting technology i. The optimization is solved in two steps: first we solve for optimal a given i, for $i = 1, 2$, and the first-order condition is

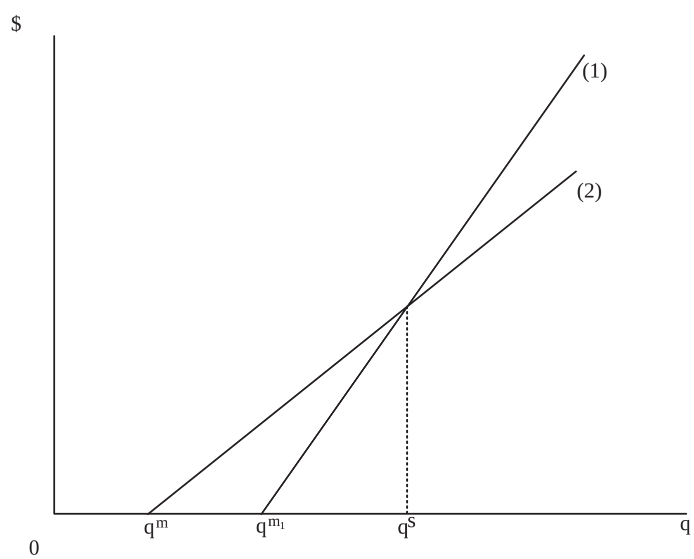
$$Pf_e h(i, \alpha) = W$$

or $Pf_e = W/h(i, \alpha)$, where $W/h(i, a)$ is price of effective water. Then we compute optimal i according to

$$i = \begin{cases} 2 & \text{if} \quad \Pi(2) > \Pi(1) \\ 1 & \text{if} \quad \Pi(1) > \Pi(2) \\ 0 & \text{if} \quad \Pi(1), \Pi(2) < 0 \end{cases}$$

Both $\Pi(2)$ and $\Pi(1)$ have to be nonnegative. When land quality is high ($\alpha = 1$), $a(2) = a(1)$ and $\Pi(1) > \Pi(2)$ since $C_2 > C_1$. It can be easily shown that $\Pi_\alpha(i) > 0$, for $i = 1, 2$ (i.e., profit increases with land quality), and that, for high-quality lands, the marginal effects of quality ($\Pi_\alpha(1) > \Pi_\alpha(2)$) hold that loss in profit as land quality declines is larger under traditional technology. This is reasonable since modern technology tends to *augment* land quality, especially if we assume that the ratio $\frac{h(2,\alpha)-h(1,\alpha)}{h(1,\alpha)}$ becomes bigger as α declines. This assumption is true with respect to sprinkler or drip irrigation. Their impact on water-use efficiency compared to furrow irrigation is stronger in locations with low water–holding capacity. This assumption suggests the following technology–use pattern:

Figure 3.14: Switching quality



$$i = 0 \text{ for } \alpha \leq \alpha^m$$
$$i = 2 \text{ for } \alpha^m < q < \alpha^s$$
$$i = 1 \text{ for } \alpha > \alpha^s$$

α^s is a switching quality. It is determined solving

$$\Pi(1, \alpha^s) = \Pi(2, \alpha^s)$$

or

$$\max_a Pf[h(2, \alpha^s)a] - Va - C_2 = \max_a Pf[h(1, \alpha^s)a] - Va - Cc_1 \tag{3.47}$$

and α^m is the marginal quality, determined by solving:

$$\Pi(2, \alpha^m) = 0$$

or

$$\max_a Pf[h(2, \alpha^m)a] - Va - C_2 = 0 \tag{3.48}$$

To see how changes in price will affect marginal or switching quality, we need to perform a comparative static analysis based on (3.47) or (3.48). Differentiation of (3.48) with respect to α and P yields:

$$\frac{d\alpha^m}{dP} = \frac{\Pi_P(2) + \Pi_a(2)a_P}{\Pi_\alpha(2) + \Pi_a(2)a_\alpha}$$

Since first-order conditions are met at α^m, $\Pi_a(2) = 0$, and

$$\frac{d\alpha^m}{dP} = \frac{\Pi_P(2)}{\Pi_\alpha(2)} = -\frac{y(2)}{Pf_e h_\alpha(2)a(2)} < 0 \tag{3.49}$$

Note that $Pf_e h = W$. Therefore, define

$$\Psi(i) - \frac{h_\alpha(i)}{h(i)}\alpha \equiv \text{ elasticity of irrigation efficiency with respect to quality}$$

Using this definition, (3.49) becomes

$$\frac{d\alpha^m}{dP} = -\frac{\alpha^m}{W}\frac{y(2)}{\Psi(2)a(2)}$$

Similarly, differentiation of (3.47) with respect to α and P yields:

$$\frac{d\alpha^S}{dP} = \frac{\Pi_P(2) + \Pi_P(1)}{\Pi_\alpha(2) + \Pi_\alpha(1)}$$

Noting that, since $h(\alpha, 1) = \alpha$, $\Psi(1) = 1$, we get:

$$\frac{d\alpha^S}{dP} = -\frac{y(2) - y(1)}{W[\Psi(2)a(2) - a(1)]} \tag{3.50}$$

It can be argued that $\Psi(2) < 1$, and we know that $y(2) > y(1)$. In most cases $a(1) > a(2)$; therefore, $\frac{d\alpha^s}{dP} > 0$. Namely, increase in price will increase adoption by increasing the switching land quality. Let us assume that $g(\alpha)$ is the land quality density function; $g(\alpha)\Delta\alpha$ denotes the amount of land with qualities between $\alpha - \frac{\Delta\alpha}{2}$ and $\alpha + \frac{\Delta\alpha}{2}$ for small $\Delta\alpha$. Thus, aggregate output supply and input demand are given by:

$$Y[P, V, c_1, c_2] = \int_{\alpha^m}^{\alpha^s} y(2, \alpha)g(\alpha)d\alpha + \int_{\alpha^s}^{1} y(1, \alpha)g(\alpha)d\alpha \tag{3.51}$$

$$A[P, V, c_1, c_2] = \int_{\alpha^m}^{\alpha^s} a(2, \alpha)g(\alpha)d\alpha + \int_{\alpha^s}^{1} a(1, \alpha)g(\alpha)d\alpha. \tag{3.52}$$

The Leibnitz rule is useful in analyzing the properties of the aggregate supply response. Using this rule:

$$\begin{aligned} Y_P \quad &= \underbrace{\frac{d\alpha^S}{dP}\left[y(2, \alpha^S) - y(1, \alpha^m)\right] g(\alpha_S)}_{+} \underbrace{- \frac{d\alpha^m}{dP} g(2, \alpha^m)}_{+} \\ &\underbrace{+ \int_{\alpha^m}^{\alpha^S} y_P(2, \alpha)g(\alpha)d\alpha}_{+} \underbrace{+ \int_{\alpha^m}^{1} y_P(1, \alpha)g(\alpha)d\alpha}_{+} \end{aligned}$$

The first two elements reflect changes in the extensive margin, and the other two reflect changes in the intensive margin. All elements are positive which means that supply is positive. The extensive margins are especially important when there is a relatively large amount of land having switching or marginal qualities. One can use similar procedures to evaluate Y_V, A_P, and A_V.