When Promoters Like Scalpers

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April 1, 2003

Abstract

If a monopoly supplies a perishable good, such as tickets to a performance, and is unable to price discriminate within a period, the monopoly may benefit from the potential entry of resellers. If the monopoly attempts to intertemporally price discriminate, the equilibrium in the game among buyers is indeterminate when the resellers are not allowed to enter, and the monopoly's problem is not well defined. An arbitrarily small amount of heterogeneity of information among the buyers leads to a unique equilibrium. We show how the potential entry of resellers alters this equilibrium.

JEL classification numbers: L12, D42, D45, D82

keywords: intertemporal price discrimination, scalpers, coordination game, common knowledge

1 Introduction

A perishable-good monopoly that can charge different prices in different periods but cannot price discriminate within a period may benefit from the existence of a resale market. We illustrate the effects of resellers in a market where a monopoly sells perishable tickets that have no value after the event. For specificity, we discuss tickets for a concert or other performance, which are resold by ticket agencies, brokers, "scalpers" (United States), or "touts" (Great Britain).

We study two versions of a two-period model. Consumers have common knowledge about the value of a ticket in one version, and they lack common knowledge in the other. We restrict our attention to situations where scalpers do not harm and possibly benefit the monopoly. Relevant examples include ticket agencies and brokers who explicitly receive cooperation from monopoly ticket providers.

In our model, some consumers have a relatively high willingness to pay ("high types"), while others have a lower willingness ("low types"). We assume that the monopoly cannot distinguish between the two types of consumer except by observing their behavior over time. Consequently, the monopoly cannot price discriminate if it sells all its tickets in a single period. It can discriminate over time by setting a relatively high price in the first period in which it sells tickets, and then selling the remaining tickets at a discount in the second period.

By incurring a transaction cost, scalpers can distinguish between consumers at a moment in time and hence can price discriminate within a period. For example, the scalper may be able to determine customers' types by standing in the parking lot and observing their cars or clothes or by talking to them. The potential entry of scalpers changes the monopoly's pricing problem and the equilibrium of the game.

In the one-period model, the effect of scalpers on the equilibrium is straightforward. Provided that scalpers' transaction costs are sufficiently low, the monopoly uses them as its agents. It sells all of the tickets to scalpers at a price between the high and low willingness to pay. Scalpers resell as many tickets as possible to high types at a relatively high price, and unload the remaining tickets to low types at a lower price (Rosen and Rosenfield (1997), DeSerpa (1994)). That is, the monopoly requires that the scalpers buy a "bundle" of tickets; some they can sell at a high price and others they have to dump.

Our primary objective is to study the role of scalpers in situations where a perishable-good monopoly is able to intertemporally price discriminate. For example, the price of advance

tickets may differ from the price of tickets at the gate. In a two-period game where consumers have common knowledge about payoffs, the outcome of the game is not well-defined, regardless of whether scalpers can enter.

When scalpers cannot enter, buyers engage in a coordination game. For a range of first-period prices, there are two equilibrium sales levels in this coordination game. Since we cannot assign a unique monopoly payoff to every first-period price, the monopoly problem is not well-defined.

If scalpers are able to enter the model, there is an interval of prices for which there is no equilibrium in the game among buyers. However, the possibility of scalpers entering the market enables the monopoly to obtain profits at least as great as the maximum equilibrium level in the absence of scalpers. Scalpers lower consumer surplus, and may increase or decrease social welfare. If we restrict attention to prices for which there exists an equilibrium to the game among buyers, then the monopoly either induces all high types to buy in the first period and scalpers do not enter in equilibrium; or the monopoly sells all its tickets to scalpers in the second period, using them as its agent.

The game with common knowledge – with or without scalpers – is not an adequate model of intertemporal price discrimination of a perishable good. It does not predict the outcome for every possible first-period price, so we cannot fully characterize the equilibrium price and sales path. Consequently, we obtain only limited comparisons of the equilibrium with and without scalpers. In addition, the conclusion that the monopoly uses scalpers to sell either all tickets or no tickets is not empirically plausible. Monopolies sometimes sell some advance tickets to high-type customers, and sell the remaining tickets through agents.

We modify the model in a "plausible" manner to remedy these limitations: we replace the assumption of common knowledge about payoffs with the assumption that agents have heterogeneous information. In this model, buyers do not know the true value of the ticket in the first period, and each buyer has a private assessment of the value. For example, buyers assess the quality of the performance based on knowledge of the identity of the participants (the director, performers, team members). Buyers do not know the exact assessment that others have made, but they know something about the distribution of those assessments. In the second period, an important piece of information becomes common knowledge. For example, potential customers read critics' reviews, find out which actors are in the cast, or learn the identities of playoff teams. When this information becomes available, all buyers know the

value of the ticket; they all know that all other buyers know the value, and they know that others know that others know, *ad infinitum.*

The heterogeneity of information in the first period – the lack of common knowledge about payoffs – induces a unique equilibrium in the first-period game among buyers when scalpers cannot enter. The monopoly problem is well defined: There is a unique payoff corresponding to every price. The fact that the lack of common knowledge about payoffs induces a unique equilibrium in coordination games has been noted by, *inter alia*, Carlsson and Damme (1993), Morris and Shin (1998) and Rubinstein (1989). We contrast this unique equilibrium (when scalpers are prohibited) to the "corresponding" (possibly not unique) equilibrium in the game where scalpers can enter. At least for small levels of uncertainty, scalpers increase monopoly profit. For some parameter values, the monopoly sells some tickets to high types in the first period, and uses scalpers as its agent for the remaining tickets in the second period.

The literature on scalping is small, and the literature on ticket sales in general is only slightly larger. Previous analyses of scalpers use one-period models. Except for Swofford (1999), the papers on scalping ignore price discrimination. Most of these take the price set by the monopoly as given and they ignore how scalpers affect the market.[1] Theil (1993) assumes that the monopoly underprices tickets (for exogenous reasons) and then examines whether scalpers raise or lower surplus in a one-period model. He emphasizes that the presence of scalpers may reduce the probability that others can buy low-price tickets (but he does not discuss price discrimination). Swofford (1999) explains why different views toward risk, different cost functions, or different abilities to price discriminate may make scalping profitable. Happel and Jennings (1995) discuss anti-scalping laws without using a formal model. In an empirical study, Williams (1994) finds that scalpers increase the average National Football League ticket price by nearly $2, a result that is consistent with scalping benefiting the monopoly.

Two additional papers are more closely related to ours than are these papers on scalping. Peck (1996) shows that fixed prices and rationing may be used in incomplete markets where lotteries cannot be used. Peck observes in passing that the presence of scalpers may determine whether an efficient market exists. Van Cayseele (1991) shows that a monopoly with the ability to commit may engage in intertemporal price discrimination in a market with rationing. As in

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[1]Some papers on ticket sales discuss price discrimination by the monopoly (but generally do not mention scalpers), e.g.,Barro and Romer (1987), Baumol and Baumol (1997), Bertonazzi, Maloney, and McCormick (1993), DeSerpa (1994), Gale and Holmes (1993), DeGraba and Hohammed (1999), Huntington (1993) and Lott and Roberts (1991).

our model, prices are higher in the first period and then fall. Our model differs from his in that we assume that the monopoly cannot price discriminate but that the scalpers can.

2 One-period model

We begin with a one-period model in which a monopoly that cannot price discriminate benefits by having scalpers sell tickets. Each potential customer wants to buy one ticket. There are H high-type buyers who are willing to pay up to p^h for a ticket and L low-type buyers whose willingness to pay is p^l. The difference in willingness to pay between the two types of customers is $D \equiv p^h - p^l > 0$. The monopoly has T tickets available and faces excess demand of $E \equiv H + L - T > 0$. All agents are risk neutral.

In the absence of scalpers, the monopoly sells all its tickets at either p^h or at p^l because it cannot distinguish between customers. To eliminate the uninteresting case where the monopoly sells to only high types at the high price, we assume that it earns more by selling all tickets at the low price.

Assumption 1 $p^l T > p^h H$.

Selling all tickets at the low price is not socially efficient. High types must compete with low types to buy a ticket and hence not all high types succeed in buying a ticket. However, high types who are lucky enough to obtain a ticket receive consumer surplus of $p^h - p^l$.

Suppose that the monopoly is able to set aside s tickets for scalpers. The scalpers price discriminate and extract all potential surplus from buyers. However, scalpers incur a transaction cost. A scalper that sells a ticket for price p receives revenue net of transactions costs of ϕp, where $\phi < 1$. That is, the scalper's transaction cost per ticket is $(1 - \phi)p$.[2]

If the monopoly sells all the tickets to scalpers ($s = T$), then scalpers are able to sell H tickets at price p^h and dump the rest, $T - H$, on low types at p^l. Due to free entry, risk-neutral scalpers earn zero expected profit, so that the monopoly sells each of its T tickets to them at price

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[2]It is reasonable to assume that transactions costs are higher for high-priced tickets because of the increased search costs needed to find a high-type buyer. The assumption that these costs are proportional to the price of the ticket is not needed in this one-period model. However, the assumption greatly simplifies the computations in our two-period models in Sections 4.2 and 5.2.

$$p = \frac{\phi[p^h H + p^l(T - H)]}{T}. \tag{1}$$

If the scalpers price discriminate efficiently (ϕ is close to 1), the price that they pay is an average of the high and the low price. If scalpers cannot price discriminate efficiently (ϕ is small), it never pays the monopoly to sell to them. If $p > p^l$, then the monopoly prefers to sell all its tickets to scalpers.

Because the scalpers can price discriminate, all high types obtain tickets if the monopoly sells to scalpers. Consequently, the equilibrium with scalpers avoids the inefficiency that results when some high types fail to obtain tickets in the no-scalper equilibrium. However because scalpers incur a transaction cost, the scalper equilibrium is less efficient than the equilibrium that would occur in the absence of transaction costs. High types do not obtain consumer surplus in this equilibrium.

The scalpers allow the monopoly to "bundle." Some sports teams bundle by requiring their customers to buy tickets to all preseason and regular season games. If customers' demands for preseason and regular season tickets are negatively correlated, the team earns a higher profit by selling only bundles of preseason and regular season tickets rather than selling them separately.

In our model, scalpers are forced to buy tickets for both types of consumers. The scalpers make a profit selling tickets to high types and lose money selling to low types. As with many models of imperfect price discrimination, a bad is bundled with the good in order to facilitate surplus extraction (Chiang and Spatt 1982). Thus, by selling to only scalpers, the monopoly is able to bundle or price discriminate.

3   Two-period model with complete information

We now want to examine the role of scalpers in a market where a perishable-good monopoly can intertemporally price discriminate. We use a two-period model, which provides the simplest framework for describing intertemporal price discrimination. The second (and last) period occurs shortly before the performance is about to begin; after that time the tickets are worthless. The monopoly sets the price p_i in period i. All agents are risk neutral and they have complete information about consumers' valuation of tickets. We relax the second assumption in the next section. In both sections, we consider two extreme cases: there are no scalpers or an unlimited number of scalpers. We consider the set of subgame perfect equilibria in which the monopoly

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chooses each period's price at the beginning of that period; consumers decide whether to buy in a given period; and scalpers (when they are present) decide whether to participate.

The monopoly cannot distinguish among buyers within a period. In particular, it cannot discriminate between ordinary consumers and scalpers (unlike in the model in the previous section).[3] The monopoly can charge different prices in different periods, as in Dudey (1996) and Rosen and Rosenfield (1997). In each period consumers take the current price and the actions of all other agents as given. In the first period, consumers have rational point expectations about the second-period price.

We assume, without loss of generality, that the first-period monopoly price is strictly greater than p^l. The monopoly never wants to set a price lower than p^l. The payoff it obtains by setting $p_1 = p^l$ is never greater, and may be less, than the payoff it obtains by setting $p_1 > p^l$ and then using the price p^l in the second period. Therefore, setting $p_1 = p^l$ is a dominated strategy. The inequality $p_1 > p^l$ means that no low types buy in the first period. In the second period low types buy if and only if they can obtain a ticket at a price less than or equal to p^l.

In the first period, an endogenously determined fraction α of high types choose not to buy (i.e., to "wait"). The high types' payoff from buying in the first period equals the difference between their valuation and the price, $p^h - p_1$. The expected payoff from waiting equals the probability of getting a ticket in the second period, times the difference between p^h and the price they will have to pay. High types who wait to buy must compete with low types and possibly scalpers in the second period.

The equilibrium value of α depends on the first-period price and on whether scalpers are allowed to enter in the second period. If $0 < \alpha < 1$ in equilibrium, then all high types must be indifferent between the buying or waiting in the first period. If no high type buys in the first period ($\alpha = 1$), then $p^h - p_1$ must be less than or equal to the expected benefit of waiting. If all high types buy in the first period ($\alpha = 0$), then $p^h - p_1$ must be greater than or equal to the expected benefit of waiting.

As usual in a multi-period game, we solve the game starting in the last period. We begin by examining how the second-period equilibrium changes if scalpers are allowed to enter in that period. We then examine the first-period equilibrium as a function of the first-period price. We explain why scalpers never enter in the first period; in equilibrium, only high types buy tickets in the first period. We characterize the high types' equilibrium response to the first-period price

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[3]The results are unchanged if the monopoly is allowed to set aside a block of tickets for scalpers.

and then present welfare results.

3.1 The second-period equilibrium

The second-period equilibrium depends on whether scalpers are allowed to enter. We assume that all buyers pay the price p_2 in the second period and that the monopoly is not able to discriminate among high- and low-type buyers and scalpers within a period. Let S be the endogenous number of scalpers who try to buy tickets, s is the endogenous number of scalpers who actually obtain tickets, and L^* is the endogenous equilibrium number of low types that try to buy a ticket from the monopoly. Individuals who are indifferent between buying and not buying break the tie by trying to buy a ticket. Thus, $L^* = L$ if $p_2 \leq p^l$, and $L^* = 0$ if $p_2 > p^l$.

Given L^* and s, the probability that a high type is able to buy a ticket from the monopoly in the second period is

$$\theta = \frac{T - (1-\alpha)H - s}{\alpha H + L^*}. \tag{2}$$

The numerator of θ is the number of remaining tickets after $(1-\alpha)H$ were sold in the first period and scalpers obtain s tickets. The denominator is the total number of low types and high types trying to obtain a ticket from the monopoly.

We have (see the Appendix for all proofs)

Lemma 1 *Under Assumption 1, the optimal second-period price is* $p_2^* = \max\left\{p^s\left(\alpha\right), p^l\right\}$ *where*

$$p^s(\alpha) \equiv \phi\frac{(T-H)\,p^l + \alpha p^h H}{T - (1-\alpha)\,H}. \tag{3}$$

If $p^s\left(\alpha\right) > p^l$, *scalpers obtain all the tickets remaining after* $(1-\alpha)\,H$ *were bought in the first period:* $s = T - (1-\alpha)\,H$.

By setting $\alpha = 1$, we obtain the special case of a one-period model (compare Equations (1) and (3)). Again, the one-period model shows that the monopoly uses scalpers as its agents because of their ability to price discriminate.

The function $p^s(\alpha)$ in Equation (3) is increasing in α. We define $\hat{\alpha}$ as that value of α that satisfies $p^s(\alpha) = p^l$:

$$\hat{\alpha} \equiv \frac{(1-\phi)\left(\frac{T}{H} - 1\right)p^l}{(\phi p^h - p^l)} > 0. \tag{4}$$

A necessary and sufficient condition for $\hat{\alpha} < 1$ is that $\phi > T/(T + DH)$. If $\alpha = \hat{\alpha}$, the monopoly is indifferent between selling all the tickets to scalpers or to ordinary buyers. We

assume that the monopoly breaks the tie by selling to ordinary buyers. The monopoly does not sell to scalpers if $\alpha \leq \hat{\alpha}$.

3.2   The first-period equilibrium

To determine the effect that scalpers have on the equilibrium, we first consider the equilibrium where scalpers are not permitted in the market and then the one in which they may enter. When scalpers are not able to enter, the high-type buyers play a coordination game in the first period for a range of prices. The game has two equilibria: Either no high types or all high types buy in the first period. We define the "limit price," $\bar{p}$, as the supremum of prices at which there exists an equilibrium where all high types buy in the first period. We define the "choke price," $\tilde{p}$, as the infimum of prices at which there exists an equilibrium where no high types buy in the first period. *The limit price in this setting is higher than the choke price.* We show that the buyers' behavior is indeterminate for any first-period price that satisfies $\tilde{p} < p_1 < \bar{p}$. Consequently, the monopoly is not able to solve its profit maximization problem because it cannot calculate expected quantity demanded over some range of prices.

We then consider the first-period equilibrium in the game where scalpers are allowed to enter the market. With no loss in generality, we assume that scalpers do not enter in the first period. If the monopoly were to set the first-period price low enough to induce the scalpers to enter, they would buy all of the tickets. The monopoly can achieve the same level of profit by selling all tickets to scalpers in the second period rather than in the first period. That is, it can set the first-period price so high that no one buys in the first period, and then sell all the tickets to scalpers in the second period. Selling all of the tickets to scalpers in the first period and selling all of the tickets to them in the second period are equivalent. However, the monopoly may do better by setting a first-period price that is high enough to discourage scalpers from entering, but low enough to induce some high types to buy in the first period.

The possibility that scalpers enter creates a credible threat that high types will receive 0 surplus if they do not buy in the first period. For prices below the limit price, this threat eliminates the equilibrium where no high types buy in the first period. Thus, the potential for scalpers to enter – whether or not they actually appear in equilibrium – removes the first-period indeterminacy for prices $\tilde{p} < p_1 < \bar{p}$. However, for some range of prices there exists no equilibrium with scalpers. The introduction of scalpers removes one source of indeterminacy but creates another.
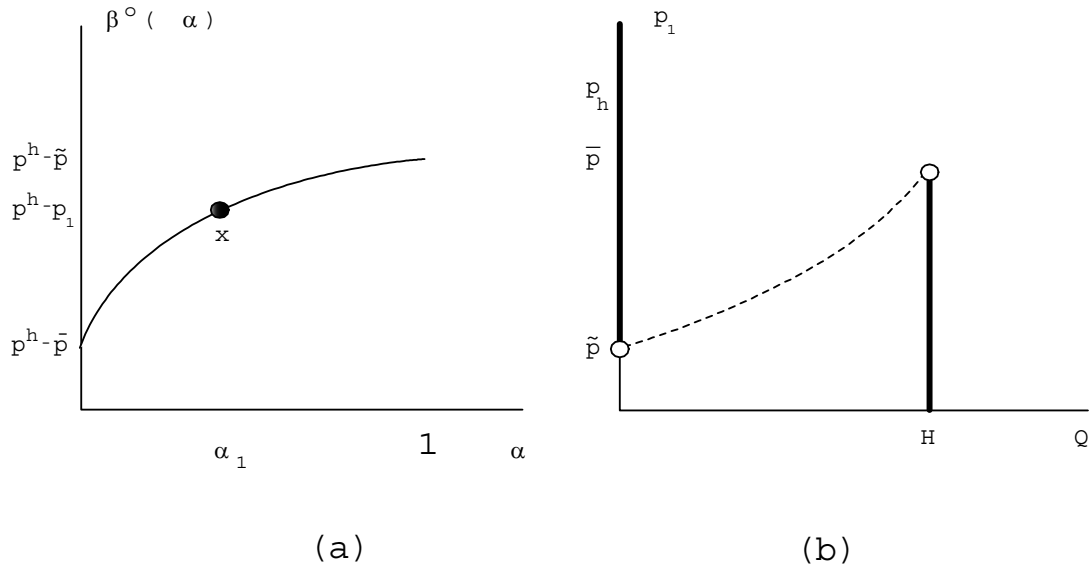
$\beta^{\circ}(\alpha)$ axis with values $p^h - \tilde{p}$, $p^h - p_1$ at point X, $p^h - \bar{p}$; horizontal axis α_1, 1, α — label (a)

P_1 axis with p_h, $\bar{p}$, $\tilde{p}$; horizontal axis H, Q — label (b)

Figure 1: The market without scalpers. (a) Benefit of waiting. (b) Demand

3.2.1 The market without scalpers

We start by examining the market without scalpers. High-type buyers decide whether to buy in the first period or wait until the second period with the hope of obtaining a cheaper ticket. We define the high-type buyer's expected benefit of waiting (not buying in period 1) when scalpers are excluded as $B^0(\alpha)$. Hereafter the superscript "0" means that we assume that scalpers are prohibited from entering the market. We also simplify our notation by adopting the normalizations $T = p^l = 1$, so $p^h = D + 1$.

The expected benefit of waiting equals the consumer surplus, D, obtained from buying at the lower price times the probability, θ, that the high-type buyer will be able to obtain a ticket at this price in the second period. Because the equilibrium values in the second period are $p_2^* = p^l = 1$ and $L^* = L$, the probability is $\theta = [1 - (1 - \alpha)H]/[\alpha H + L]$. Thus, the expected benefit to a high type from waiting is the increasing, concave function

$$B^0(\alpha) = D\frac{1 - (1 - \alpha)H}{\alpha H + L}. \tag{5}$$

Figure 1a graphs $B^0(\alpha)$. If the monopoly sets the low choke price $\tilde{p}$, and if all high types wait to buy ($\alpha = 1$), then the surplus from buying in this period equals the expected benefit of

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waiting: $p^h - \tilde{p} = B^0(1)$. If the monopoly charges the limit price, $\bar{p}$, and if all high types buy in this period ($\alpha = 0$), then $p^h - \bar{p} = B^0(0)$. Using these definitions and our normalizations, we have

$$\bar{p} \equiv 1 + \frac{DE}{L} > \tilde{p} \equiv 1 + \frac{DE}{E+1} \tag{6}$$

Any point on the $B^0(\alpha)$ curve for $0 \leq \alpha \leq 1$ is an *unstable* Nash equilibrium. For example, consider point x in Figure 1a. This point corresponds to first-period price p_1 (where $\tilde{p} < p_1 < \bar{p}$) and a fraction of high types who wait, α_1, with $B^0(\alpha_1) = p^h - p_1$. The proposed equilibrium pair (p_1, α_1) is unstable: agents who believe that a positive measure of high types will deviate from the proposed equilibrium would want to follow that deviation. (A "deviation" means that high types do the opposite of what they are "instructed" to do in equilibrium; they wait when they are supposed to buy, or buy when they are supposed to wait.) If the measure $\mu > 0$ of high types deviate from the proposed equilibrium by buying rather than waiting, then $\alpha = \alpha_1 - \mu < \alpha_1$. With this deviation, all high types prefer to buy in the first period. Similarly, if the measure $\mu > 0$ of high types deviate from the proposed equilibrium by waiting rather than buying, all high types strictly prefers to wait. Thus, a small deviation from the proposed equilibrium generates an increasingly large deviation in the same direction, until a boundary is reached (all wait or all buy). By a similar argument, the two points $(\alpha, p_1) = (0, \bar{p})$ and $(\alpha, p_1) = (1, \tilde{p})$ are unstable; in these cases, we need only consider "one-sided deviations".

Using Figure 1a, we can derive the first-period demand correspondence in the absence of scalpers, Figure 1b. At prices $\tilde{p} < p_1 < \bar{p}$, there are two stable equilibrium demands in the first period, $Q = H$ (where $\alpha = 0$) and $Q = 0$ (where $\alpha = 1$) represented by the heavy vertical lines, and an unstable equilibrium set of demands represented by a dashed curve. (Because the set of unstable equilibrium demands is rising, this "demand curve" is upward sloping.) As more agents buy in the first period, the chance of getting a ticket at the low price in the second period diminishes, making it more attractive to buy early. An additional purchase in the first period lowers both demand and supply by one unit in the second period. The net effect is to lower the probability of being able to buy a ticket in the second period.

Thus, an increase in the measure of agents taking an action (buying or not buying), increases the benefit to other agents of taking the same action. Due to this externality, buyers play a coordination game. At any first-period price that satisfies $\tilde{p} < p_1 < \bar{p}$, the high types' expected payoff is greater in the equilibrium where no one buys ($\alpha = 1$). In the equilibrium where all high types buy, their payoff is $p^h - p_1$, whereas their expected payoff is $B^0(1) > p^h - p_1$ in the

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equilibrium where no one buys.

Efficiency is greater in the equilibrium where all high types buy. In the absence of scalpers, the only source of inefficiency is that some high types might end up without tickets. Only a fraction $\theta < 1$ of high types obtain tickets when $\alpha = 1$, but they all obtain tickets when $\alpha = 0$.

Figure 1b illustrates why we call $\tilde{p}$ the choke price and $\bar{p}$ the limit price. In Figure 1b, $\tilde{p}$ is the greatest lower bound of the set of prices at which there is a stable equilibrium demand of 0. Similarly, $\bar{p}$ is the least upper bound of prices at which there is a stable equilibrium in which all high types buy.

We summarize the discussion above as:

Proposition 1 *When scalpers are not permitted to enter the market, the equilibrium is indeterminate for first-period prices $\tilde{p} < p_1 < \bar{p}$. At those prices there are two possible equilibria, $\alpha = 1$ and $\alpha = 0$. Buyers prefer the first equilibrium but social welfare (efficiency) is greater in the second. All high types buy in the first period if $p_1 \leq \tilde{p}$ and none buy if $p_1 \geq \bar{p}$. The second-period price is $p^l = 1$, regardless of the first-period price.*

3.2.2 The market with scalpers

Now, we describe the equilibrium set when the monopoly is allowed to sell to scalpers in the second period. By Lemma 1, the high type's expected value of waiting is

$$B(\alpha) = \left\{ \begin{array}{c} B^0(\alpha) \text{ for } \alpha \leq \hat{\alpha} \\ 0 \text{ for } \alpha > \hat{\alpha} \end{array} \right\}, \tag{7}$$

where $\hat{\alpha}$ is the critical value of α, above which the monopoly sells only to scalpers.

If $\hat{\alpha} \geq 1$ it never pays the monopoly to sell to scalpers, so their ability to enter the market is irrelevant. The only interesting case is when $\hat{\alpha} < 1$, as we hereafter assume. We restate this assumption and Assumption 1 as:

Assumption 2 $\frac{1}{D+1} > H > \frac{1-\phi}{\phi D}$.

If $\alpha > \hat{\alpha}$, high types who did not buy in the first period have to deal with scalpers in the second period, and they capture no surplus. Figure 2a graphs $B(\alpha)$ under Assumption 2, and Figure 2b shows the related demand correspondence. The price $\hat{p}$ is the solution to $p^h - \hat{p} = B(\hat{\alpha})$.
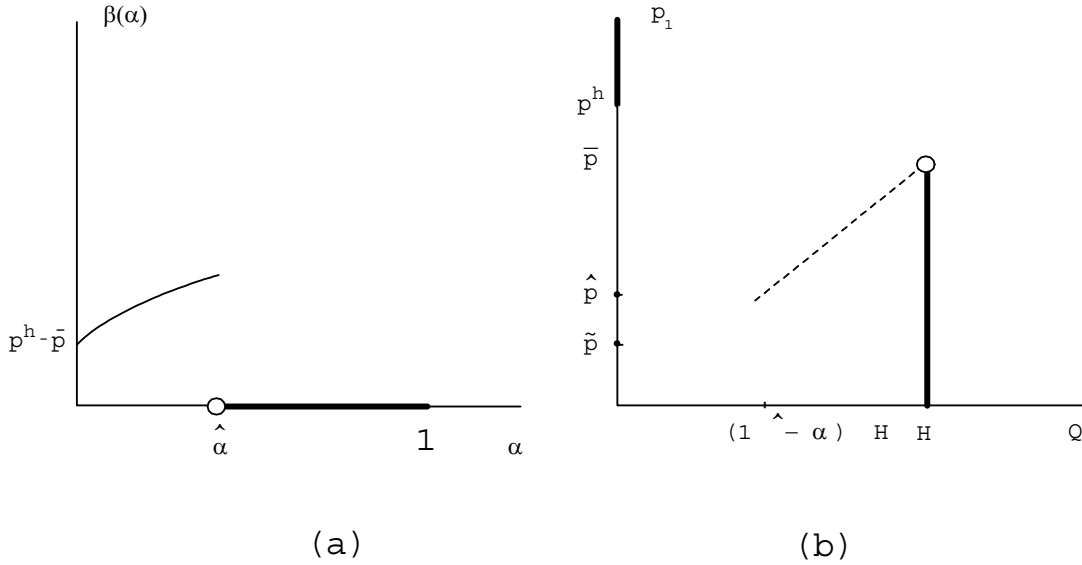
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Figure 2: The market with scalpers. (a) Benefit of waiting. (b) Demand.

If scalpers can enter the market, then $\alpha = 1$ is not an equilibrium for prices $p_1 \leq \bar{p}$. At any price $p_1 < \bar{p}$, high types strictly prefer to buy in the first period rather than wait, regardless of whether $\alpha = 0$ or $\alpha = 1$. Interior values of α ($0 < \alpha < 1$) cannot be *stable* equilibria, for the same reason as in the model without scalpers. At $p_1 = \bar{p}$, high types prefer to buy in the first period if $\alpha = 1$, and they are indifferent as to when they buy if $\alpha = 0$. However, $\alpha = 0$ is not a stable equilibrium at $p_1 = \bar{p}$, because other high types would want to mimic the deviation if a positive measure of buyers were to deviate by waiting.

We now show that there is no equilibrium where the first-period price satisfies $\bar{p} \leq p_1 < p^h$. If $\bar{p} < p_1 < p^h$, then in equilibrium it cannot be the case that $\alpha \leq \hat{\alpha}$. When $\bar{p} < p_1 < p^h$ and $\alpha \leq \hat{\alpha}$, the payoff of waiting is strictly greater than the payoff of buying in the first period. High types who were "supposed to buy" would want to deviate by waiting. Similarly, it cannot be the case that $\alpha > \hat{\alpha}$ because then the payoff of buying is strictly greater than the payoff of waiting. Again, if $p_1 = \bar{p}$ there is no stable equilibrium. If $p_1 = p^h$, on the other hand, the only stable equilibrium is $\alpha = 1$.

If the monopoly sets the first-period price at $\bar{p} - \delta$ (where δ is an arbitrarily small positive number), it sells to all high types in the first period, and its total revenue from sales in both periods is $(\bar{p} - \delta) H + (T - H)$. If the monopoly sets the first-period price above p^h, it sells all tickets to scalpers in the second period, and obtains the revenue $p^s(1)$. Using Equation (3),

we find that the monopoly prefers to sell to scalpers rather than setting $p_1 = \bar{p} - \delta$ if and only if

$$\phi > \frac{DH(L + H - 1) + L}{L(1 + HD)}. \tag{8}$$

We summarize these results in the following:

Proposition 2 *Under Assumption 2, all high types buy in the first period if $p_1 < \bar{p}$ (the limit price) and none buy if $p_1 \geq p^h$. The monopoly prefers to sell all tickets through scalpers rather than selling to high types at the limit price if and only if scalpers are sufficiently efficient; i.e., if and only if ϕ satisfies Equation (8).*

3.3 A comparison

The monopoly's expected demand function is indeterminate over a range of prices with or without scalpers. However, the source of the problem is different in the two settings. In the absence of scalpers, there are two equilibrium levels of demand for a range of prices $\tilde{p} < p_1 < \bar{p}$. At these prices, the relation between the monopoly's profit and price is a correspondence rather than a function. Consequently, we cannot determine the monopoly's optimal first-period price. With scalpers, the equilibrium, conditional on the price, is unique if it exists. However, for a range of prices, $\bar{p} \leq p_1 < p^h$, the equilibrium does not exist. For these prices, monopoly profits are not defined. Nonetheless, we can show that the ability of scalpers to enter the market increases monopoly profits, reduces high type consumer welfare, and has ambiguous effects on efficiency.

We now show that scalpers do not reduce and may increase monopoly profits. If the monopoly charges $\bar{p} - \delta$ and scalpers cannot enter, the supremum monopoly profit is $\bar{p}H + (1 - H)$ when all high types buy. When scalpers can enter, the monopoly is guaranteed approximately the supremum level of profit, because the threat of scalpers eliminates the equilibrium where all high types wait. If ϕ satisfies the inequality (8), the monopoly does even better by selling to scalpers.

Scalpers reduce consumer welfare. We noted in Proposition 1 that consumer welfare without scalpers is greatest in the equilibrium where all high types wait ($\alpha = 1$). Their surplus is smallest when the monopoly charges slightly less than $\bar{p}$ and all high types buy in the first period. If the inequality (8) is not satisfied, the monopoly can charge (approximately) $\bar{p}$ and eliminate the equilibrium $\alpha = 1$. Consumers get the minimum payoff they would have received in the absence of scalpers. If the inequality (8) is satisfied, consumers receive zero surplus.

With scalpers, all high types get tickets, and the only social loss arises from transactions costs. A larger ϕ corresponds to a smaller transactions cost. If ϕ does not satisfy (8), scalpers do not enter in equilibrium, so the equilibrium level of transaction costs is zero. Therefore, the supremum of efficiency loss arises when the inequality in (8) is replaced by an equality, so that inefficient scalpers enter. The efficiency loss is $1 - \phi$ times the scalpers' revenue, which equals

$$\left(1 - \frac{DH(L + H - 1) + L}{L(1 + HD)}\right)\left(p^h H + (1 - H)\right).$$

Without scalpers, the efficiency loss is greatest in the equilibrium $\alpha = 1$. The loss equals the number of high types who do not get tickets, $(1 - \theta)H$, times the efficiency loss for each ticket bought by a low type rather than a high type, D:

$$DH\frac{H + L - 1}{H + L}.$$

The difference in the efficiency loss in these two cases is a complicated function of the parameters. However, the maximum efficiency loss in the absence of scalpers increases with L. There is a smaller chance that a high type obtains a ticket because of the greater competition for tickets. To illustrate that the ability of scalpers to enter may increase or decrease the efficiency loss, we let $p^l = 1$, $p^h = 2$, $T = 1$, and $H = 0.2$, so that $L > 0.8$ to ensure that there is excess demand. For these parameters, the presences of scalpers increases the maximum efficiency loss if and only if $0.8 < L < 1.7$. Here, scalpers benefit the monopoly and harm high-type consumers. Scalpers may reduce social welfare if they are only "moderately efficient" – if (8) is "barely satisfied" – and if the excess demand for tickets (at price p^l) is relatively small.

4 Two-period model without common knowledge

We now drop the assumption that high types have common knowledge about the value of a ticket. The absence of common knowledge removes the indeterminacy and nonexistence of equilibria that we described in the previous section. The change also makes the predictions of the model more plausible.

We first show that, in the case where scalpers are prohibited, there is a unique equilibrium for a given first-period price. In this equilibrium, high types buy if and only if they receive a signal of quality that exceeds a critical threshold. This threshold is an increasing function of the first-period price. We refer to this type of equilibrium as a "threshold equilibrium." We have

a weaker result in the case where scalpers are allowed to enter. In that case, we cannot show that the high types' equilibrium behavior (conditional on first-period price) is unique; however, we show that there is a unique equilibrium *within the class of threshold equilibria*. We can compare these two equilibria in order to determine the effect of scalpers.

We begin with a description of the high types' first-period uncertainty about the value of a ticket. The true value for type i is γp^i, $i = h, l$. Before sales in the first period, potential customers view γ as a uniformly distributed random variable. The support of this distribution is wide enough that there exist "dominance regions." That is, for the range of prices that the monopoly might use, the support of the prior distribution includes small enough values of γ such that not buying is a dominant strategy for high types; the support also includes large enough values of γ such that buying is a dominant strategy.

Each high-type buyer receives a signal η about the quality of the performance (and hence the value of the ticket); η is drawn from a uniform distribution:

$$\eta \sim U[\gamma - \epsilon, \gamma + \epsilon],$$

where ϵ is a measure of the amount of uncertainty or heterogeneity of information. The distribution of η is common knowledge, but the value of the individual's signal is private information, and γ is unknown to all buyers. Whether the individual decides to buy or to wait and attempt to obtain a cheaper ticket in the second period depends on the first-period price, the individual's private signal, and this person's beliefs about what other agents will do.

We assume that buyers do not regard p_1 as a signal about quality. In a more general model, buyers would recognize that the price reflects the monopoly's information about quality, this inference would be informative (would change the equilibrium), and the monopoly would understand the extent to which the price reveals information.[4] However, our simplifying assumption that p_1 is uninformative is correct provided that buyers' posterior distribution on quality after observing p_1 (and before obtaining their private signal) is uniform and contains the two dominance regions.

At the beginning of the second period, the true value of γ is revealed, and the fraction of

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[4]This generalization would make the model extremely complicated, probably intractable. In addition, allowing price to be an informative signal would create another source of indeterminacy. In general, the requirement that the buyers have consistent beliefs about the monopoly's information is not strong enough to result in a unique equilibrium. Thus, allowing this generalization would undermine our reason for studying the model without common knowledge.

high types who waited, α, is also public information. Thus, the analysis of the second period is the same as in the complete information game, except that γp^i replaces p^i, $i = h, l$. The outcome in the second period depends on α and on whether scalpers can enter.

4.1 The buyers' problem without scalpers

If scalpers cannot enter in the second period, buyers know that the second-period price will be $\gamma p^l = \gamma$. In period 1, high-type buyers receive a signal, observe the first-period price, and decide whether to buy.

Let $\pi(\eta)$ be a decision rule; $\pi(\eta)$ is the probability that a high type who receives the signal η decides to wait (i.e., not buy in the first period). This class of decision rules includes mixed strategies, but we show that the unique *equilibrium decision rule* is a pure strategy. If the equilibrium decision rule is $\pi(\eta)$, the fraction of high types who do not buy when the value of the unknown parameter is γ is[5]

$$\alpha = \alpha\left(\gamma; \pi\right) = \int_{\gamma-\epsilon}^{\gamma+\epsilon} \pi(\eta)d\eta. \tag{9}$$

If the value of the unknown parameter is γ, then the fraction $\alpha(\gamma; \pi)$ of high types wait, and the payoff in the second period is $\gamma B^0(a(\gamma; \pi)) \geq 0$. The posterior distribution of γ, conditional on the signal η, is uniform over $[\eta - \epsilon, \eta + \epsilon]$. Given a signal η and given that the buyer expects other high types to use the equilibrium strategy π, the expected value of waiting is

$$V^0(\eta; \pi) = \frac{1}{2\epsilon} \int_{\eta-\epsilon}^{\eta+\epsilon} \gamma B^0(a(\gamma; \pi))d\gamma. \tag{10}$$

The expected value of buying in the first period is $\eta p^h - p_1$ The *advantage of waiting* is the difference in the expected benefit of waiting and the expected value of buying the first period:

$$A^0(\eta, p_1; \pi) = V^0(\eta; \pi) - \left(\eta p^h - p_1\right). \tag{11}$$

The function $\pi(\eta)$ represents a general strategy. We now consider a particular type of strategy – a threshold strategy – that we denote $I_k(\eta)$. This threshold strategy is a step function:

$$I_k(\eta) = \left\{ \begin{array}{l} 1 \text{ if } \eta < k \\ 0 \text{ if } \eta \geq k \end{array} \right\}. \tag{12}$$

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[5]This expression is valid for values of γ more than ϵ distance from the boundaries of the support of the prior for γ. Although this qualification applies to the subsequent integrals in the text, we do not repeat it. However, we are careful about these limits of integration in the proofs.

The parameter k is the threshold signal. If $\pi(\eta) = I_k(\eta)$, then high types buy if and only if they receive a signal greater than or equal to k, and the fraction of high types who wait is (using Equation (9))

$$\alpha(\gamma; I_k) = \begin{cases} 1 \text{ if } \gamma + \epsilon < k \\ \frac{k-\gamma+\epsilon}{2\epsilon} \text{ if } \gamma - \epsilon \le k \le \gamma + \epsilon \\ 0 \text{ if } k < \gamma - \epsilon \end{cases}. \tag{13}$$

We use a variation of a proof in Morris and Shin (1998) to show that the unique equilibrium decision rule $\pi(\eta)$ is a step function $I_k(\eta)$, and that the threshold k is unique. We characterize the equilibrium value of k as a function of an arbitrary first-period price, p_1, the amount of uncertainty, ϵ, and the other parameters of the model. We state the result in terms of functions ρ^0 and ϖ^0, which depend only on exogenous parameters. The Appendix defines these functions.

Proposition 3 *If scalpers are unable to enter in the second period and Assumption 1 holds, there is a unique equilibrium to the game with uncertainty. In this equilibrium, high types buy in the first period if and only if they receive a sufficiently high signal. That is, $\pi^*(\eta) = I_k$. The critical signal k^{0*} is given by the formula*

$$k^{0*} = -\frac{\varpi^0 \epsilon + p_1}{\rho^0}; \quad \rho^0 < -1, \quad \varpi^0 < 0. \tag{14}$$

This threshold is an increasing function of the first-period price and a decreasing function of the uncertainty parameter, ϵ.

4.2 The buyers' problem with scalpers

We now turn to the equilibrium where scalpers are allowed to enter in the second period. In the second period, when the quality, γ, and the number of high types who do not yet have tickets, αH, are common knowledge, the equilibrium is the same as in Section 2. The monopoly wants to sell to scalpers if and only if $\alpha > \hat{\alpha}$, defined in Equation (4). Our assumption that γ scales both p^h and p^l implies that $\hat{\alpha}$ is independent of γ.

For an arbitrary strategy $\pi(\eta)$, Equation (9) gives the fraction of high types who wait. If a high type receives a signal η and other high types use the strategy π, the expected value of waiting is

$$V(\eta; \pi) = \frac{1}{2\epsilon} \int_{\eta-\epsilon}^{\eta+\epsilon} \gamma B(a(\gamma; \pi)) d\gamma. \tag{15}$$

Equation (7) defines the function $B(\cdot)$. The expected advantage of waiting is

$$A(\eta, p_1; \pi) = V(\eta; \pi) - \left(\eta p^h - p_1\right). \tag{16}$$

Because $B(\alpha) \leq B^0(\alpha)$ and the inequality is strict for $\alpha > \hat{\alpha}$, it follows that $A(\eta, p_1; \pi) \leq A^0(\eta, p_1; \pi)$ and that the inequality holds strictly for some values of η. High types wait in the first period if and only if the value of waiting is positive. Therefore, *for a given strategy π and a given price p_1, the potential entry of scalpers in the second period may increase first-period sales.* Thus, it is reasonable to expect that the ability of scalpers to enter causes the first-period demand function to shift out. However, the potential presence of scalpers changes the equilibrium decision rule π^*, so the comparison is not straightforward.

If $\pi = I_k$ then α is given by Equation (13). In this case, using Equation (4) we have

$$\alpha \leq \hat{\alpha} \iff \gamma \geq k + \epsilon(1 - 2\hat{\alpha}) \equiv \hat{\gamma}. \tag{17}$$

When agents use the strategy I_k, high types who do not buy in the first period obtain positive surplus in the second period (scalpers do not enter) if and only if the true quality exceeds a threshold level $\hat{\gamma}$. If γ is less than this threshold, many agents wait because they receive low signals in the first period. Consequently, a large number of high types want to buy in the second period. Therefore, the monopoly sells to scalpers so as to capture the potential surplus of the remaining high types (less transaction costs).

Proposition 3 showed that when scalpers are prohibited from entering the market, the unique equilibrium strategy is a threshold strategy. The threshold strategy has two reasonable characteristics. First, it is *monotonic* in the signal: $[\pi(\eta) - \pi(\eta')]/[\eta - \eta']$ has the same sign for all $\eta \neq \eta'$. For example, if agents are willing to buy when they receive a particular signal, they will also buy when they receive a higher signal. Second, the candidate is a *symmetric pure strategy*: Agents who receive the same signal behave in the same way and they do not randomize.

The only other monotonic symmetric pure strategy reverses the inequalities, so that $\pi(\eta) = 0$ for $\eta \leq k$ and $\pi(\eta) = 1$ for $\eta > k$. However, this alternative cannot be an equilibrium.[6] Thus, I_k is the only candidate within the class of monotonic symmetric pure strategies.

We have the following description of the equilibrium when scalpers are allowed to enter. This description uses ϖ and ρ, functions of exogenous parameters defined in the Appendix.

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[6]Under the alternative, a high type who receives a sufficiently high signal is certain that no other high types will buy, so the expected benefit of waiting is $V^0(\eta, \pi) = \eta B^0(1)$. The expected cost of waiting is $\eta p^h - p_1$, which is greater than $\eta B^0(1)$ for large η. Hence, a high type who receives a very high signal does not want to wait. Thus, the alternative is not an equilibrium.

Proposition 4 *Suppose that scalpers are able to enter in the second period, Assumption 2 holds, and we restrict the equilibrium to be a member of the class of monotonic symmetric pure strategies. That is, we assume that high types use threshold strategies. Under these assumptions, there exists a unique equilibrium threshold, k^* (conditional on p_1). In this equilibrium, high types buy in the first period if and only if they receive a signal $\eta \geq k^*$. The equilibrium threshold k^* is linear in p_1 and ϵ.*

$$k^* = -\frac{\varpi \epsilon + p_1}{\rho}; \quad \rho < -1 \tag{18}$$

When scalpers are allowed to enter the market, $B(\alpha)$ is nonmonotonic. This non-monotonicity means that the proof used for Proposition 3 does not carry over to the case where scalpers are permitted. We cannot rule out the possibility that mixed strategies exist, and therefore we cannot show that the equilibrium is unique in this case.[7] In contrast, when scalpers are prohibited the equilibrium is unique (Proposition 3). When scalpers are permitted, we have a weaker result: uniqueness within the class of threshold strategies.

4.3 The demand function

When the payoffs are not common knowledge ($\epsilon > 0$), the first-period demand function is piecewise linear. For prices above the choke price, $\alpha = 1$, so demand is 0. For prices below the limit price, $\alpha = 0$, so demand is H. For intermediate prices, demand equals $[1 - \alpha(\gamma; I_k)] H$. The function $\alpha(\gamma; I_k)$, given by Equation (13), is linear in k. The equilibrium value of k, given by either Equation (14) or (18) depending on whether scalpers can enter, is linear in p_1. Therefore, the inverse demand is linear in p_1 at intermediate prices. Solving for p_1 in the equations $\alpha = 0$ and $\alpha = 1$ gives the limit price p^{L0} and the choke price p^{C0} when scalpers are not allowed to enter:

$$p^{L0} = \left(\rho^0 - \varpi^0\right)\epsilon - \gamma\rho^0; \quad p^{C0} = -\left(\varpi^0 + \rho^0\right)\epsilon - \gamma\rho^0. \tag{19}$$

(We obtain the limit and choke price when scalpers are allowed to enter by replacing ρ^0 and ϖ^0 by ρ and ϖ.) As $\epsilon \to 0$, $p^{L0} \to p^{C0}$. That is, as the amount of uncertainty becomes small, the first-period demand curve becomes flatter at prices above the limit price. In the limit as

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[7]The claim contained in Step b.i of the proof of Proposition 3 does not follow if the payoff function $B(\alpha)$ is nonmonotonic. Thus, although the ability of scalpers to enter in the second period leads to a unique equilibrium under common knowledge about payoffs, it may lead to nonuniqueness of equilibria without common knowledge.

$\epsilon \to 0$, demand is perfectly elastic at the limit price, which is $-\gamma \rho^0$ when scalpers cannot enter the market and $-\gamma \rho$ when scalpers are allowed to enter.

We can compare these results to those in the model where high types have common knowledge. Equation (6) gives the values of the limit and choke prices ($\bar{p}$ and $\tilde{p}$) under certainty when scalpers are prohibited from entering the market. For any price below $\bar{p}$, there is an equilibrium in which all high types buy in the first period. For any price below $\tilde{p}$, this equilibrium is unique. Thus, $\bar{p}H + (1 - H)$ is the monopoly's supremum of the set of equilibrium payoffs under certainty, when scalpers are prohibited. By charging $\tilde{p}$, the monopoly is guaranteed the reservation level of profit $\tilde{p}H + (1 - H)$.

In order to compare the outcomes with and without common knowledge, we set $\gamma = 1$. We have

Proposition 5 *Suppose that scalpers cannot enter the market and that the monopoly knows the true quality (γ). (i) Given a small amount of buyer uncertainty ($\epsilon \approx 0$), the profit of a monopoly that charges the limit price is higher than its reservation level under common knowledge ($-\rho^0 > \tilde{p}$), but lower than the supremum under common knowledge ($-\rho^0 < \bar{p}$). (ii) Greater uncertainty (larger ϵ) lowers the limit price, ($\rho^0 - \varpi^0 < 0$).*

Simulations suggest that the first part of Proposition 5 also holds when scalpers can enter, but we have not been able to show this result analytically.

We can also compare the limit prices in the models with and without the potential entry of scalpers:

Proposition 6 *For a small level of uncertainty ($\epsilon \approx 0$), the limit price when scalpers are allowed to enter is higher than the limit price when scalpers are prohibited from entering:$-\rho > -\rho^0$.*

Under common knowledge, scalpers have no effect on the limit price, but they remove the indeterminacy of the equilibrium (Proposition 2). The potential for scalpers eliminates the possibility that sales will be low in the first period, but has no effect on the maximum equilibrium level of profits unless scalpers actually enter in equilibrium. With an arbitrarily small amount of uncertainty, the demand functions with or without scalpers are well defined, so there is a unique equilibrium for a given price; the monopoly that knows γ faces no uncertainty about the equilibrium. However, scalpers increase the limit price, thereby increasing profits under limit pricing, at least for small levels of uncertainty (Proposition 6).

5 The monopoly's problem with buyer uncertainty

For any first-period price, the buyers' lack of common knowledge about the value of the ticket induces a unique equilibrium if there are no scalpers, and a unique equilibrium within a restricted class if there are scalpers. In the interest of simplicity, we consider only the case where the monopoly (but not buyers) knows the value of γ. We first study the monopoly's problem without scalpers, and then with scalpers.

5.1 The monopoly's problem without scalpers

Monopoly revenue is

$$R^0 = (1 - \alpha) H p_1 + \gamma \left[1 - (1 - \alpha) H \right] = (1 - \alpha) H (p_1 - \gamma) + \gamma \tag{20}$$

where α is given by (13) and k is given by (14).

We need only consider first-period prices between the limit and the choke price, i.e. prices that satisfy $p^{L0} \leq p_1 \leq p^{C0}$. Since all high types buy at $p_1 < p^{L0}$, revenue increases over that range of prices. For $p_1 > p^{C0}$ first-period demand is 0, so revenue is constant over that range.

For first-period prices strictly between the limit and the choke price, the monopoly revenue is a concave quadratic function. Define $\breve{p}$ as the price that maximizes R^0 *ignoring the constraints* $0 \leq \alpha \leq 1$:

$$\breve{p} = 0.5 \left[(1 - \rho) \gamma - (\rho + \varpi) \epsilon \right].$$

Because revenue is increasing at prices below the limit price and revenue is constant at prices above the choke price, the monopoly sets $p_1 = p^{C0}$ if and only if $\breve{p} \geq p^{C0}$. Using the definitions of $\breve{p}$ and p^{C0}, we can rewrite this last inequality as $-[\varpi + \rho]/[1 + \rho] > \gamma/\epsilon$. This equality is never satisfied, since $\varpi < 0$ and $\rho < -1$. Therefore, the monopoly always makes some sales in the first period: It never sets the choke price.

The monopoly sets $p_1 = p^{L0}$ if and only if $\breve{p} \leq p^{L0}$, and it sets $p_1 = \breve{p}$ if and only if $\breve{p} > p^{L0}$. Using the definitions above, we restate these conclusions in the following proposition.

Proposition 7 *The monopoly uses the limit price (sells to all high types in the first period) if the amount of uncertainty is small. It sets the price $\breve{p}$ (sells to some, but not to all high types in the first period) if the amount of uncertainty is sufficiently large. The optimal first-period price*

is

$$p_1^* = \left\{ \begin{array}{l} p^{L0} \text{ if } \frac{1.5\rho^0 - .5\varpi^0}{.5(\rho^0+1)} \leq \frac{\gamma}{\epsilon} \\ \check{p} \text{ if } \frac{\gamma}{\epsilon} < \frac{1.5\rho^0 - .5\varpi^0}{.5(\rho^0+1)} \end{array} \right\}, \tag{21}$$

with the critical ratio $[1.5\rho^0 - .5\varpi^0]/[.5(\rho^0+1)] > 0$ and independent of ϵ.

To illustrate that under reasonable circumstances the monopoly wants to choose a price greater than the limit price, which induces some but not but not all high types buy in the first period, we calculate the equilibrium value of α for the following:

Example 1 *Let $D = 1 = \gamma$ and $E = .5$ (i.e. the total number of potential buyers is 50 percent greater than the number of seats). The value $D = 1$ and Assumption 1 imply that $H < .5$. Figure 3 shows the equilibrium value of α for $0.1 < H < 0.5$ and $0.1 < \epsilon < 0.4$.*
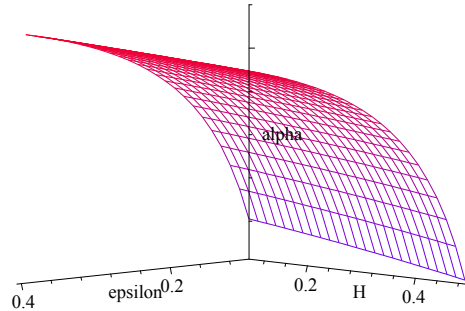

Figure 3: Equilibrium α without scalpers for $D = 1 = \gamma$, $E = .5$

The only possible source of inefficiency in this model is that some high types might end up without tickets. The fraction of high types who do not buy tickets in either the first or the second period is $\alpha(1 - \theta)$, where θ is given by Equation (2) with $s = 0$ and $L^* = L$. The number of high types who do not obtain tickets is $\alpha(1 - \theta)H$, and the expected social loss for each ticket that goes to a low type rather than to a high type is γD. Therefore, the expected social loss (the amount of inefficiency) is

$$\Omega \equiv \alpha(1 - \theta)HD\gamma = \alpha\left(1 - \frac{1 - (1-\alpha)H}{E + 1 - (1-\alpha)H}\right)HD\gamma.$$

The social loss Ω is increasing in α.

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For parameter values such that the monopoly limit prices, $\alpha = 0$, the equilibrium is efficient, so $d\Omega/d\epsilon = 0$. However, a change in ϵ changes the limit price and hence monopoly profit and the welfare of high types. When the monopoly does not limit price, a change in ϵ changes α, leading to a change in efficiency:

Proposition 8 *(i) For values of ϵ small enough that they satisfy $[1.5\rho^0 - .5\varpi^0]/[.5\,(\rho^0 + 1)] \leq \gamma/\epsilon$, monopoly profit is decreasing in ϵ and welfare of high types is increasing in ϵ. (ii) For values of ϵ large enough that they satisfy $[1.5\rho^0 - .5\varpi^0]/[.5\,(\rho^0 + 1)] > \gamma/\epsilon$, the level of efficiency (social welfare) is decreasing in ϵ.*

5.2   The monopoly's problem with scalpers

With scalpers, the monopoly's revenue is[8]

$$
\begin{aligned}
R\,(p_1) &= p_1\,(1 - \alpha)\,H + \gamma\,[1 - (1 - \alpha)\,H]\max\{p^s(\alpha), 1\} \qquad (22)\\
&= p_1\,(1 - \alpha)\,H + \gamma\max\left[1 - (1 - \alpha)\,H,\ \phi\left(1 - \left(1 - \alpha p^h\right)H\right)\right].
\end{aligned}
$$

The first term is the revenue from sales to high types in period 1, and the second term is the revenue from sales to scalpers or to high and low types in period 2. The second line of the equality uses the definition of p^s in Equation (3). The equilibrium value of α, a function of p_1, is obtained using Equations (13) and (18).

Proposition 7 shows that the monopoly wants to limit price in the absence of scalpers when ϵ is small, and Proposition 6 shows that scalpers increase the limit price when ϵ is small. With scalpers, the monopoly is able to limit price, but it is not necessarily optimal to do so. Its profit under the optimal policy is at least as great as it would be if it were to limit price. Consequently, we have

Corollary 1 *For small ϵ, the ability of scalpers to enter increases monopoly profit and reduces welfare of high types.*

The monopoly could use four types of strategies in the first period. It could use the limit price or the choke price, p^L or p^C, in which case it either sells to all high types or no high

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[8]The definition of the equilibrium second-period price under scalpers, Equation (4), shows that if both p^h and p^l are scaled up by γ, then p^s is also scaled up by γ. Therefore, we can factor out γ in the expression for second-period profit.

types. We define the entry price p^E as the price that results in the number of first-period sales that leaves the monopoly indifferent between selling to scalpers or directly to final buyers in the second period. That is, p^E is the price that induces $\alpha = \hat{\alpha}$, as defined in Equation (4). For $p^L < p_1 < p^E$, there are some high types remaining in the second period, but too few for the monopoly to want to use scalpers. For $p^E < p_1 < p^C$, some high types buy in the first period, but enough remain for the monopoly to want to use scalpers in the second period.

The complexity of the second-period revenue function makes it difficult to obtain analytic results. However, the following example illustrates that when ϵ is large it is optimal to sell to some but not all high types in the first period, and to induce scalpers to enter in the second period.

Example 2 *Let $D = L = 1$, $H = .4$, and $\phi = .9$. The high type's willingness to pay is twice the low type's willingness, there are enough low types to exactly fill the venue, enough high types to fill 40% of the venue, and the scalper captures 90% of the rent from price discrimination. The monopoly uses the limit price if $\epsilon < 0.12\gamma$ and sets p_1 such that $p^E < p_1 < p^C$ if $\epsilon > 0.12\gamma$. It is never optimal to use either the choke price or a price between p^L and p^E. For $\epsilon = 0.182\gamma$, the monopoly payoff at the local maximum without scalpers is 1.21γ and the payoff at the local maximum with scalpers is 1.28γ, a 6% increase in profit, so the monopoly induces scalpers to enter. The fraction of high types who wait is $\alpha = 0.7$.*

6   Summary and conclusions

The presence of resellers may aid the monopoly provider of a nondurable good in several ways. In a one-period market with full information, scalpers, touts, bucket shops, or other resellers may enable a monopoly that cannot price discriminate to earn nearly the price-discrimination level of profit. The lower the scalpers' transaction costs, the closer the monopoly's profit is to the maximum that the monopoly could achieve if it could perfectly price discriminate. The scalpers allow the monopoly to effectively bundle the tickets for both types of customers.

In a two-period model with common knowledge, the monopoly's cannot solve its maximization problem because it does not know a portion of its expected demand function. This indeterminacy arises with or without scalpers, but the source of the problem differs. In the absence of scalpers, there are two possible equilibrium outcomes for prices between the choke and limit prices. Here, we cannot determine the monopoly's optimal first-period price. With

scalpers, the equilibrium conditional on the price is unique if it exists. However, the equilibrium does not exist for a range of prices where monopoly profit is not defined. Monopoly profit when scalpers can enter is no less – and may be greater – than the maximum equilibrium profit in the absence of scalpers. The monopoly sells all the tickets or none of the tickets to scalpers. When price discriminating scalpers enter, the welfare of customers with high willingness to pay falls, and social welfare may rise or fall.

With almost common knowledge of payoffs, there is a unique equilibrium for a given price when scalpers cannot enter the market. If the amount of uncertainty is small but not zero (and scalpers cannot enter), the monopoly wants to sell to all customers with a high willingness to pay in the first period (i.e. to limit price). The resulting level of monopoly profit is greater than its reservation level under certainty, but less than the highest possible profit under certainty. Greater uncertainty lowers the monopoly's limit price and benefits consumers. The ability of scalpers to enter the market increases the limit price, increasing monopoly profits and reducing consumer welfare, at least for small levels of uncertainty.

Many jurisdictions have anti-scalping laws (Williams 1994). We identified a plausible circumstance (a small degree of heterogeneity of information) where these laws benefit consumers without reducing efficiency. However, the model is sufficiently complicated that other possibilities also arise. For example, if the amount of uncertainty is sufficiently large and scalpers are prohibited, the monopoly sells to only a fraction of people with high willingness to pay in the first period. This outcome is not socially efficient. If scalpers are permitted to enter and their transactions cost is negligible, they are a near-perfect agent for price discrimination so the outcome is approximately efficient. In this case, prohibiting scalpers lowers social welfare.

In our model, the ability of scalpers to enter does not benefit consumers. This negative result is a consequence of our assumption that scalpers capture all of the surplus when dealing with buyers. If this surplus were shared, the potential entry of scalpers into the second period market might benefit some consumers with high reservation prices.

7 Appendix:Proofs

Proof. (Lemma 1) In view of Assumption 1, $p_2 \geq p^h$ is never optimal, so we need only consider second-period prices less than p^h. At $p_2 < p^h$ all high types try to obtain tickets from the monopoly. If the monopoly sets $p_2 = p^l$ its revenue in the second period is $p^l [T - (1 - \alpha) H]$ and $L^* = L$.

If $p_2 > p^l$, then $L^* = 0$. If it is optimal to set $p_2 > p^l$ then the monopoly must sell some tickets to scalpers: $s > 0$. (Otherwise the monopoly revenues are less than $p^h \alpha H$, which by Assumption 1 is less than $p^l (T - (1 - \alpha) H)$, the level of revenue obtained by setting $p_2 = p^l$.)

If $p_2 > p^l$ and $s > 0$, it must be the case that $s > (1 - \theta)\alpha H$. If the last inequality did not hold, second period monopoly revenue is again less than $p^h \alpha H$. Therefore, if it is optimal for the monopoly to set $p_2 > p^l$, scalpers have tickets left over after selling to high types. In other words, scalpers sell $(1 - \theta)\alpha H$ tickets to the high types who did not buy in the first period and who were unable to obtain discounted tickets from the monopoly in the second period. Scalpers sell their remaining tickets to low types.

The zero-profit condition for scalpers is therefore

$$\phi \left[p^h (1 - \theta)\alpha H + p^l (s - (1 - \theta)\alpha H) \right] - p_2 s = 0. \tag{23}$$

Substituting Equation (2) and $L^* = 0$ into (23) and rearranging, we have

$$p_2 = \phi p^h - \frac{\phi D (T - H)}{s} \tag{24}$$

Equation (24) shows that the price that is consistent with 0 profits for scalpers is an increasing function of sales to scalpers, s. When the monopoly sells more tickets to scalpers and fewer to high types, it increases the likelihood that scalpers will be able to sell to high types. Consequently, scalpers are willing to pay more for a ticket. If the monopoly wants to sell to scalpers, it would like to sell them all the remaining tickets. That is, if it is optimal to set $p_2 > p^l$, then $s = T - (1 - \alpha)H$ is the optimal level of sales to scalpers. Substituting this value of s into Equation (24) and rearranging gives equation (3).

We now need to show that the monopoly can achieve $s = T - (1 - \alpha)H$ by setting $p_2 = p^s(\alpha)$. When $L^* = 0$ and S scalpers and αH high types compete for tickets, the probability that any individual obtains a ticket is $[T - (1 - \alpha) H]/[S + \alpha H]$, so the number of tickets that scalpers obtain is

$$s = \frac{T - (1 - \alpha) H}{S + \alpha H} S.$$

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When $p_2 = p^s(\alpha)$, $S = \infty$ is consistent with 0 profits for scalpers, and $S = \infty$ implies that $s = T - (1 - \alpha)H$.

By setting $p_2 = p^s(\alpha)$ the monopolist is able to insure that scalpers capture all remaining tickets. Obviously, if the monopoly is able to discriminate by selling directly to scalpers, it can achieve the same outcome. ■

Proof. (Proposition 3) We prove the proposition in two parts. In part (a) we show that there exists a unique equilibrium within the class of step functions and that k^* is increasing in the first-period price. In part (b) we follow an argument in Morris and Shin (1998)

Part (a): Within the class of equilibrium candidates of the form of Equation (12), there exists a unique equilibrium.

Step (i): Substituting Equation (13) in Equation (11), with $\pi = I_k$, we learn that $A^0 > 0$ for sufficiently small η and $A^0 < 0$ for sufficiently large η.

Step (ii): Next we establish that

$$\frac{\partial A^0}{\partial \eta} < 0. \tag{25}$$

To obtain this inequality, we consider the three cases where $\eta < k - \epsilon$, $k - \epsilon \leq \eta \leq k + \epsilon$, and $k + \epsilon < \eta$. For each of these cases, a straightforward calculation (details omitted) establishes that $\partial A^0 / \partial \eta < 0$. Since a buyer waits if and only if $A^0 > 0$, Equation (25) implies that the optimal decision rule of a high type is to buy if and only if that person receives a sufficiently high signal, given that the person believes that other high types are buying if and only if they receive a sufficiently high signal.

Step (iii): Next we show that there is a unique solution to the equation $A^0(k, p_1; I_k) = V^0(k; I_k) - (kp^h - p_1) = 0$. That is, there is a unique value of k such that I_k is the optimal decision rule for a high type, when all other high types use I_k. Integrating the expression in Equation (10) we obtain

$$\begin{aligned}
V^0(k; I_k) &= Z^0 k + \varpi^0 \epsilon \tag{26} \\
Z^0 &= D\left(1 - \ln(H + L) + \frac{1-L}{H}\ln(H + L) - \frac{\ln L}{H}(1 - L - H)\right) \\
\varpi^0 &= \frac{D}{H}[2H + (H + 3L - 1)(\ln L - \ln(L + H)) \\
&\quad + \frac{2}{H}(1 - L)L(\ln(L + H) - \ln L) - H]. \tag{27}
\end{aligned}$$

Because this function is linear in k, A^0 is linear in k, and there exists a unique solution to $A^0(k, p_1; I_k) = 0$.

Step (iv) We use Equation (26) to write

$$A^0(k, p_1; I_k) = \rho^0 k + \varpi^0 \epsilon + p_1, \tag{28}$$
$$\rho^0 \equiv \frac{(1 - L - H)\left[\ln(L + H) - \ln(L)\right]}{H}D - 1 < -1$$

Since $1 < L + H$ (there are more high and low type buyers than there are seats), $\rho < 0$ so A^0 is strictly decreasing in k. Therefore, k^*, the unique solution to $A^0(k, p_1; I_{k^*}) = 0$ is increasing in p_1.

(Step v) We want to show that $\varpi^0 < 0$. The formula above for ϖ^0 implies that $d\varpi^0/dD = -E\omega/H^2$, where (using $L = E + 1 - H$)

$$\omega \equiv (2 + 2E - H)(\ln(E + 1) - \ln(E + 1 - H)) - 2H.$$

$d\varpi^0/dD$ has the opposite sign as ω and $\varpi^0 = 0$ when $D = 0$. Therefore, in order to show that $\varpi^0 < 0$ for $D > 0$, it is sufficient to show that $\omega > 0$. Using the definition of ω, we see that $\omega = 0$ for $H = 0$. Therefore, it is sufficient to show that ω is increasing in H. We have

$$\frac{d\omega}{dH} = \frac{\varphi}{-E - 1 + H}$$
$$\varphi \equiv E\ln(E + 1) - E\ln(E + 1 - H) + \ln(E + 1) - \ln(E + 1 - H)$$
$$- H\ln(E + 1) + H\ln(E + 1 - H) - H.$$

Since $-E - 1 + H < 0$, it is sufficient to show that φ is negative for $H > 0$. Because $\varphi = 0$ when evaluated at $H = 0$, it is sufficient to show that φ is decreasing in H. We have

$$\frac{d\varphi}{dH} = \ln(E + 1 - H) - \ln(E + 1) < 0.$$

Part (b) Here we summarize the argument in Morris and Shin (Morris and Shin 1998) that shows that I_k is the unique equilibrium.

Step (i) (This is Lemma 1 in (Morris and Shin 1998).) For any two strategies $\pi(\eta)$ and $\pi'(\eta)$ such that $\pi(\eta) \geq \pi'(\eta)$, we have $\alpha(\gamma; \pi) \geq \alpha(\gamma; \pi')$ from Equation (9). Since $B(\alpha)$ is an increasing function, we conclude that $V(\eta; \pi) \geq V(\eta; \pi')$.

Step (ii) (This argument is the last part of Lemma 3 in (Morris and Shin 1998).) Given any equilibrium $\pi(\eta)$, define the numbers $\bar{\eta}$ and $\tilde{\eta}$ as

$$\bar{\eta} = \sup\{\eta \mid \pi(\eta) > 0\}$$
$$\tilde{\eta} = \inf\{\eta \mid \pi(\eta) < 0\}.$$

By these definitions,

$$\tilde{\eta} \leq \bar{\eta}, \tag{29}$$

which is Equation 6 in (Morris and Shin 1998). By continuity, when the signal $\tilde{\eta}$ is received, the advantage of buying must be at least as high as the advantage of waiting, so $A(\tilde{\eta}; \pi) \leq 0$ (Equation 7 in (Morris and Shin 1998)). Since $I_{\tilde{\eta}} \leq \pi$, Step (b.i) implies that $A(\tilde{\eta}; I_L) \leq A(\tilde{\eta}; \pi) \leq 0$. From Step (a.iv) we know that $A^0(k, p_1; I_k)$ is decreasing in k and that k^* is the unique solution to $A^0(k, p_1; I_k) = 0$. Thus, $\tilde{\eta} \geq k^*$ (Equation 8 in (Morris and Shin 1998)). A symmetric argument shows that $\bar{\eta} \leq k^*$. Thus, $\bar{\eta} \leq \tilde{\eta}$. Given this inequality and Equation (29), $\bar{\eta} = k^* = \tilde{\eta}$. Thus, I_k is the unique equilibrium. ∎

Proof. (Proposition 4) Step (i) (Existence of threshold equilibrium.) By assumption, we have restricted the set of possible equilibria to be in the set of threshold strategies. We explained in the text why it cannot be an equilibrium threshold strategy to wait when the signal is high. Consequently, we need only to show that there exists a threshold strategy in which agents wait if the signal is low. That is, if a particular high type believes that all other high types are using strategy I_k, the agent wants to buy if and only if that person receives a sufficiently high signal.

To demonstrate this claim, we study the function $A(\eta, p_1; I_k)$ over three intervals: $\eta < \hat{\gamma} - \epsilon$, $\eta > \hat{\gamma} + \epsilon$, and $\hat{\gamma} - \epsilon \leq \eta \leq \hat{\gamma} + \epsilon$. If $\eta < \hat{\gamma} - \epsilon$, then $V(\eta; I_k) = 0$ so $A(\eta, p_1; I_k) = -(\eta p^h - p_1)$, which is positive for sufficiently low η and is strictly decreasing in η. In the second region, where $\eta > \hat{\gamma} + \epsilon$, $B(\alpha(\gamma; I_k) = B^0(\alpha(\gamma; I_k))$ for all possible values of γ. Therefore over this region $A(\eta, p_1; I_k) = A^0(\eta, p_1; I_k)$. In the proof of Proposition 3.a.i (see Equation (25)) we showed that this function is decreasing in η and is negative for sufficiently large η. Finally, in the third case where $\hat{\gamma} - \epsilon \leq \eta \leq \hat{\gamma} + \epsilon$, we know that

$$A(\eta, p_1; I_k) = \frac{1}{2\epsilon} \int_{\hat{\gamma}}^{\eta + \epsilon} B^0(\alpha(\gamma; I_k)) d\gamma - \left(\eta p^h - p_1 \right),$$

where the lower limit of integration is $\hat{\gamma}$ rather than $\eta - \epsilon$ because $B = 0$ for $\gamma < \hat{\gamma}$. We conclude that

$$\frac{\partial A}{\partial \eta} = B^0(\alpha(\eta + \epsilon; I_k)) - p^h < B^0(1) - p^h < 0,$$

where the first inequality follows from the monotonicity of B^0 in α, and the monotonicity of α in γ. The second inequality follows from the definition of $B(1)$. Thus, we have shown that the optimal response to the strategy I_k is to buy if and only if the signal is sufficiently large.

Step (ii) (Uniqueness of threshold equilibrium.) Now we show that there is a unique solution to the equation $A(k, p_1; I_k) = V(k; I_k) - \left(k p^h - p_1 \right) = 0$. From Equation (17), we know

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that

$$1 > \hat{\alpha} > 0 \Longleftrightarrow k - \epsilon < \hat{\gamma} < k + \epsilon.$$

Since the first pair of inequalities is true by assumption, we know that $k - \epsilon < \hat{\gamma} < k + \epsilon$. Therefore

$$A(k, p_1; I_k) = \frac{1}{2\epsilon} \int_{\hat{\gamma}}^{k+\epsilon} B^0(\alpha(\gamma; I_k)) d\gamma - \left(kp^h - p_1 \right). \tag{30}$$

For the purpose of comparing the results in the case with and without scalpers, we replace the lower limit of integration with

$$\hat{\gamma} = k + \epsilon - 2\epsilon\beta; \quad \beta \equiv (1 - \phi) \frac{1 - H}{H(\phi D + \phi - 1)} \tag{31}$$

and we rewrite the integral in Equation (30) as

$$\frac{1}{2\epsilon} \int_{k+\epsilon(1-2\beta)}^{k+\epsilon} B^0(\alpha(\gamma; I_k)) d\gamma = \varpi\epsilon + Zk \tag{32}$$

where

$$\begin{aligned}
\varpi &= \frac{-D}{H^2}[H \ln L + H^2 \ln (H\beta + L) - 2L \ln (H\beta + L) - H \ln (H\beta + L) - 2L^2 \ln L \\
&\quad - 3LH \ln L + 2H\beta - H^2 \ln L + 2L^2 \ln (H\beta + L) + 2L \ln L - 3H^2\beta + H^2\beta^2 \\
&\quad + 3HL \ln (H\beta + L) - 2HL\beta] \\
Z &= D\left(\frac{1}{H}(-1 + L + H) \ln L + \frac{1}{H}(-L - H + 1) \ln (H\beta + L) + \beta \right)
\end{aligned}$$

Since $\hat{\gamma} > k - \epsilon$, it must be the case that $\beta < 1$. Also, if we set $\beta = 1$ rather than the value given in Equation (31), Equation (32) gives the expected benefit of waiting in the absence of scalpers. We use these facts below.

Substituting Equation (32) into (30) and noting that $p^h = D + 1$, we find that the advantage of waiting is

$$\begin{aligned}
A(k, p_1; I_k) &= \varpi\epsilon + \rho k + p_1 \tag{33} \\
\rho &= Z - D - 1
\end{aligned}$$

Because A varies linearly with ρ, there is a unique value of k that solves $A(k, p_1; I_k) = 0$.

(iii) Finally, we need to show that $\rho < -1$. We know that ρ is a function of β and that $\rho_{|\beta=1} = \rho^0 < -1$ by Proposition 2. We know that $d\rho/d\beta = D[1 - H + H\beta]/[H\beta + L] > 0$ because $H < T = 1$. Since $\beta < 1$, we conclude that $\rho < \rho^0 < 1$. ∎

Proof. (Proposition 5): (i) We first show that $-\rho^0 > \tilde{p}$, where $\tilde{p} = D + 1 - D/[H + L]$ (using Equation (6)). We know that

$$
\begin{aligned}
-\rho^0 - \tilde{p} &= \left(-(1 - L - H) \frac{\ln(H + L) - \ln L}{H} - 1 + \frac{1}{H + L} \right) D \\
&= D \frac{L + H - 1}{H} \psi; \\
\psi &\equiv \left(\ln \frac{H + L}{L} - \frac{H}{H + L} \right).
\end{aligned}
$$

Next, we treat ψ as a function of H, and use the facts that $\psi(0) = 0$ and $d\psi/dH = H/(H + L)^2 > 0$ to conclude that ψ (and thus $-\rho^0 - \tilde{p}$) is positive for all $H > 0$.

We now show that $-\rho^0 - \bar{p} < 0$, where $\bar{p} = D(L + H - 1)/L + 1$ (using Equation (6)). We note that

$$
\begin{aligned}
-\rho^0 - \bar{p} &= \frac{DE}{HL} \upsilon \\
\upsilon &\equiv L \ln(H + L) - L \ln L - H.
\end{aligned}
$$

Again, we treat υ as a function of H and use the facts that $\upsilon(0) = 0$ and $d\upsilon/dH = -H/(H + L) < 0$ to conclude that υ (and thus $-\rho^0 - \bar{p}$) is negative for all $H > 0$.

(ii) Finally, we need to show that $\rho^0 - \varpi^0 < 0$. Using the formulae for ρ^0 and ϖ^0 we have

$$
\begin{aligned}
\rho^0 - \varpi^0 &= \frac{\tau}{H^2} \\
\tau &\equiv 2DE(-H + 1 + E)(\ln(E + 1) - \ln(E + 1 - H)) - H^2 - 2DEH.
\end{aligned}
$$

Thus it is sufficient to show $\tau < 0$. At $D = 0$ it is obvious that $\tau < 0$, so it is sufficient to show that $d\tau/dD < 0$. We have

$$
\begin{aligned}
\frac{d\tau}{dD} &= 2E\varsigma \\
\varsigma &\equiv (1 - H + E)(\ln(E + 1) - \ln(E + 1 - H)) - H.
\end{aligned}
$$

Thus, it is sufficient to show that $\varsigma < 0$. Since $\varsigma = 0$ when $H = 0$, it is sufficient to show that ς is decreasing in H. We have

$$
\frac{d\varsigma}{dH} = \ln(E + 1 - H) - \ln(E + 1) < 0.
$$

∎

Proof. (Proposition 6): We already established the inequality $\rho < \rho^0$ in the proof of Proposition 4. ∎

Proof. (Proposition 7): Equation (21) merely restates the result in the paragraph preceding the proposition. Thus, we need only confirm that the critical ratio $(1.5\rho^0 - .5\varpi^0)/[.5(\rho^0 + 1)] > 0$. The denominator is negative because $\rho^0 < -1$ from Proposition 3.a.iv. Since $\varpi^0 < 0$, we know that $1.5\rho^0 - .5\varpi^0 < 1.5\rho^0 - \varpi^0 < \rho^0 - \varpi^0$. In the proof of Proposition 5.ii, we confirmed that $\rho^0 - \varpi^0 < 0$. Thus, both the numerator and the denominator or the critical ratio are negative, so the ratio is positive. ∎

Proof. (Proposition 8): (i) Because $\rho^0 - \varpi^0 < 0$ and given the definition of the limit price (Equation (19), the limit price decreases with ϵ. Combining this fact and Proposition 6, we confirm part (i).

(ii) For small values of ϵ such that $[1.5\rho - .5\varpi]/[.5(\rho + 1)] > \gamma/\epsilon$, we know from Proposition 6 that the monopoly uses the price $\breve{p} = 0.5[(1-\rho)\gamma - (\rho + \varpi)\epsilon]$. Substituting this price into Equation (14) and then substituting the result into Equation (13), we determine that the equilibrium level of α is

$$\alpha^* = -\frac{1}{4}\frac{\varpi\epsilon + \gamma + \gamma\rho - 3\epsilon\rho}{\epsilon\rho}. \tag{34}$$

Using this equation and previous definitions, we find that

$$\frac{d\alpha^*}{d\epsilon} = \frac{1}{4}\gamma\frac{1+\rho}{\epsilon^2\rho} > 0,$$

where the inequality follows because $\rho < -1$. Since an increase in ϵ leads to an increase in the equilibrium number of high types who wait (α^*), efficiency falls. ∎

Proof. (Corollary 1): For small ϵ, the monopoly limit prices in the absence of scalpers (Proposition 6). With scalpers, it is feasible to limit price, and scalpers increase the limit price (for small ϵ) by Proposition 5. Therefore , scalpers strictly increase monopoly profits when ϵ is small. ∎

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