Stochastic Models and Option Values
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Estimating Structural Resource Models When Stock Is Uncertain: Theory and Its Application to Pacific Halibut

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Economists and policymakers frequently face the problem of making decisions about stochastic systems but oftentimes do not directly observe the most important elements. Common examples include managing fisheries when one does not directly observe the stock of fish, making energy policy when unproved reserves are not known, restricting immigration without a clear knowledge of the number of undocumented aliens, and designing a policy to fight drug addiction when the supply of drugs is not observable. All of these policy problems have an unobservable component which is critical to understanding the behavior of the system being studied. A combination of the methods of maximum likelihood and the Kalman filter provides a way to estimate the parameters of the stochastic difference equations that govern the evolution of resource stocks. Much of the problem of regulating fisheries stems from the great variance in the fish stock from season to season. Environmental factors (such as water temperature and fishing) explain some of the apparent changes in year-to-year fish stocks, but a good deal of the variance cannot be explained by deterministic means and is, therefore, taken as stochastic.

In the next section of this paper, we set out the theory of the Kalman filter/maximum likelihood method of estimation in a simple fishery setting. We contrast our view with the more common estimation practice. Section 3 of the paper applies these ideas to a real fishery problem, the Pacific Halibut fishery. The model in section 3 is a good deal more complex than that in the second section. The final section includes some conclusions about the

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halibut example and some suggestions for other areas in agricultural and resource economics where application of the filter idea can enhance stock estimates and thus policy decisions.

The maximum likelihood Kalman filter

Since one cannot observe the actual stock of fish in the sea, it is natural to use a statistic one can observe as a proxy. The proxy that should be chosen depends upon the regulation of the fishery. We will begin this section with a presentation of a simple, dynamic fishery model. The production function in the model is chosen so that yield-per-unit effort is an indicator of the fish stock. That model is then expanded to include a quota on total fishery harvest. When the quota is harvested, the season is over and the season length is measured in days. In this regulated model, it is catch-per-day-per-unit effort that proxies the unobserved stock. Since most fisheries are regulated, these changes are not trivial.

The simplest model of an unregulated, open access fishery determines the (stochastic) time path of fishing effort, E; catch, h; and fish, x, as functions of (exogenously) given prices for fish, p, and costs of maintaining one unit of effort, c.[1] The simplest model, which is to say a too simple model, begins with a law, $F()$, governing the growth of the fish stock. The model ignores age classes, predators, etc., to achieve this simplicity.

$$x_{t+1} = F(x_t - h_t, \beta) + w_t\,, \tag{1}$$

where w is a normally distributed mean zero variate with variance W, β is a parameter vector, and t is time in years. All the additive error terms in this paper are assumed uncorrelated with each other and are assumed to be serially uncorrelated.

The second equation of this model is a standard equation of yield-effort models,

$$\begin{aligned} h_t &= kE_t x_t + E_t v_{yt}\,, \qquad \text{or} \\ y_t &= \frac{h_t}{E_t} = kx_t + v_{yt}\,. \end{aligned} \tag{2}$$

Equation (2) states that yield-per-unit effort, y, is proportional to stock with constant of proportionality, k. The random error in the first form

[1] The model generalizes to n-dimensions. In all that follows, one could interpret x as a vector of state variables and y as a vector of measurement variables. All the arguments remain the same.

of equation (2) has variance proportional to effort. In the second form of the equation, the random error, v_{yt}, is distributed normal with mean zero and variance, V_y. Equation (2) is called a measurement equation because the dependent variables are observable and depend on the unobservable stock, x. Equation (2) is just a production function giving output, h, as a function of a single input, E, and an uncontrollable variable, x, that plays a role formally equivalent to technical progress. Again, an additive normal error is assumed. Any other production function that maintains the role of x would also work, assuming one knew the coefficients. Finally, annual effort is modeled as

$$E_{t+1} = E_t + \delta(py_t - c)E_t + v_{et}\,, \tag{3}$$

where v_{et} is a random error term. This last equation embodies the notion that positive profits per unit effort (the term inside parentheses) lead to entry of effort at rate δ. Negative profits lead to a decrease in effort. It is the naive long-run model of the firm: Positive profits mean (slow) entry of firms and effort. A more elaborate version of the simple model would come from modeling the components of effort: boats and effort per boat and allowing the components to vary. One would then amend equation(2) so that price equals marginal cost. As it stands, an interpretation is that each firm finds it profitable to fish exactly 1 unit of effort which is presumably equivalent to the maximum amount they can fish. Smith (1968) is usually credited with the three-equation dynamic version of this model. His model is more elaborate and allows for crowding and for differing production functions. Obviously, it does not matter whether one uses the primal $h(E)$ point of view or the dual $c(h)$ formalism, though there is certainly fishery literature that argues this choice is meaningful (Fullenbaum, Carlson, and Bell, 1971). In short, this is the simplest dynamic model that could be called a "fishery".

When a fishery is regulated, a slightly different model is appropriate. Abstracting quite a bit, Pacific Halibut are regulated by means of a quota on total catch of the fishery, q. The quota is enforced by closing the fishery when the quota is met. The fraction of the potential fishing season during which harvesting is allowed is called σ. We shall make the extreme assumption that, as soon as the fishery is closed, the boats just sit in port earning nothing. Equation (1) is unchanged by this regulation since it is merely a statement of the biology. Equation (2) needs some work:

$$\begin{aligned} q = h &= kEx\sigma + v_y E\sigma\,, \quad \text{or} \\ y &= \frac{h}{E\sigma} = kx + v_y\,, \end{aligned} \tag{2r}$$

modifies equation (2) by taking the effective effort as $E\sigma$ or effort times the percent of the season it is used. The adjustment equation for effort depends upon per boat profits. Regulation leaves costs unaffected, but per boat revenues are now $(\sigma p y)$, so

$$E_{t+1} = E_t \delta(\sigma_t p y_t - c) E_t + v_{et} \,. \tag{3r}$$

Equations (1), (2r), and (3r) are the simple regulated fishery. The entry equation (3r) is particularly naive. It assumes that firms act in accord with instantaneous profits and not rational expectations about their present value of profits (Berck and Perloff, 1984).

Typically, one observes all the variables except stock, x, and wishes to estimate the parameters, δ and k, and whatever parameters are in the biomass size function, F. Also, one should be estimating a regulated model, rather than the more popular unregulated one, because fisheries are regulated.

Estimation with unknown stock

The parameters of the equations representing the simple fishery and the fish stock can be estimated by a combination of the Kalman filter and maximum likelihood. In this section, we will explain how this procedure is done in simplified terms and for the simplified model. We follow the presentation in Meinhold and Singpurwalla (1983). More standard and more detailed explications can be found in Harvey (1981) or Gelb (1974). The models presented in this section differ from the estimated models primarily in the number of measurement equations, the choice of additive rather than multiplicative errors, and the need for an extended filter in the empirical work.

Equation (1) is called a state equation because it describes the evolution of the unobservable state variable, fish stock. Its value is never known. The best estimate of stock at time t, given all observations (on y and h) up to and including $t-1$, is denoted $x_{t|t-1}$. This is shorthand for a normal random variate with mean $\hat{x}_{t|t-1}$ and variance $P_{t|t-1}$. One then observes y_t. This new information leads to a revised estimate of the fish stock. This new best estimate, given information through time t, is denoted $x_{t|t}$; and it is again a normal random variate with mean $\hat{x}_{t|t}$ and variance $P_{t|t}$. Given β, W, and V_y, or estimates of them, the Kalman filter is an algorithm to determine $x_{t|t}$ when one knows $x_{t-1|t-1}$ and realizations of the variables, y_t, which depend on x_t. The filter also gives the variance of $x_{t|t}$, denoted $P_{t|t}$, which is determined from $P_{t-1|t-1}$ and y_t.

For the simplest case, which is all we shall describe here, assume that $\hat{x}_0$ and $P_{0|0}$ are the mean and variance of x at time zero.[2] To keep the example simple, also assume that the stock function, F, in equation (1) is linear. The filter will give an estimate $x_{1|1}$ in terms of $x_{0|0}$ and y_1. Similarly, one gets estimates for time 2 from those of time 1 and so on. Therefore, it suffices to consider the general case of getting $x_{t|t}$ from $x_{t-1|t-1}$.

At time $t-1$, which is to say before y_t is observed, the estimate of x_t, called $x_{t|t-1}$, is $F(x_{t-1|t-1} - h_{t-1}, \beta)$. This estimate is a normal random variable because it is a linear function of a normal random variable and some constants. To be explicit, let $x_{t|t-1} = F + w = \beta_0 + \beta_1(x_{t-1|t-1} - h_{t-1}) + w$. Its mean is $\beta_0 + \beta_1(\hat{x}_{t-1|t-1} - h_{t-1})$. Its variance is the variance of w, called W, plus $\beta_1 P_{t-1|t-1}\beta_1'$. The latter term is the contribution of the randomness of $x_{t-1|t-1}$ to the randomness of $x_{t|t-1}$. For later use, let this variance be called

$$R_t = P_{t|t-1} = \beta_1 P_{t-1|t-1}\beta_1' + W\,. \tag{4}$$

Since $x_{t|t-1}$ summarizes the beliefs about x prior to observing y, it is a prior expectation in the Bayesian sense.

The next piece of the filter is to make use of a measurement equation which is the equivalent of conducting an experiment. The yield-per-unit effort is predicted using $\hat{x}_{t|t-1}$ and is compared to observed yield-per-unit effort. The yield-per-unit effort in period t is just y_t. At $t-1$, one's beliefs about y_t are a consequence of equation (2),[3]

$$y_{t|t-1} = kx_{t|t-1} + v_y\,. \tag{5}$$

The point forecast of y_t is just the mean of (5),

$$\hat{y}_{t|t-1} = k\hat{x}_{t|t-1}\,. \tag{6}$$

From (5) and (6), the error in predicting y is the normal random variate,

$$e_{t|t-1} = y_t - \hat{y}_{t|t-1} = k\big(x_{t|t-1} - \hat{x}_{t|t-1}\big) + v_y\,, \tag{7}$$

where $\mathbf{E}[e_{t|t-1}] = 0$. The variance of the prediction error in catch-per-unit effort is $V_y + kR_tk'$, where V_y is the variance in the error of the yield

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[2] The quantities x and y could equally be interpreted as vectors. Then k is a matrix with the number of columns equal to the dimension of x and the number of rows equal to the dimension of y. Its transpose is k'. Similarly, the V's, P's, etc., are conformable matrices.

[3] One could do this equally with equation (2r).

equation. One other statistic that will be needed is the covariance of $e_{t|t-1}$ and $x_{t|t-1}$.

$$\operatorname{cov}\left(x_{t|t-1}\,, e_{t|t-1}\right) = \mathbf{E}\left[\left(e_{t|t-1} - 0\right)\left(x_{t|t-1} - \hat{x}_{t|t-1}\right)\right] = k'R_t\,, \tag{8}$$

where $\mathbf{E}[\ ]$ is the expectation operator. The variables $x_{t|t-1}$ and $e_{t|t-1}$ are correlated normal random variates, so they are jointly normal with the following distribution:

$$\begin{pmatrix} e_{t|t-1} \\ x_{t|t-1} \end{pmatrix} = N\left(\begin{pmatrix} 0 \\ \hat{x}_{t|t-1} \end{pmatrix}, \begin{pmatrix} V_y + kR_tk' & R_tk' \\ kR_t & R_t \end{pmatrix}\right). \tag{9}$$

Since observing the error in predicting catch-per-unit effort is the same as observing catch-per-unit effort, the posterior distribution of x (which is $x_{t|t}$) is just the same as $(x_{t|t-1}|e_t)$ or the conditional distribution of $x_{t|t-1}$, given the observed e_t. To summarize, equation (9) gives the joint distribution of the normal variates, $e_{t|t-1}$ and $x_{t|t-1}$. We seek the conditional distribution of $x_{t|t-1}$, given $e_{t|t-1}$. The conditional distribution for a joint normal can be found in a standard text or the article by Meinhold and Singpurwalla (1983). From the formula for the conditional distribution, $x_{t|t}$ is normally distributed with mean and variance given by

$$\begin{aligned} \hat{x}_{t|t} &= b_0 + b_1\left(\hat{x}_{t-1|t-1} - h_{t-1}\right) + R_tk'\left(V_y + kR_tk'\right)^{-1}e_t\,, \\ P_{t|t} &= R_t - R_tk'\left(V_y + kR_tk'\right)^{-1}kR_t\,. \end{aligned} \tag{10}$$

Equation (10) gives the mean of $x_{t|t}$ as a function of e_t — the observed observation error. Equation (10) is the Kalman filter for this simple fishery model. To recapitulate: For any set of parameters, β, δ, k, etc., and any $x_{0|0}$ and $P_{0|0}$, one can calculate all $x_{t|t}$'s by use of equation(10). That calculation is simply the algebra of conditional expectations and nothing more.

One estimates equations (1) to (3) by the maximum likelihood method. Equation (3) can be estimated by ordinary least squares, quite apart from the others. Unless one assumes contemporaneous correlation of the errors or the inclusion of endogenous variables (such as current price), as we will do later, the Gauss-Markov theorem assures that ordinary least squares is best. The other two equations are estimated by Kalman filter/maximum likelihood.

The maximum likelihood method requires the choice of parameters (β, k, x_0, P_0) to maximize the likelihood of what one observes. The only observable variable is y_t, and the likelihood of the sample is just the product of the likelihoods of the T observations.

A typical observation consists of y_t and $\hat{x}_{t|t-1}$. The likelihood of y is the same as $kx_{t|t-1} + v_y$ which is a normal variate with mean $k\hat{x}_{t|t-1}$ and variance $V_y + kR_tk'$. Letting u_t be the observed residual $u_t = y_t - k\hat{x}_{t|t-1}$, the likelihood of the t-th observation is

$$L_t = \frac{1}{\sqrt{2\pi(kR_tk' + V_y)}} e^{-u_t\left(V_y + kR_tk'\right)^{-1} u_t} . \tag{11}$$

The likelihood of the sample is just $L(x_{0|0}, P_{0|0}, \delta, \beta, k, V_y, W) = \prod_{t=1,T} L_t$ which is the likelihood of observing all T observations of y. In practice, for any set of parameters, one evaluates L by first using the filter, equation (10), to find all of the x's and P's. The x's and P's are used to calculate R_t and u_t. Finally, the L_t's are calculated and multiplied together from $t = 1, \ldots, T$ (or the logs of L_t are summed) to get the likelihood function. The maximum likelihood method is to use numerical methods to find the values of the parameters that maximize the likelihood function. We were not able to find algebraic expressions for the derivatives of this function with respect to the parameters, so we have no proof that the function is concave or that convergence is guaranteed.

The Kalman filter can be easily extended to use sampling information as well as catch-per-unit information. Halibut stocks are predicted by an age-cohort analysis. Let γ_t be the International Pacific Halibut Commission (IPHC) stock estimates made by this method alone, and assume that v_γ is their error with variance V_γ. Thus,

$$\gamma_t = x_t + v_\gamma \tag{12}$$

is a second measurement equation. The mechanics of the filter are as before except that V is now a matrix of v_y and v_γ, and u_t is a vector. One could enlarge this model to the belief that γ is a linear function of the true stock without undue computational burden. This combination of the filter and the sampling information gives the minimum mean square error way to use both cohort and catch-per-unit effort data. The actual series we have from IPHC already combines both of these methods in a nonoptimal fashion, so we do not pursue this any further.

In contrast to these methods, the methods in the literature are to either ignore the dynamic and stochastic nature of the fish stock (e.g., Bell, 1972) or make a clever substitution to eliminate the fish stock and ignore its stochastic nature (e.g., Spence, 1973). There is little to be said in favor of assuming fish stocks to be in equilibrium over a long sample period, so there is little to be said for estimations of yield-effort curves based on that assumption. The problem with Spence's ingenious method is that he suppresses the error in the measurement equation.[4] On the other hand, dispensing with an error term before one makes felicitous substitutions is certainly well within the reduced form tradition of econometrics (add your error terms when it is convenient). Short of the filter techniques proposed here, Spence's method is certainly the next best.

In the next section we apply this Kalman filter/maximum likelihood algorithm to a more realistic model of the Pacific Halibut fishery.

Application to the Pacific Halibut fishery

The IPHC was established in 1923 by a treaty between Canada and the United States to rehabilitate and maintain Pacific Halibut stocks at or near maximum sustainable yield. The fishery consists of four separately managed areas. Since the IPHC cannot directly observe the stock of halibut, it relies on changes in catch-per-unit effort and age composition studies to manage the resource. The management tools used by IPHC are gear restrictions, size limits, the regulation of incidental catch (IC), and an annual quota on total catch. Although halibut are exploited by a variety of vessel types that are shared with other fisheries, only one type of gear — longline skate (a setline) — has been in use since the early days of the fishery. The biology of the fishery is such that fishermen exploit a large number of year classes simultaneously. For this reason, Crutchfield (1981) states that the halibut fishery is "ideally characterized by the traditional biomass-fishery model".

The model employs annual data on the halibut fishery from 1936 through 1982 for the two most important areas numbers 2 and 3. Table 1 gives the

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4 Spence writes his state equation as $x_{t+1} = F(x_t) - c_t$, where F is natural growth. In filter terms, an observation equation would be Spence's catch equation, $c_t = F(x_t)[1 - e^{-\lambda E_t}]$, with catch as a function of fishing effort and a parameter, λ. Let $z_t = c_t[1 - e^{-\lambda E_t}]^{-1}$; then $z = F$ and one estimates $z_{t+1} = F[z_t e^{-\lambda E_t}] + w$. This appears to successfully eliminate the unobservable from the equation. Spence reaches this simple result by writing $z = F$ rather than $z = F + v$; that is, he ignores the error in measuring stock by catch per (a function of) unit effort. Including this error in the measurement equation leads to a much more complicated result. Put differently, one does not know stock. It is a random variable derived from a stochastic process, and z provides only an estimate of it. If one carries through the algebra, $z_{t+1} = F[(z_t - v_t)e^{-\lambda E_t}] + w_{t+1} + v_{t+1}$. Even if one does not bother much with the problems caused by nonlinear F, this is still an autocorrelated, lagged dependent variable problem in need of some attention.

Table 1: Pacific Halibut Fishery Model.

BIOMASS

$$\text{Ln}(\text{biomass}_{2,t}) = s_1 + s_2 \cdot \text{Ln}(\text{biomass}_{2,t-1} - \text{catch}_{t-1} - IC_{2,t-1}) + w_t\,.$$

CATCH

$$\text{Ln}(\text{catch}_2/\text{day}_{2,t}) = c_1 + c_2 \cdot \text{Ln}(\text{biomass}_{2,t}) + c_3 \cdot \text{Ln}(\text{effort}_2/\text{day}_{2,t}) + v_{ct}\,.$$

EFFORT

$$\text{Ln}(\text{effort}_2/\text{day}_{2,t}) = \text{ef}_1 + \text{ef}_2 \cdot \text{Ln}(\text{biomass}_{2,t}) + \text{ef}_3 \cdot \text{Ln}(\text{halprice}_t) + \text{ef}_4 \cdot \text{Ln}(\text{saplepr}_t) + \text{ef}_5 \cdot \text{Ln}(\text{salmonpr}_t) + v_{et}\,.$$

HALIBUT PRICE

$$\text{Ln}(\text{halprice}_t) = h_1 + h_2 \cdot \text{Ln}(\text{catch}_{2,t} + \text{catch}_{3,t} + \text{catch}_{4,t}) + h_3 \cdot \text{Ln}(\text{pincome}_t) + h_4 \cdot \text{Ln}(\text{holdings}_t) + v_{ht}\,.$$

Notes: Subscripts indicate area and year.
IC is incidental catch.

equations of the complete model which is a good deal more complicated than the simplified model of section 2. Table 2 gives the definitions of all the variables.

The first equation is the equation for the evolution of the unobservable fish biomass (thousand metric tons, t.m.t.) Unlike the simple model, the state equation is not linear in the state variable, biomass. The functional form chosen for the stock equation allows slower growth when the biomass is larger (which the linear form does not), but it still does not capture the backward bending part of the growth curve or allow for a maximum biomass. Catch is what is caught by boats trying to catch halibut, while incidental catch, taken as exogenous, is what is caught by boats targeting other species. The stock equation is a feasible improvement over the linear form.

The nonlinearity of the biomass equation leads to filters that are based on the probability density function of biomass. This will likely result in nonlinear filters that are burdensome to implement. To maintain compu-

Table 2: Pacific Halibut Data and Source.

Variable	Description	Source
Catch	Quantity of halibut caught by the longline fleet in management areas 2, 3, and 4. Expressed in round weight metric tons.	Myhre, 1977; IPHC Annual Reports, 1977 to 1982.
Incidental catch	Quantity caught by other fishers in management areas 2, 3, and 4.	Myhre, 1977; IPHC Annual Reports, 1977 to 1982.
Effort	Number of longline skates used to catch halibut in management areas 2 and 3. Expressed in 100 skates.	Myhre, 1977; IPHC Annual Reports, 1977 to 1982.
Season length	Number of days between the opening and closing of the season for management areas 2 and 3.	Skud, 1977; IPHC Annual Reports, 1977 to 1982.
Halibut price	Average exvessel price of Pacific Halibut for areas 2, 3, and 4. Deflated using the Implicit Price Deflator for GNP (IPD) with base year = 1972.	IPHC Annual Report, 1982; U.S. President, 1970 and 1984.
Salmon price	Average exvessel price of salmon, all species, North Pacific Ocean. Deflated using IPD with base year = 1972.	United States Department of Commerce, 1975; Orth, et al., 1981; Fisheries of the United States, 1976 to 1982.
Sablefish price	Average exvessel price of sablefish, North Pacific Ocean. Deflated using IPD with base year = 1972.	NMFS-Fishery Statistics of the United States, 1939 to 1956; U.S. Department of Commerce, 1974.
Cold storage holdings	Beginning of season holding of frozen Pacific Halibut expressed in round weight metric tons.	NMFS-Fishery Statistics of the United States; NMFS-Fishery Industries of the United States
Per capita income	U.S. Per Capita personal disposable income in 1972 dollars.	U.S. President, 1969, 1980, 1984.
Halibut biomass	IPHC estimates of Pacific Halibut Biomass from cohort and catch-age analysis, management areas 2 and 3.	Deriso and Quinn, 1983; Hoag and McNaughton, 1978; Quinn, et al., 1985.

tational simplicity, the "extended" Kalman filter is derived from the linearized state equation. It maintains filter linearity in the state (biomass) variable. First, we treat the state variable as the natural log of biomass. Its equation is linearized through a first-order Taylor series expansion about the mean of biomass, $\hat{x}_{t-1|t-1}$.

The size of the error introduced by linearizing the logged biomass equation depends on the size of catch relative to biomass. As catch increases relative to biomass, the error increases; and this is compounded by the degree of nonlinearity of biomass in the true biomass growth equation. In the case of the Pacific Halibut fishery, direct and incidental catch is a small proportion of the biomass (roughly 15 percent), so the error in estimating biomass via the extended Kalman filter is expected to be small. Although other methods of dealing with nonlinear state equations exist, they are more complicated and may not reduce error to the extent that would justify the additional effort needed to implement them. According to Gelb (1974), the extended Kalman filter "has been found to yield accurate estimates in a number of important practical applications".

The catch-per-day equation is just the classic equation of yield-effort fishery economics generalized to permit arbitrary, but constant, elasticities of catch per day with respect to stock and effort per day.

Effort per day in the halibut fishery is modeled as dependent upon current market forces. Current biomass and exvessel price directly affect current profitability, and sablefish and salmon prices give the opportunity cost of fishing. The gear used for fishing halibut (skate soaks) is not so specialized as to preclude the same vessels switching from one fishery to another. It is believed that this intraseason switching of target species is the main form of exit and entry in this fishery. Thus, current variables are expected to have a large effect on effort per day.

In all, there are six measurement equations — three each for management areas 2 and 3 of the Pacific Halibut fishery. Each management area also has a biomass equation. There is also an exvessel price equation to represent the demand for halibut at dockside. The parameters of these equations are estimated using the full information-Kalman filter/maximum likelihood technique. The technique of the previous section is generalized to include two state variables (log stock in each area), an extended filter, seven endogenous variables, and six measurement equations. The likelihood for an observation is just as in full information/maximum likelihood method except that the term kR_tk' (the variance induced by the uncertainty about the stock) is added to the usual variance covariance matrix.

The estimates of stock biomass are constructed from the logged biomass estimates and their variances, which are obtained from the Kalman filter. Since the conditional and updated estimates of the log of stock biomass in each time period are distributed normal with mean ν_t and variance P_t, the conditional and updated estimates of stock in each time period, x_t, are distributed log-normal with a mean of

$$\mathbf{E}(x_t) = e^{\nu_t + (1/2)P_t} \tag{13}$$

and a variance of

$$V(x_t) = e^{2\nu_t + P_t}\left(e^{P_t} - 1\right). \tag{14}$$

Results

The parameter estimates of the model obtained from the numerical optimization using the Kalman filter and the Davidon-Fletcher-Powell optimization method are presented in Table 3. The standard errors were computed by means of a bootstrap,[5] but the histograms of the replicated parameter estimates tell a somewhat different story. The histograms do not appear to be symmetric and generally indicate that parameter values are very much less likely to be zero than might be concluded from the t ratios.

A parameter estimate of one associated with the exponent on the biomass equation is the demarcation between higher stock giving higher and lower growth. One cannot reject the simple model of linear growth for area 2, though one can reject it for area 3. The catch-per-day equation in area 2 is nearly linear in biomass and in effort per day (and they are not significantly different from unity in area 3) so, again, the very simple model seems acceptable. Effort per day is increasing in biomass and halibut price, which is as one would expect, but the effects of competing species are of uncertain sign (and this is true in both areas). Finally, the demand curve slopes down and, when one examines the bootstrap replicates, significantly so. Holdings depress price while per-capita income increases it, which is as it should be. Neither of these latter two variables are significantly different from zero. In summary, the parameter estimates are about what one should expect and are much closer to the estimates from a naive model than one would have first thought likely.

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[5] The 92 percent, bias corrected, centered confidence intervals were also computed this way, using the method suggested by Efron (1982).

Table 3: Parameter Estimates of the Pacific Halibut Fisher Model.

Parameter	Estimate	Standard deviation	92 percent C.I.[a]	
Area 2 biomass				
Intercept — s_1	0.304	0.242	0.032	0.309
Escapement 2 — s_2	0.956	0.050	0.931	1.08
Area 3 biomass				
Intercept — s_1	1.43	0.726	1.40	5.05
Escapement 3 — s_2	0.727	0.100	0.278	0.735
Area 2 catch per day				
Intercept — c_1	−4.15	20.35	−86.0	−0.397
Biomass 2 — c_2	1.03	3.75	0.364	2.14
Effort 2 per day — c_3	0.876	0.428	0.008	0.981
Area 3 catch per day				
Intercept — c_1	−0.049	3.66	−6.42	2.16
Biomass 3 — c_2	0.643	0.893	−0.046	1.06
Effort 3 per day — c_3	0.751	0.619	0.172	2.14
Area 2 effort per day				
Intercept — ef_1	−17.41	20.7	−82.8	−2.07
Halibut price — ef_2	1.47	0.779	0.736	3.11
Biomass 2 – ef_3	2.78	3.60	0.968	14.8
Sablefish price — ef_4	0.083	0.426	−0.388	1.09
Salmon price — ef_5	−0.981	0.516	−0.980	1.14
Area 3 effort per day				
Intercept — ef_1	−7.30	6.00	−25.1	−6.0
Halibut price — ef_2	1.08	1.34	0.614	6.64
Biomass 3 — ef_3	1.23	0.598	0.874	2.84
Sablefish price — ef_4	−0.536	0.291	−0.832	−0.379
Salmon price — ef_5	0.156	0.892	−0.930	2.57
Exvessel halibut price				
Intercept — h_1	12.8	12.8	9.17	74.4
Total catch — h_2	−0.831	0.839	−5.45	−0.650
Income — h_3	0.430	1.04	−1.08	0.839
Holdings — h_4	−0.124	0.131	−0.187	0.375

[a]Bias corrected 92 percent central confidence interval about the point estimate.

The biomass estimates

Estimates of Pacific Halibut biomass (Table 4) for management area 2 and management area 3 are a byproduct of the Kalman filter/maximum likelihood estimation. This subsection presents the estimates from the Kalman filter. These estimates are compared to biomass estimates obtained from a recent IPHC publication (Quinn, Deriso, and Hoag, 1985). The IPHC biomass estimates were derived from catch-age analysis.

Figure 1 is a plot of the updated biomass estimates for area 2 against time in years. Estimates of area 2 of biomass from Quinn, Deriso, and Hoag are also plotted on this graph. The pattern of biomass estimates from the Kalman filter methodology strongly resembles the pattern of biomass estimates given by IPHC. According to IPHC estimates, the peak biomass occurs in 1955. The Kalman filter biomass estimates increase dramatically from 1943 to 1954 and decrease sharply from 1955 to 1960. The IPHC biomass estimates begin to rise again in 1980 while the Kalman filter biomass estimates begin to rise in 1977.

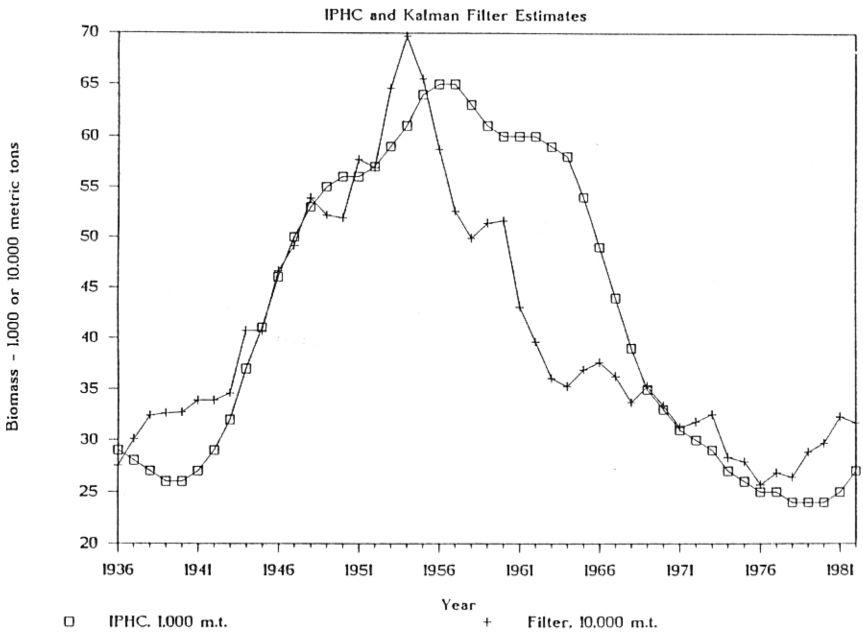

Figure 1: Halibut Biomass in Area 2.

For area 2, the regression of IPHC estimates (IPHC2) on the Kalman filter estimates (BIO2) resulted in the following relationship (standard deviation in parentheses):

$$\begin{aligned} \mathrm{BIO2}_t &= 110 + 6.9\,\mathrm{IPHC2}_t\,, \\ &\quad\;(24)\quad(.54) \\ R^2 &= .78;\ n = 47;\ \text{residual standard error} = 55.3. \end{aligned} \tag{15}$$

Estimation of the model parameters and variance matrix using the Kalman filter/maximum likelihood technique resulted in biomass estimates that are 10 times larger than those of the IPHC. This discrepancy becomes apparent immediately in the Kalman filter recursions by examining the conditional and updated biomass estimates and variances for 1936. Given the bioeconomic model, the biomass estimates have very low variances. As a result, the IPHC estimates do not fall in the confidence intervals of the Kalman filter biomass estimates.

Table 4, column 3, presents the updated estimates of area 3 biomass. Figure 2 presents a plot of the Kalman filter biomass estimates and the IPHC estimates (Quinn, Deriso, and Hoag, 1985) against time in years. Both patterns exhibit two peaks and two valleys. The IPHC biomass series peaks in 1946 and 1961 while the Kalman filter series peaks in 1952 and 1960. Both series declined beginning in 1962. The biomass begins to increase again in 1976 according to the IPHC series and in 1975 according to the Kalman filter series. The Kalman filter series predicts a much faster increase in biomass after 1979 than does the IPHC series.

The regression of IPHC biomass estimates (IPHC 3) on the Kalman filter biomass estimates (BIO 3) resulted in the following relationship (standard deviation in parentheses):

$$\begin{aligned} \mathrm{BIO3}_t &= 25 + .79\ \mathrm{IPHC3}_t\,, \\ &\quad\;(12)\quad(.18) \\ R^2 &= .29;\ n = 47;\ \text{residual standard error} = 27. \end{aligned} \tag{16}$$

The Kalman filter biomass estimates are approximately the same magnitude as the IPHC estimates. While the explanatory power of equation (16) is not as great as in equation(15), the estimates of biomass in area 3 after 1975, predicted by the Kalman filter model, agree with the increases in biomass after 1975 that were noted throughout the fishery in 1980 through 1986 (van Amerongen, 1985; *Alaska Fisherman's Journal*, 1984 and 1986).

The biomass estimates for both areas follow the pattern of IPHC biomass estimates. Area 2 biomass estimates from the Kalman filter are about 10

Table 4: Pacific Halibut Biomass Estimates in Area 2 and Area 3, 1936–1982 (in round weight per 1,000 metric tons).

Year	Biomass (t/t) area 2	Biomass (t/t) area 3
1936	274.83	47.82
1937	301.19	51.07
1938	323.68	53.11
1939	325.85	54.96
1940	327.39	56.86
1941	338.91	65.11
1942	338.63	62.09
1943	345.65	64.56
1944	407.12	58.00
1945	406.73	67.30
1946	465.85	81.84
1947	490.99	82.41
1948	539.13	102.56
1949	521.47	95.14
1950	519.19	95.66
1951	576.91	113.75
1952	569.22	127.99
1953	646.30	121.78
1954	697.27	116.97
1955	655.42	88.57
1956	587.30	78.40
1957	525.63	69.68
1958	499.34	78.29
1959	513.94	103.89
1960	516.30	112.72
1961	431.01	100.47
1962	396.33	97.43
1963	361.30	103.45
1964	352.85	88.74
1965	368.65	73.36
1966	375.76	88.10
1967	363.38	62.39
1968	337.12	52.07
1969	352.72	56.77
1970	334.27	55.42
1971	313.45	43.96
1972	317.82	40.63
1973	325.18	29.42
1974	282.88	23.37
1975	278.89	26.37
1976	257.39	28.40
1977	268.01	38.26
1978	263.87	43.07
1979	287.65	53.69
1980	296.53	98.89
1981	323.30	131.54
1982	316.73	175.82

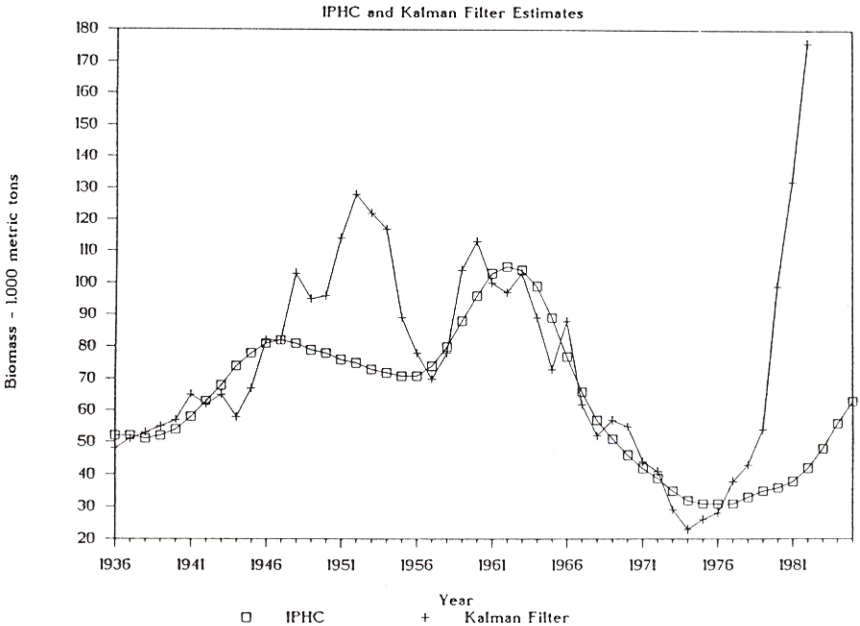

Figure 2: Halibut Biomass in Area 3.

times larger than the IPHC estimates although both series follow the same trend over time. In area 3, the IPHC and the Kalman Filter biomass estimates have the same magnitude. The time series of both estimates show two peaks, two valleys, and a sharp increase in biomass from 1980 to the present. For area 3, the filter estimates are much closer to the post-1975 fishing experience than are the IPHC estimates.

Maximum sustainable yield

The IPHC is charged with developing and maintaining halibut stocks at a level which provides maximum sustainable yield (MSY) to the fishery (Bell, 1978). The IPHC uses estimates of MSY and annual surplus production (ASP) to set annual quotas for management areas 2, 3, and 4. MSY is the maximum harvest that can be caught on a sustained basis without depleting the population. To achieve MSY, the quotas are set equal to 75 percent of ASP. ASP is the maximum potential change in biomass from the previous year to the current year. ASP in the current year is equivalent to the catch in the previous year plus the change in biomass from the previous year to the current year. When catch is held below (above) ASP, then the biomass increases (decreases). This method is appropriate when stocks are below the level necessary to achieve MSY. To ensure that the quotas are below ASP, the ASP is multiplied by 0.75.

The ASP, MSY, Maximum Sustainable Catch (MSC), and the catch that maximizes the present value of revenues to the fishery were calculated using the estimated model. The results for each area along with the actual quotas set by IPHC are presented in Table 5 for the years 1984, 1985, and 1986. The area 2 biomass is below MSY. However, it is close to the level that provides the highest revenues. The 75 percent ASP and the revenue maximizing catch in area 2 are approximately equal and substantially greater than the actual quota recommended by IPHC. The area 3 biomass is above MSY. The allowable catch recommended by the 75 percent ASP and the revenue maximization of area 3 provides for a reduction in biomass to levels at and below MSY, respectively. Both catch levels are significantly above the actual quota.

Revenues in both areas can be increased by raising the quotas without compromising the productivity of the fishery. In both management areas, a policy that restricts catch to achieve MSY will not benefit halibut fishers. This result explains the events of the mid-1980s when unprecedented increases in catch-per-unit effort resulted in record volumes of halibut caught and a severe depression in halibut prices and income. The current problem in the Pacific Halibut fishery is the accurate forecasting and control of incidental catch of halibut. The size of the incidental catch has a large impact on the biomass and the recommended quota. The degree to which IPHC regulations will benefit fishers in the future will depend on their ability to recommend quotas consistent with maximum revenue, or income, and their ability to estimate and control the amount of incidental catch.

Conclusions and extensions

Filter methods were shown to provide a reasonable basis for estimation of fishery models. They are equally useful for other economic cases of unobserved stocks.

Extension: Other examples

Other examples of important stochastic stock problems in resources and agriculture include aliens, crop acreage, and undiscovered reserves of an

Table 5: Comparison of Recommended and Actual Quotas (1,000 metric tons).

Year/ Area	75 percent ASP	Maximum revenue	Actual quota	Biomass under actual quota
1984				
Area 2	20.4	19.4	9.1	352.7
Area 3	39.4	52.1	13.7	256.9
1985				
Area 2	20.7	20.1	11.8	368.8
Area 3	36.4	35.2	17.5	291.3
1986				
Area 2	21.0	19.9	13.9	382.6
Area 3	33.8	26.2	21.0	318.0
Area		Maximum sustainable yield		
Area 2		732.3		
Area 3		211.5		
		Maximum sustainable catch		
Area 2		32.2		
Area 3		57.7		

exhaustible resource. The method could also be used to estimate aggregate capital. Each of these models is sketched in turn.

To make an estimate of resources which remain to be found in some areas, a planner could reason as follows. The prior is $(\hat{x}_0, P_0)$ — the quantity of the resource to be discovered and its variance. It might just as well be a vector of the types of resources and their covariances. Again, h_t is found with exploration effort, E_t, so the observation equation is $h_t = f(x_t, E_t)$ and the state equation is simply $x_{t+1} = x_t - h_t$. The measurement equation has the same justification as the fishing measurement equation: It is easier to find an exhaustible resource when there is more of it to be found. The state equation is the exhaustible resource state equation. Assume that the measurement equation is just the familiar $h/E = kx$. Substitute the definitions of $\hat{x}_{t|t-1}$ and e_t into equation (10), the filter; and subtract h_t from both sides to get

$$\hat{x}_{t|t} - h_t = \hat{x}_{t|t-1} + R_t k(V + kR_t k')^{-1}(y_t - k\hat{x}_{t|t-1}) - h_t \,. \qquad (17)$$

Equation (17) is the estimate of the mean stock remaining to be discovered after the findings of period t, that is, $\hat{x}_{t+1|t}$. Since $y_t = h_t/E_t$, $(\partial \hat{x}_{t|t+1})/(\partial h_t) > 0$ whenever

$$R_t k'(V + kR_t k')^{-1} E_t^{-1} > 1 . \tag{18}$$

Looking at (9), this can be re-expressed as (19)

$$\frac{\text{cov}\left(e_{t|t-1}, x_{t|t-1}\right)}{\text{var}\left(e_{t|t-1}\right)} > E . \tag{19}$$

Increasing discoveries increases one's estimate of stock when surprises in discovery per unit effort are highly correlated with stock and when the absolute value of effort or the variance of stock is low.

The multivariate expansion of this model could include good and bad grades of the resource. The discovery equation would then have two k's, one for each grade. Discovery effort would result in both good and bad grades being discovered, in proportion to their difficulty to discover, k, and their abundance, x. This gives a model without the usual odious assumption that good grades are discovered first.

Undocumented crop acreage is a real problem in California. Marijuana is often alleged to be an important (sometimes the important) cash crop of the northern timber growing regions. The natural agricultural model is a stock adjustment model. Unobservable production, x, is a function of past production, observable price, and observable apprehension expenditure. The latter variable represents a very severe cost to the grower. In addition to expenditure on enforcement, one also observes enforcement success — tons seized. Let y be tons seized per dollar of enforcement effort. As before, $y = kx$ is the observation equation. There are many problems with this example — particularly rapid technical progress in crop cultivation and detection avoidance and equally rapid progress in detection through aerial surveillance.

Undocumented aliens are definitely not directly observable. Torok and Huffman (1986) examined U.S.-Mexican trade in winter vegetables and undocumented immigration. Their model shows that the same work force picks tomatoes in both countries so that the United States will import either labor or tomatoes. To estimate their structural model, they substitute apprehensions and apprehension effort for the actual unobservable stock of laborers and supply of labor. Their model could be cast in the filter mode by adding a state equation and treating the stock of those undocumented in the other equations. A plausible state equation would be: The stock of Mexican agricultural laborers (undocumented) is determined by the past stock and wage and unemployment differentials between the United States and Mexico. Undocumented immigration is a complicated matter. Both the actual number of aliens who do not return home and the size of the network that safely imports them contribute to what would be measured as stock. The observation equations are the demand and supply of undocumented labor. The demand equation in their model (simplified) is $\ln(A) = c_0 + c_1 \ln(P) + c_2 \ln(w)$, where A is apprehensions, P is tomato price, and w is the wage rate. The c's are constant. The model is derived from a simple yield effort model: $\ln(A) = k \ln(BP) + \ln(N)$, where BP is apprehension effort by the Immigration and Naturalization Service and N is the stock of undocumented laborers. (They also assume that labor supply is proportional to N, so the demand equation for apprehensions really is the labor demand equation.) The natural way to expand the model would be to avoid the substitution of apprehensions for labor quantity and write $\ln(N)+k_1 \ln(BP) = c_0+c_1 \ln(P)+c_2 \ln(w)$ and $\ln(A) = k_2 \ln(BP)+\ln(N)$ as the two observation equations. The two different k's allow for the difference between labor supply and the stock of undocumented laborers. A third observation equation would be the labor supply equation which can be handled in a similar fashion.

The last example is the capital stock. The state equation is $K_t = K_{t-1} - \delta K_{t-1} + I_t$ or capital is depreciated at the unknown rate, δ, and replenished by investment, I. The usual method for constructing such a sequence is to take K_0 as some reasonable estimate and infer δ from depreciation data for various industries. Although the resultant numbers are used as the capital stock, they are clearly estimates with substantial probable error. Viewed this way, $Y = F(K, L, M)$; the aggregate production function in terms of capital (labor and materials) is just a measurement equation for the capital stock. The filter will produce stock estimates, estimates of the production function, and estimates of the reliability of the stock estimates.

Conclusions

Unobservable stochastic variables play a major role in many applied economic fields. In fishery economics, the major explanatory variable is the unobserved fish stock. Sampling methods based upon population dynamics are almost always supplemented with information inferred from the economic activity of the harvesting agents. In this paper, we used just the information from economic activity to infer stock. Our Pacific Halibut biomass estimates generally agree in pattern, though not magnitude, with the estimates made by IPHC. In the immediate postsample period, the filter estimates for area 3 provide more accurate predictions of halibut catch-per-unit effort than the IPHC estimates. The experience in the fishery was closer to our expectations than it was to those of the IPHC. We view this as providing some evidence for the utility of this method.

On the other hand, the vast disparity in our stock estimates and those of the IPHC suggests that our estimates ought to be called effective stock, that is, the unobserved variable that correlates well with fishing success.

This paper also provides several other examples of unobserved variables in economics. In each case the Kalman filter/maximum likelihood approach is a promising method for preserving the stochastic variability and endogeneity of the model during estimation. This method provides more information on the dynamics of the unobserved variable than has been available or used in past studies.

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