(Appendix to: When Promoters Like Scalpers) Global strategic complementarity in a global games setting

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Here we describe the relation between our (imperfect information) models with and without scalpers, and the models in two other papers that relax the assumption of global strategic complementarity in a global games setting. In the absence of scalpers, actions are global strategic complements; the advantage of taking a particular action increases with the measure of other agents who take the same action. When scalpers can enter, actions are strategic complements or substitutes depending on the measure of agents who take the action; in other words, actions are not global strategic complements. In Goldstein and Pauzner (2003)'s model of bank runs, actions are "one-sided strategic complements", a term that is defined below. Karp, Lee, and Mason (2003) study a model in which actions could be either strategic complements or substitutes.

The four panels in Figure 1 show – for four different games – the advantage of taking a particular action, as a function of the measure (α) of agents taking that action, holding fixed the value of a payoff-relevant variable γ and other parameters (such as the first period price in the ticket game, and the first period interest rate in the bank-run game). We denote the advantage of taking the action as Δ. (This function is conditional on α and γ and is therefore distinct from the function A – an expectation over γ and α– used in the text. For example, in the absence of scalpers, $\Delta = B^0 - \left(\gamma p^h - p_1\right)$; with scalpers, $\Delta = B - \left(\gamma p^h - p_1\right)$.)

Figure 1a graphs $\Delta\left(\alpha; \gamma.p_1\right)$ (the advantage of waiting) in the ticket game when there are no scalpers, taking as given γ and the first period price. The graph is strictly increasing for $\alpha < 1$. The dashed line shows the advantage of waiting given a larger value of γ. Figure 1b shows $\Delta\left(\alpha; \gamma.p_1\right)$ (the advantage of waiting) in the ticket game with scalpers. That function increases for $\alpha < \hat{\alpha}$ and then drops discontinuously. Provided that $\gamma p^h > p_1$ (as Figure 1b assumes) the agent loses positive surplus by waiting; therefore, $\Delta < 0$ for $\alpha > \hat{\alpha}$ (where the second period surplus is 0). Again, the dashed line shows that a larger value of γ reduces the advantage of waiting. For all four panels we assume that the value of γ is at a level such that the advantage of taking the action is positive for some range of α and negative for some other range.

Now we consider the payoffs in two other games. Figure 1c, adapted from Goldstein and Pauzner (2003), shows the graph of the advantage of "running on a bank", i.e. withdrawing funds from a bank "early", as a function of α. Here α is the measure of agents who run on the bank. An increase in the economic fundamental, γ, increases the chance that an investment will succeed in the second period, making it possible for the bank to pay a high return to agents who

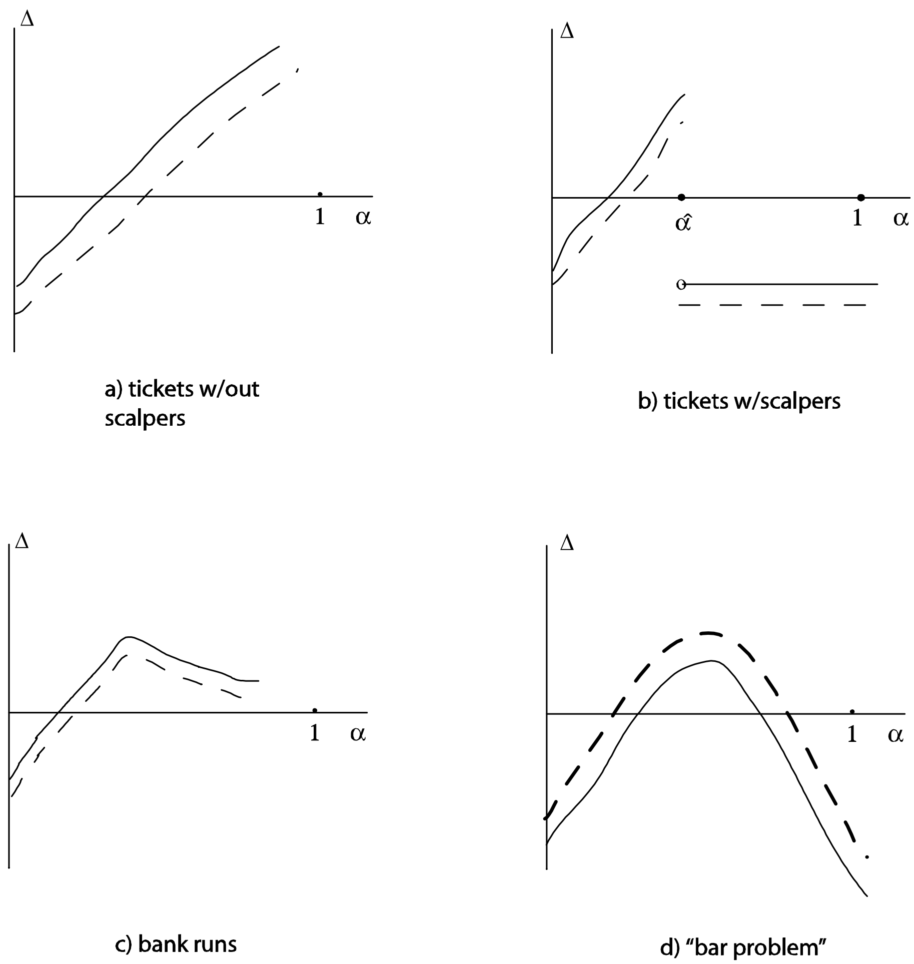

Figure 1: The benefit of a particular action as a function of α. (Dashed line shows benefit for higher γ.)

did not withdraw their deposits early. If few agents run on the bank (α is small) the optimal action is to not withdraw funds from the bank ($\Delta < 0$); if enough agents run on the bank, the bank is certain to default. In that case, $\Delta > 0$.

For α sufficiently large the slope of Δ is negative. For large α it remains optimal to withdraw funds, but the advantage of doing so decreases with α. Over this range of α the funds in the bank are insufficient to meet the demands of all the agents who try to withdraw their deposit. The bank's funds are rationed, so an increase in the number who try to withdraw decreases the amount that any individual receives. Therefore Δ is decreasing in α for large α. The fact that Δ is not monotonic means that the assumption of "global strategic complements" does not hold.

However, the graph of Δ has a single crossing, and it remains positive over the range where it decreases. Goldstein and Pauzner (2003) refer to this as "one-sided strategic complementarity". The dashed line shows the advantage of running on the bank given a stronger economic fundamental.

Figure 1d, adapted from Karp, Lee, and Mason (2003), shows the advantage of going to a bar rather than staying home and reading a book. Agents do not like bars that are nearly empty or very crowded, so (for a range of values of γ, including the value shown in the figure) $\Delta < 0$ for α close to 0 or close to 1. Agents' utility of going to the bar also depends on the quality of the band, given by γ. In this setting actions are strategic complements for small α and strategic substitutes for large α.

Goldstein and Pauzner (2003) show that there exists a unique equilibrium (a threshold equilibrium) in the bank run game despite the lack of global strategic complementarity. Karp, Lee, and Mason (2003) show that in the bar problem there is no threshold equilibrium – in fact no equilibrium that is monotonic in the signal – when the degree of strategic substitutability is high and ϵ is small. Thus, Figures 1b – 1d illustrate three departures from the global complementarity assumption, and three kinds of results: (b) a unique threshold equilibrium but no general uniqueness result, (c) a unique equilibrium, a threshold equilibrium, and (d) no threshold equilibrium. The "one-sided strategic complementarity" is crucial for Goldstein and Pauzner's proof of uniqueness; since this assumption does not hold for the games illustrated by figures 1b or 1d, the absence of a uniqueness proof in those cases is perhaps not surprising. However, Figures 1b and 1d both have two disjoint intervals over which $\Delta < 0$ (and therefore both violate the single-crossing property), so it may be surprising that there is a unique threshold equilibrium in one case and no threshold equilibrium in the other.

In order to explain this difference, we use Goldstein and Pauzner's intuitive explanation of the existence of a (unique) threshold equilibrium. This argument is valid for the games in Figures 1a and 1c. The explanation is also useful for the game in Figure 1b, because it shows that a local condition for existence is satisfied. However, the explanation completely breaks down for the game in Figure 1d. Goldstein and Pauzner's intuitive explanation is therefore useful in helping to understand why the games in Figures 1b and 1d have such different equilibria, despite the apparent similarities of the two payoff functions.

In all four games there exists a unique signal, that we denote as k, such that if an agent believes that all other agents are using the threshold strategy $I_k(\eta)$ defined in Equation (11),

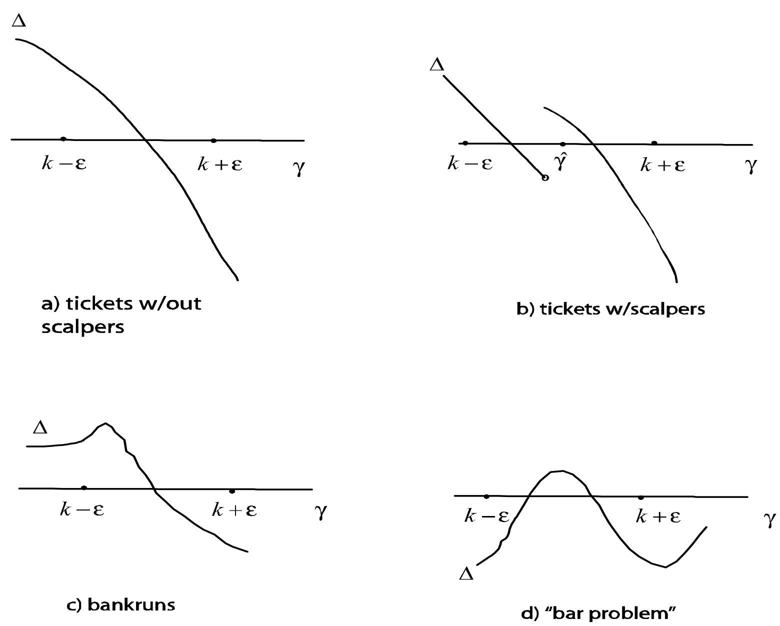

Figure 2: The benefit of an action as a function of γ, under a threshold strategy.

the agent who observes $\eta = k$ is indifferent between the two actions. Thus, in each of the four games there is a unique *candidate* threshold equilibrium. If all agents use this threshold strategy, the function $\alpha(\gamma; I_k)$ is given by Equation (12). No agent takes the action if $\gamma < k - \epsilon$ and all agents take the action if $\gamma > k + \epsilon$.

In order for the candidate to be an equilibrium, it must be the case that the expectation of Δ (conditional on the belief that all agents use the threshold strategy) is 0 conditional on the signal $\eta = k$ and that the expectation is negative (respectively, positive) for $\eta > k$ (respectively, $\eta < k$). This requirement implies that the graph of the conditional expectation of Δ (as a function of the signal) is decreasing in the neighborhood of k.

The four panels of Figure 2 show the graphs of Δ as a function of the state of nature γ, under the assumption that all agents use the threshold strategy $I_k(\eta)$. (The level of the threshold differs in the different games.) In the ticket game without scalpers (Figure 2a), where $\Delta = \gamma B^0 - \left(\gamma p^h - p_1\right)$, all agents wait for sufficiently low γ, and the advantage of waiting declines linearly with γ at the rate $B^0(1) - p^h$. For γ slightly above $k - \epsilon$, some agents receive a signal above k and decide to buy, reducing the chance of getting a ticket in the second period for those who wait. Thus, increases in γ lead to a faster reduction in the benefit of waiting. (The function Δ becomes concave.)

In the ticket game with scalpers, under the threshold strategy we have

$$\Delta = \left\{ \begin{array}{cc} -\left(\gamma p^h - p_1\right) & if \quad \gamma < \hat{\gamma} \\ B^0\left(\alpha\left(\gamma\right)\right) - \left(\gamma p^h - p_1\right) & if \quad \gamma > \hat{\gamma} \end{array} \right\}.$$

where $\hat{\gamma}$ is defined in Equation (16) and $\alpha\left(\gamma\right)$ is given by Equation (12). The function has a discontinuous upward jump at $\hat{\gamma}$ (Figure 2b).

In the model with bank runs, for very bad states of nature all agents run on the bank. The advantage of running on the bank is constant with respect to the state of nature (Figure 2c). For γ slightly above $k-\epsilon$, some agents receive a signal that induces them to not run on the bank. By keeping their deposits in the bank they increase the payment to those agents who withdraw their deposits, thereby increasing the advantage of running on the bank. Therefore $\frac{\partial \Delta}{\partial \gamma} > 0$ for γ slightly above $k-\epsilon$. When the economic fundamental becomes sufficiently strong, the increased probability that the investment will be successful reduces the attraction of making an early withdrawal; over this region $\frac{\partial \Delta}{\partial \gamma} < 0$, as shown.

When agents use a threshold strategy in the bar problem, they decide to go the bar for sufficiently large γ (e.g., a very good band). If the congestion effect is strong enough and ϵ is small enough, there is an interval of $\gamma > k+\epsilon$ over which $\Delta < 0$, as shown in Figure 2d. Over this interval the quality of the band (the value of γ) is not great enough to compensate for the amount of congestion.

By construction, the integral of Δ over $(k-\epsilon, k+\epsilon)$ is 0 for all four panels of Figure 2 (since agents who observe $\eta = k$ are indifferent between the two actions). Recall that in all four games, the posterior distribution of γ (conditional on the signal) is uniform over the support $[\eta-\epsilon, \eta+\epsilon]$. We are now in a position to describe how the value of this integral changes as the signal changes. For concreteness, consider an agent who observes a signal η' slightly larger than k. In Figures 2a - 2c, the integral of Δ over $(\eta'-\epsilon, \eta'+\epsilon)$ is negative, since, as the signal increases from k to η', we take away positive area and add negative area to the integral. Thus, given the signal $\eta' > k$, the expected value of waiting in the ticket game and the expected value of withdrawing funds in the bank-run game is negative, as required by equilibrium. The converse holds for signals slightly below k. This argument establishes that the expectation of Δ (conditional on the belief that all agents use the threshold strategy) is a decreasing function of the signal in the neighborhood of $\eta = k$. (We explained above why this negative slope is a requirement for equilibrium.)

In panels (a) and (c) the argument in the previous paragraph holds for all $\eta' > k$, not only in the neighborhood of k. This argument, together with the observation that there is a unique candidate threshold equilibrium, implies that there is a unique threshold equilibrium for the games described in panels (a) and (c) . (Goldstein and Pauzner's argument that there are no other types of equilibria in the bank run game is distinct from the argument summarized here.) The upward discontinuity in panel (b) prevents extending the graphical argument in the ticket game with scalpers away from the neighborhood of $\eta = k$. That is, the argument only shows that the slope of the expectation of Δ is negative at $\eta = k$ – a necessary condition for the existence of a threshold equilibrium. The argument cannot be used to show that the expectation of Δ as the same sign as $k - \eta$.

However, the graphical argument is not valid for the game in panel (d), even for signals in the neighborhood of k. When the signal increases from k to η', taking expectations of Δ involves both adding and subtracting a negative area to the integral. In this case, there is no reason to think that the expected payoff has the correct monotonicity. Karp, Lee, and Mason (2003) confirm that there is no threshold equilibrium in the bar game when congestion is large and ϵ is small.

References

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