Trade and Resource Policy with Overlapping Generations*

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Abstract

Trade changes incentives to protect an open-access natural resource. In an OLG setting, the capital asset market transfers policy-induced future gains and losses to the current asset owner. The asset market creates incentives for agents currently alive to protect the natural resource under autarchy. Trade reverses these incentives. In a dynamic political economy, resource policies in both the open loop and Markov Perfect equilibria protect the resource under autarchy; in the open economy these policies exacerbate the open-access distortion, harming the resource. The difference arises from the interplay of the asset market and general equilibrium effects. Trade generally lowers welfare.

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1 Introduction

Trade liberalization changes incentives to protect a natural resource, and potentially changes the endogenous enforcement of *de jure* property rights. This possibility is particularly important for small resource-rich countries with imperfect property rights. The body of theory that addresses this issue has largely ignored general equilibrium relations between factor and product markets, and the role of asset prices. We emphasize these two considerations, imbedding a general equilibrium model into an overlapping generations (OLG) setting. Policies that preserve a resource stock, e.g. forests or fisheries, change real factor returns, and also generate capital gains or losses. The links between policy, resource stocks, factor returns, and asset prices differ in open and closed economies. By understanding the manner in which trade alters these links, we see how trade changes incentives to protect a resource, thereby changing endogenous policy.

Opening a closed economy alters the effect of resource protection on real factor returns and on asset prices, simply because commodity prices are endogenous in a closed economy but exogenous in a small open economy. In a dynamic political economy, resource-protecting policies emerge endogenously in a closed economy. In an open economy, agents adopt policies that exacerbate weak property rights. The paper thus identifies a new and potentially important negative consequence of trade.

The productivity of many resources, such as forests and fisheries, depends on the level of the resource stock. Where there are property rights to a resource, its asset price reflects future payoff streams. Asset owners care about that price, even if they do not care about future consumption flows. The absence of property rights to the natural resource, and the consequent absence of a price for its stock, eliminates this link between future productivity and current conservation incentives.

Economies with weak property rights to a natural resource likely contain other productive assets enjoying strong property rights. The difference across assets of property rights might be due to custom or to enforcement problems. For example, local custom might support common property rights to a forest, but not to mining equipment. Because it is easier to build a wall around a factory than a fishery, enforcement problems are more challenging for the fishery.

Due to the general equilibrium link between conservation policy and real factor returns, resource policy can affect even the price of assets not used in the resource sector. People who own those assets, care about their prices, regardless of their intrinsic concern for the resource stock. To make this point as simply as possible, we adopt a two-sector Ricardo Viner model in which one sector uses the resource stock and the other uses privately owned capital. Both sectors use mobile labor. The resource stock affects the labor allocation, thus affecting the nominal wage and return to capital. Future resource stocks affect future returns to capital, thus affecting the current price of capital. Conservation policy that affects future resource stocks thus affects current owners of capital. This link is absent for workers, who cannot indenture themselves or their progeny. The relation between trade and incentives to conserve the resource turns on this asymmetry between owners of capital and labor.

If people currently alive apply the same discount rate to their successors' and their own utility, i.e. if they have a perfect bequest motive, then the OLG model collapses to the Infinitely Lived Agent model. We emphasize the other extreme case, where people care about their own future utility, but not that of their descendants. As a baseline, we also consider the social planner (= altruist agent) outcome.

To examine the effect of trade on endogenous policies, we imbed a political economy model into a dynamic game. In each period, the political economy equilibrium returns a resource tax or subsidy that maximizes the joint welfare of currently living agents. The political economy equilibrium in a period depends on the relation between the current policy and the price of the single privately owned asset, capital. That relation depends on future policies, which emerge from the political economy equilibria involving future agents. The outcome depends on the trade regime.

We are interested in a subgame perfect equilibrium to this sequential game. To obtain intuition, we first consider the analytically tractable (but not subgame perfect) open loop Nash equilibrium. We show that in a closed economy, the open loop equilibrium policy in each period is a tax that protects the natural resource. In the small open economy, the equilibrium policy is a subsidy that exacerbates the lack of property rights. We then use numerical methods to study a Markov Perfect

Equilibrium (MPE); we find the same qualitative features as in the open loop setting.

A tax (or subsidy) enables agents to perfectly control current extraction. However, agents in the current period cannot choose future taxes. The endogenous policy is the equilibrium to a sequential game. Our political economy setting does not allow current agents to mandate consumption transfers from the not-yet-born to themselves, e.g. by using social security or bonds. Asset markets are the sole mechanism for transferring consumption from the future to the present. (Resource conservation transfers consumption from the currently living to the not-yet-born.) The ability of currently living agents to determine current extraction using a tax, does not imply that they could also create property rights to the resource. Meaningful property rights require an allocation of the resource not only between the generations currently living, but also among all future generations. Currently living agents cannot bind their successors to respect an allocation, for essentially the same reason that current agents cannot bind their successors to adopt a particular tax.

In summary, we model an economy with two productive assets, privately owned capital and an open-access resource. For reasons outside the model, society respects property rights for the former but not the latter. People alive in a period can govern their own use of the resource stock, but their incentive for doing so depends on their expectations of future behavior, and those expectations depend on the trade regime.

Related literature Bulte and Barbier (2005) and Ruta and Venables (2013) review literature on trade in resource-rich countries with imperfect property rights. Much of this literature compares the consequences of trade under open access and under perfect property rights (Brander and Taylor, 1997a, 1998). Trade creates standard gains, but for a resource exporter with open access, trade exacerbates over-exploitation of the resource. In the short run, with a predetermined resource stock, only the standard gains occur, so trade improves short run welfare. Over time, the trade-induced fall in the open-access resource stock creates a welfare loss. If the world price is only slightly higher than the closed economy price, then the standard gain from trade is small, but the worsening open-access problem is non-negligible. In this case, trade reduces long run aggregate welfare. A related strand of literature studies trade with common property rather than open-access (Chichilnisky, 1994; Brander and Taylor, 1997b; and Karp et al., 2001).

Copeland (2005) describes how corruption and special interest groups influence the evolution of property rights. The trade regime affects both the incentives and the costs of enforcing *de jure* property rights. Trade therefore can affect the level of *de facto* property rights. Where private and social benefits of protecting property rights differ, private enforcement decisions are not socially optimal (de Meza and Gould, 1994). In this second-best setting, trade induces a change in *de facto* property rights that might lower welfare (Hotte et al., 2000; Karp, 2005). Even if a social planner chooses the level of enforcement, a higher resource price induced by trade might increase incentives to poach, thereby increasing enforcement costs and possibly lowering welfare (Copeland and Taylor, 2009). With endogenous *de facto* property rights, trade has ambiguous welfare effects, regardless of whether a social planner or private agents make enforcement decisions.

Most of the papers above are either static or consist of a sequence of static equilibria. The exceptions, Hotte et al. (2005) and Copeland and Taylor (2009), use dynamic optimization models, but both consider an infinitely lived agent – private owners in the former and a social planner in the latter. These papers consider only the steady state to the optimal control problems. Van Long (2010) surveys natural resource applications of dynamic games, concluding that this literature relies too heavily on the infinitely lived agent framework and largely fails to consider endogenous policy evolution.[1]

Our political economy model of endogenous policy builds on insights and techniques used in the macro literature that studies the provision of a public good and/or redistribution across generations (Hassler et al. 2003, 2005, 2007; Conde-Ruiz and Galaso, 2005; Klein et al., 2008; Bassetto, 2008). We extend Karp and Rezai (2013) in order to compare the open and closed economies.

[1]Natural resource and environmental applications of OLG models begin with Kemp and Long (1979). Most subsequent contributions study sustainability and the correction of environmental externalities under a social planner (Howarth and Norgaard, 1992; Mourmouras, 1991; John and Pecchenino, 1994; Howarth, 1998; Bovenberg and Heijdra, 1998; Laurent-Lucchetti and Leach, 2011). These papers study only closed economy models.

A small but growing empirical literature examines the relation between environmental outcomes and asset prices, e.g. for stocks (Konar and Cohen, 2001) or houses (Chay and Greenstone, 2005). A related literature measures the relation between regulation and firm profits (Lin, 2008, Bushnell et al, 2013). These empirical papers measure direct connections between the environmental outcome or policy and the asset price or profit level. The links that we study are indirect, operating only through general equilibrium (factor market) channels, and are therefore likely to be harder to measure econometrically. In addition, the environmental outcomes or policies in the empirical settings above are (plausibly) exogenous to the asset price or profit level, whereas in our setting, the policies are endogenous.

2 Model

We describe the model and show that resource protection has the same qualitative effect on real returns to capital and labor in the closed economy, but opposite effects in an open economy. This difference is the basis for the trade regime's role in determining agents' incentives to protect the resource stock (Section 3). Understanding those incentives helps explain the relation between the trade regime and endogenous policies in the subgame perfect setting (Section 4).

2.1 Fundamentals

The economy consists of a manufacturing sector, M, and a resource sector, F. Capital and labor stocks are constant, both normalized to 1. Labor is mobile across sectors, capital is specific to sector M, and the resource stock, x_t, is specific to sector F. With L_t units of labor in sector F, production equals $F_t = \gamma L_t x_t$ with $\gamma > 0$, and $M_t = (1 - L_t)^{\beta}$ with $0 < \beta < 1$; there are profits (rent) in sector M, but not in the resource sector due to open-access. Manufacturing is the numeraire good, and p_t is the price of F in period t. In a closed economy, the price is a function of the resource stock, x_t, and the resource tax, T_t: $p_t = p\left(T_t, x_t\right)$. In a small open economy, p_t is a constant, P. We use p_t to denote the price in the generic case, and when no confusion results, we suppress time subscripts.

Policy and factor returns Open access means that too much labor moves to the resource sector. Society can improve the allocation by imposing an ad-valorem tax, T_t, on production of good F (a "resource tax"). Resource-sector workers receive revenue $\gamma p_t(1-T_t)L_t x_t$. Society returns the tax revenue, $R_t = \gamma p_t T_t L_t x_t$, in a lump sum, giving the fraction $\chi \in [0,1]$ to the young generation and $1-\chi$ to the old. (Later we allow χ to vary over time.) If the tax is negative (a subsidy), then $R_t < 0$, and the policy has a fiscal cost; χ determines the generations' share of this cost.

In a diversified economy, workers are indifferent between working in either sector. An open economy might specialize in the manufacturing sector, but, because of manufacturing rents, it never specializes in the resource sector. The equilibrium wage is $w_t = p_t(1-T_t)\gamma x_t$ if the economy is diversified, and $w_t = \beta$ if the economy specializes; $p_t(1-T_t)$ is the relative producer price of good F. Given a labor allocation, the equilibrium rental rate is $\pi_t = (1-\beta)(1-L_t)^{\beta}$. The price of the manufacturing asset, capital, is σ_t.

Dynamics Agents live for two periods and consume both goods in both periods. The old agent owns the manufacturing asset, and the young agent owns labor. The young worker divides income, $w_t + \chi R_t$, into current consumption and saving for retirement, achieved by purchase of shares (s_t). The old agent spends all her income, obtained from tax revenue and renting for a period and then selling her assets, $(1-\chi)R_t + \pi_t + s_{t-1}\sigma_t$. An agent who consumes $c_{F,t}$, and $c_{M,t}$ units of F and M obtains utility $u(c_{F,t},\ c_{M,t}) = \frac{1}{\mu} c_{F,t}^{\alpha} c_{M,t}^{1-\alpha}$ with scaling parameter $\mu = \alpha^{\alpha}(1-\alpha)^{1-\alpha}$. Agents spend a constant share, α, on the resource good. Their indirect utility function, v, is linear in expenditures, e: $v(e,p) = p^{-\alpha}e$.

A young agent who buys s_t shares of capital in period t spends $e_t^y = w_t + \chi R_t - s_t\sigma_t$ on consumption. In the next period, she spends $e_{t+1}^o = (1-\chi)R_{t+1} + s_t(\pi_{t+1} + \sigma_{t+1})$. Using $v(e,p) = p^{-\alpha}e$, her savings decision solves

$$\max_{s_t} p_t^{-\alpha}(w_t + \chi R_t - s_t\sigma_t) + \frac{1}{1+\rho} p_{t+1}^{-\alpha}\left((1-\chi)R_{t+1} + s_t(\pi_{t+1} + \sigma_{t+1})\right),$$

where $\rho > 0$ is the pure rate of time preference. With positive savings ($s_t > 0$), the agent's optimality condition is

$$p_t^{-\alpha}\sigma_t = (1+\rho)^{-1} p_{t+1}^{-\alpha} \left(\pi_{t+1} + \sigma_{t+1}\right). \tag{1}$$

Equilibrium supply and demand of assets are equal, so $s_t = 1\ \forall t$. The simplicity of equation (1) is due to agents' infinite intertemporal elasticity of substitution. We obtain, through repeated substitution, the price equation for capital:

$$\sigma_t = p_t^{\alpha} \sum_{i=1}^{\infty} (1+\rho)^{-i} p_{t+i}^{-\alpha} \pi_{t+i}. \tag{2}$$

The price of capital (measured in units of good M) equals the present discounted stream of future real returns, converted to current numeraire units.

Welfare for the young and old generations, W_t^y and W_t^o, equals:

$$\begin{aligned} W_t^y &= p_t^{-\alpha}\left[w_t + \chi_t R_t\right] + \tfrac{1}{1+\rho} p_{t+1}^{-\alpha}\left[\left(1-\chi_{t+1}\right) R_{t+1}\right] \\ W_t^o &= p_t^{-\alpha}\left[\pi_t + (1-\chi_t) R_t + \sigma_t\right]. \end{aligned} \tag{3}$$

The first line of this system follows by substituting equation (1) into the young agent's maximand. This agent's welfare equals the present value utility obtained from the wage and tax revenue, and is independent of σ_t. An unanticipated change in the asset price does not affect the young generation's welfare in any period. For example, a higher current asset price reduces their current consumption expenditures and thus reduces their current utility; their utility gain in the next period, made possible by the higher next-period asset price, and higher consumption expenditures, exactly offsets the current utility loss.

The natural resource stock is predetermined in each period, but changes endogenously over time. The stock obeys a logistic growth function,

$$\begin{aligned} x_{t+1} &= x_t + r x_t \left(1 - \frac{x_t}{C}\right) - L\gamma x_t = \left(1 + r\left(1 - \frac{x_t}{C}\right) - L\gamma\right) x_t \\ &= \left(1 + \bar{r}(x_t, T_t)\right) x_t;\ \text{with } \bar{r}(\cdot) \equiv r\left(1 - \frac{x_t}{C}\right) - L(x_t, T_t)\gamma, \end{aligned} \tag{4}$$

with r the intrinsic growth rate, C the carrying capacity of the resource, and the function $\bar{r}(\cdot)$ the actual growth rate of the resource; $L = L(x_t, T_t)$ is the equilibrium amount of labor in the resource sector.

2.2 Comparative statics

The comparative statics of nominal factor returns, with respect to policy and the resource stock, are the same in the open and closed economies. The comparative statics of real returns differ in the two trade regimes. Our primary results use Cobb-Douglas utility and M-sector production functions, but the intuition for those results turns on simple and more general comparative statics. To emphasize this point, we state the comparative statics of nominal returns in a more general setting:

Lemma 1 *(Nominal factor returns) Suppose that utility is homothetic and the production possibility frontier is concave, and that the economy is diversified. Under either trade regime: (i) a small tax increases the next-period stock and has a* 0 *first order effect and a negative second order effect on current aggregate utility. (ii) A higher resource tax decreases* L *and* w *and increases* π*. (iii) A higher resource stock decreases* π *and increases* L*,* w*, and aggregate utility.*

(Appendix A.1.) These results are obvious. For example, a higher tax or a lower stock causes workers to move from sector F to M, lowering w and increasing π.

Real factor returns equal the amount of utility that an agent obtains by renting one unit of labor or one unit of capital. For general utility functions, a sufficient condition for the real wage to increase (respectively, fall) is that both w and $\frac{w}{p}$ increase (respectively, fall); the analogous sufficient condition holds for the real return to capital. In an open economy, with fixed p, real factor returns change in the same direction as nominal factor returns. This observation and Lemma 1 imply:

Corollary 1 *In a diversified open economy with homothetic utility and concave production possibility frontier, a higher tax or a lower resource stock increases the real return to capital and decreases the real return to labor.*

For the closed economy, with endogenous p, we have (Karp and Rezai (2013)):

Remark 1 *With the Cobb-Douglas specification, in the closed economy, a higher tax or a lower resource stock reduces both the real wage and the real rental rate.*

Here, real factor returns have the same qualitative response to a change in either the tax or the stock. This fact is key to our results, so it is important to understand it.

The effect of the tax on the real wage is obvious, and holds in general. A higher tax causes labor to leave the resource sector, decreasing the nominal wage and increasing the relative commodity price: w and $\frac{w}{p}$ both fall. These two changes both contribute to a lower real wage, so a higher tax lowers the real wage. The effect of the stock on the real wage involves a balance of offsetting forces. A higher stock increases w at fixed p, but the higher stock also decreases p. The higher stock always increases $\frac{w}{p}$, but it has an ambiguous effect on the nominal wage, w, and therefore has an ambiguous effect on the real wage.

Similarly, the comparative statics of the real return to capital depend on the balance between two forces, which depends on functional assumptions. For example, the tax-induced decrease in the nominal wage increases nominal profits, π. Under fairly general circumstances (including the Cobb Douglas functions), the tax-induced price increase lowers $\frac{\pi}{p}$. The effect of a higher tax on the real return to capital thus depends on the balance of these two changes. For the Cobb Douglas specification, the second effect (lower $\frac{\pi}{p}$) dominates, so the higher tax lowers the real return to capital. In other circumstances, the first effect (higher π) might dominate. Thus, in general, a higher tax has an ambiguous effect on the real return to capital. For similar reasons, a change in x has an ambiguous effect on the real return to capital.

Thus, in general, the real returns to the two factors might have the same or different qualitative responses to changes in the tax or the stock. A complete taxonomy of these changes would take us far away from our research question, and is complicated by the fact that we can vary both the M-sector production function and the utility function. Because our results rely on the real returns having the same qualitative response to changes in T or x, so it is important to establish that this feature does not rely on the Cobb Douglas specification. We therefore provide (Appendix A.2):

Proposition 1 *Consider a closed economy in which agents have identical homothetic preferences, and M-sector production has constant elasticity of substitution η. (i) If $\eta = \infty$ and some labor is employed in sector M in the initial equilibrium, then a higher tax or a lower resource stock lowers the real return to both factors. (ii) If $\eta = 0$ and capital is fully employed with a strictly positive rental rate before the change in T or x, then: (iia) A higher tax increases the real return to capital and lowers the real return to labor. (iib) A higher resource stock has ambiguous effects on the real returns to both factors.*

Given the employment assumptions in the Proposition, and the continuity in η of endogenous prices, the Proposition implies that the two factor prices change in the same direction for η sufficiently large. The minimal value of η, below which the dependence flips, also depends on the utility function. With Cobb Douglas utility, $\eta = 1$ (corresponding to Cobb Douglas production) is above this minimal value. In short, the qualitative general equilibrium effects that we rely require sufficiently high elasticity of substitution between factors in sector M; the Cobb Douglas specification simplifies all calculations, but is not critical to our results.

Table 1 summarizes the effect of a higher tax or a larger resource stock on nominal and real factor returns, given sufficiently large η, homothetic preferences, and the employment assumptions in Proposition 1. Trade fixes the commodity price, eliminating the mechanism that, in the closed economy, causes the factor prices to move in the same direction.

		nominal	real (open economy)	real (closed economy)
higher T	w	$-$	$-$	$-$
	π	$+$	$+$	$-$
higher x	w	$+$	$+$	$+$
	π	$-$	$-$	$+$

Table 1: Effect of a change in T or x on nominal and real factor returns

3 Open loop equilibrium policies

We are primarily interested in the relation between the trade regime and endogenous policies when agents are fully rational. We begin with a simpler question: How does the trade regime influence agents' incentives to choose a policy in the current period, when they take as given future taxes or subsidies? That is, here we consider an *open loop* Nash equilibrium; this equilibrium is time consistent but not subgame perfect. We find that in the closed economy, in any open loop equilibrium agents use a resource tax, protecting the resource stock and benefiting their successors. In the open economy, these agents use a resource subsidy, damaging the resource stock and harming their successors. Section 4 confirms that the Markov Perfect equilibrium shares these features.

The relative consumer price of the resource good in period t is $p_t = p(x_t, T_t)$; in the open economy $p_t = P$, the constant world price, but p_t is a function in the closed economy. From now on, to simplify exposition, we adopt the Cobb Douglas specification. The nominal value of aggregate income in period t is $Y_t = Y(x_t, T_t) = p_t F_t + (1 - L_t)^{\beta} = \pi_t + w_t + R_t$, and the aggregate utility (real national income) is $p^{-\alpha}(x_t, T_t) Y(x_t, T_t)$. National income is independent of χ_t, a parameter that merely affects the allocation of tax revenue. Agents in the current period choose the current tax and revenue allocation, T_t and χ_t.

Consistent with our assumption of no altruism, the current tax and sharing rule maximize the weighted sum of the lifetime welfare of agents currently alive. The weight on the old generation's welfare equals 1, and the weight on the young generation's welfare equals $1 + \delta$, where δ is a parameter. The optimal current revenue share and tax maximize the political preference function,

$$\begin{gathered}\tilde{W}_t \equiv W_t^o + (1 + \delta) W_t^y \\ = p_t^{-\alpha} [Y_t + \sigma_t] + \delta \left(p_t^{-\alpha} [w_t + \chi_t R_t] \right) + \tfrac{1+\delta}{1+\rho} \left[p_{t+1}^{-\alpha} \left(1 - \chi_{t+1}\right) R_{t+1} \right].\end{gathered}$$

There are several models for which the equilibrium can be obtained by maximizing a weighted sum of agents' welfare: a Nash bargaining model with transfers; a Nash bargaining model without transfers, but with particular bargaining weights (for recent applications see Rausser et al., 2010); and a probabilistic voting model (as in Lindbeck and Weibull, 1987; Perrson and Tabellini, 2000; Hassler et al., 2005). Our results do not depend on the micro-foundations of this political preference function.

Because $\tilde{W}_t$ is linear in χ_t, for $\delta \neq 0$ the equilibrium value of χ_t is on the boundary of its feasible set, which we take to be $[0, 1]$. The optimal revenue split, χ_t^*, is

$$\chi_t^* = \arg\max_{\chi_t} \tilde{W}_t = \left\{ \begin{array}{c} 0 \\ \text{indeterminate} \\ 1 \end{array} \right\} \text{ if } \left\{ \begin{array}{c} \delta R_t < 0 \\ \delta R_t = 0 \\ \delta R_t > 0 \end{array} \right\}. \tag{5}$$

The parameter δ is fixed, but R_t is endogenous, and has the same sign as T_t. Our principal result does not depend on whether χ_t^* equals 0 or 1, and therefore does not depend on the sign of δ.

Hereafter, we assume that $\delta \neq 0$, and $|\delta|$ is small. The assumption $\delta \neq 0$ eliminates the indeterminate case in equation (5), because in equilibrium $R_t \neq 0$. The assumption that $|\delta|$ is small means that agents choose the current tax or subsidy to increase the aggregate lifetime welfare of those currently alive, not to transfer income from one currently living generation to the other. Agents who tax resource extraction leave a higher stock, benefitting their successors; those who subsidize resource use harm their successors. But these selfish agents do not care about their successors. For $\delta \approx 0$, the value of χ_t has a negligible effect on the optimal tax, because χ_t (essentially) only creates a transfer between the current generations, which have (essentially) the same weight in the political preference function. In contrast, even for $\delta \approx 0$, χ_{t+1} has a non-negligible effect on the optimal tax; χ_{t+1} determines a transfer between the current and the next period young,who are not represented in the current political preference function. The next period transfer depends on x_{t+1}, which depends on T_t.

The assumption that $\delta \approx 0$ allows us to replace the political preference function $\tilde{W}_t$ with W_t:

$$W_t\left(x_t,T_t\right)\equiv\left.\tilde{W}_t\right|_{\delta=0}=p_t^{-\alpha}\left(Y_t+\sigma_t\right)+\frac{1}{1+\rho}p_{t+1}^{-\alpha}\left[\left(1-\chi_{t+1}\right)R_{t+1}\right], \tag{6}$$

the sum of three terms: the real income associated with period t production, the utility value of wealth, and the present utility value of the next period tax receipts received by the current young. The last two of these terms depend on future taxes. Hereafter, we use the function W_t rather than $\tilde{W}_t$.

In order to obtain clear and simple results, we adopt

Assumption 1 *(i) The taxes are constrained to lie in the interval* $[-\varepsilon,\varepsilon]$ *for some* $\varepsilon>0$. *(ii) For all taxes in this interval, the open economy remains diversified. (iii) For all taxes in this interval, a small decrease in current extraction, and the consequent small increase in the next period stock, also increases all future stocks outside the steady state. (iv) For all taxes in this interval, the maximand* $W_t\left(x_t,T_t\right)$ *is concave in* T_t.

Assumption 1.i is important because we want to restrict attention to small policy-induced changes in the stock. Large taxes might induce large changes in the stock. Assumption 1.ii allows us to focus on the interesting cases, where the economy is diversified. For the closed economy, we can find parameter restrictions that guarantee the monotonicity in Assumption 1.iii under BAU (and thus also for small ε). We are not able to find similarly simple parameter restrictions for the open economy, where the equilibrium depends on the world price. By adopting Assumption 1.iii we circumvent these uninteresting complications. (Extensive numerical results, described in Appendix A.5, show that with ε small, Assumptions 1.ii and 1.iii are satisfied for a wide range of parameters.) Assumption 1.iv enables us to determine the sign of the optimal policy by evaluating the derivative of welfare at a zero tax. Assumption 1 mean that the analytic results of this section tell us whether the equilibrium taxes are positive or negative, given that they are restricted to be small.

Lemma 2 *For a* ***fixed*** *sequence of future resource taxes/subsidies that satisfy Assumption 1.i–iii: (i) A current tax increases the utility value of the asset,* $p_t^{-\alpha}\sigma_t=\sum_{i=1}^{\infty}(1+\rho)^{-i}p_{t+i}^{-\alpha}\pi_{t+i}$ *in the closed economy, and decreases the utility value of the* *asset in the open economy. (ii) In both trade regimes, with ε small, an increase in the current tax increases the utility from next-period tax revenue, $p_{t+1}^{-\alpha}R_{t+1}$, if and only if $T_{t+1} > 0$.*

Proof. (i) The current tax lowers current harvest, increasing the next-period stock. By Assumption 1.i-iii, the higher next-period stock increases all future stocks. Table 1 shows that these changes increase future real returns to capital in a closed economy and decrease these real returns in the open diversified economy. The claim then follows from equation (2). (ii) The next period utility value of the tax revenue equals $p_{t+1}^{-\alpha}R_{t+1} = T_{t+1}\left[p_{t+1}^{1-\alpha}\gamma L_{t+1}x_{t+1}\right]$. Straightforward calculations (Appendix A.3) show that for small ε, $\left[p_{t+1}^{1-\alpha}\gamma L_{t+1}x_{t+1}\right]$ is an increasing function of x_{t+1}, and thus an increasing function of T_t. ■

Taking as given future taxes, we denote T_t^j as the optimal period t tax in trade regime $j \in \{\text{open, closed}\}$:

$$T_t^j\left(x_t\right) = \arg\max_{T_t} W_t\left(x_t, T_t\right). \tag{7}$$

That is, T_t^j is the best response to the future sequence of taxes, given the current stock x_t. The next result is an immediate consequence of Lemma 2:

Lemma 3 *For a **fixed** sequence of future taxes/subsidies that satisfy Assumption 1.i–iv, with ε small, a sufficient condition for $T_t^{closed} > 0$ is that $T_{t+1}^{closed} \geq 0$; a sufficient condition for $T_t^{open} < 0$ is that $T_{t+1}^{open} \leq 0$.*

Proof. Given Assumption 1.iv, the optimal current tax is positive if and only if

$$\left.\frac{dW_t\left(x_t, T_t\right)}{dT_t}\right|_{T_t=0} > 0.$$

In both the open and closed economy, the zero-tax Business as Usual (BAU) maximizes current national income. Consequently $\left.\frac{d\left(p_t^{-\alpha}Y_t\right)}{dT_t}\right|_{T_t=0} = 0$. This equality and equation (6) imply

$$\left.\frac{dW_t\left(x_t,T_t\right)}{dT_t}\right|_{T_t=0}=\left[\frac{d\left(p_t^{-\alpha}\sigma_t\right)}{dT_t}+\frac{1-\chi_{t+1}}{1+\rho}\frac{d\left(p_{t+1}^{-\alpha}R_{t+1}\right)}{dT_t}\right]_{T_t=0}.$$

In the closed economy, the first term on the right side is positive (by Lemma 2.i) and the second is non-negative if $T_{t+1}\geq 0$ (by Lemma 2.ii) Therefore, the derivative is positive if $T_{t+1}\geq 0$. Similarly, for the open economy, the first term on the right side is negative and the second term is non-positive if $T_{t+1}\leq 0$. Therefore, the derivative is negative if $T_{t+1}\leq 0$. ■

We take the equilibrium to our infinite horizon model to be the limit, as the horizon becomes large, of the equilibrium to a finite horizon model.[2] This interpretation requires modifying the asset price equation (2), replacing the upper limit ∞ by the length of the horizon, and it requires recognizing that in the last period of the finite horizon setting the young generation lives for a single period. We use the index k to denote the length of a horizon at the beginning of the game; we use the superscript $m\leq k$ to denote the number of periods to go in a game that, at the initial time, has k periods. At $m=0$, the economy is in the last period. The vector $\mathbf{T}^k\in\mathbb{R}^k$ is the list of tax levels in the k-horizon game, and the scalar ${}^mT\in\mathbf{T}^k$ is the tax used when there are m periods to go. (We place the superscript m to the left in order to avoid confusion with the index j, which denotes the trade regime.) An open loop Nash equilibrium is a trajectory of taxes, in which each element is a best response to all other elements:

Definition 1 *(OLNE) An open loop Nash equilibrium in the k-stage sequential game is a vector $\mathbf{T}^k\in\mathbb{R}^k$ with the property that the m'th element, ${}^mT\in\mathbf{T}^k$, maximizes currently living agents' joint welfare when there are m stages-to-go, given that both past and future taxes equal the corresponding elements of $\mathbf{T}^k$. An OLNE of the infinite horizon model is a limit, as $k\to\infty$, of an equilibrium $\mathbf{T}^k$.*

[2] By ruling out history-dependent strategies, the open loop setting excludes the use of "punishment" or "trigger" strategy equilibria of the type familiar in supergames. Our formalism, treating the equilibrium to the infinite horizon model as a limit, merely provides a simple way to discuss the infinite horizon game; it is not needed in order to exclude punishment or trigger strategies.

An OLNE depends on the level of the stock at the time the game begins. This initial condition and previous taxes determines the level of the stock when there are m periods to go. The optimal decision in that period depends on the current stock and future taxes, as discussed above. Under either trade regime, in the last period the asset price is 0 and the equilibrium tax is also 0, the value that maximizes current real national income. Thus, for any $k \geq 0$ and under either trade regime, we have ${}^{0}T = 0$. Using a straightforward modification of the proof of Lemma 3 (to account for the finite horizon setting), ${}^{1}T > 0$ in the closed economy, and ${}^{1}T < 0$ in the open economy. Under Assumption 1, we have by induction: ${}^{m}T > 0$ for the closed economy and ${}^{m}T < 0$ for the open economy, for all $m \leq k$. This argument leads directly to our chief analytic result:

Proposition 2 *Under Assumption 1 with small ε, the policy in every period except for the last in any k-stage OLNE: (a) involves a tax in the closed economy, and (b) involves a subsidy in the open economy. OLNE in the infinite horizon model share these characteristics, except of course that there is no "last period".*

Discussion In a closed economy OLNE, agents tax resource use; in an open economy OLNE, they subsidize resource use. Agents are not altruistic, so they chose the current policy without considering its effect on their successors' welfare. Of course, the current policy affects future stock levels, thus affecting future welfare. Future policies affect the current asset value, thus affecting current welfare.

The OLNE analysis helps to show how incentives to protect the resource differ in closed and open economies. A resource tax affects the utility associated with ownership of the asset and future tax receipts. In the open economy, a higher future stock lowers future returns to capital, thus lowering the price (and the utility value) of the asset, and encouraging current agents to subsidize resource extraction. In addition, if successors use a subsidy, then current agents anticipate fiscal costs of the policy ($R < 0$), and they understand the current young generation will pay the fraction $1 - \chi_{t+1}$ of those costs. By lowering the next period stock (thus lowering labor productivity in the resource sector), they decrease output of the resource sector in the next period. Because the subsidy is incurred for each unit of the harvest, the lower stock lowers the next period fiscal cost, thus weakly lowering the current young generation's future obligation. The closed economy reverses these incentives.

Under both trade regimes, the current policy creates capital gains or losses. The change in the asset price, induced by the resource policy, transfers some of the future costs or benefits, of the policy, to those currently alive. In the closed economy, the incentives operate to increase future welfare, i.e. to protect the resource stock. The change in the asset price enables current agents to obtain some of those future benefits. In this case, the current policy increases the "intertemporal pie", and enables those currently living to obtain some of this surplus. In the open economy, the incentives operate to shrink the "intertemporal pie", by reducing the resource stock; but those currently living nevertheless benefit from the subsidy.

The welfare effect of moving from BAU to an OLNE are ambiguous. Karp and Rezai (2013) show that in a closed economy, a sequence of small taxes increases aggregate lifetime welfare of those alive in the first period. The lifetime welfare of agents born in the future, W_t^y, equals the real income associated with the sum of their wage and share of tax revenues (equation (3)). The stock trajectory in a closed economy OLNE is above the BAU trajectory, by Proposition 2. This fact tends to increase future young generations' welfare. However, the future taxes lower the real wage of these agents, so the welfare comparison in a closed economy, between OLNE and BAU, is ambiguous. An open economy OLNE stock trajectory is lower than the BAU trajectory, lowering future young generations' welfare. However, the future subsidies to the resource sector increase their wage, again making the welfare comparison ambiguous.

4 Markov Perfect equilibrium policies

The OLNE is time-consistent but not subgame perfect: if any agent uses a non-equilibrium tax or subsidy, the trajectory of resource stocks departs from the trajectory that agents assumed when choosing their policy. The continuation of the OLNE, obtained in the initial period, therefore is not an equilibrium following the deviation. In a subgame perfect equilibrium, agents not only form point expectations about the level of successors' actions, but also form expectations about how those actions would change if the current agent were to deviate from equilibrium. We study a Markov Perfect equilibrium (MPE), in which actions and point expectations about successors' actions are conditioned on the "directly payoff-relevant state variable". Here, the only such variable is the resource stock. Using numerical methods, we obtain the MPE without restricting policies to be small or assuming concavity, dispensing with Assumption 1. We choose parameter values so that the BAU adjustment path is monotonic and the open economy remains diversified.

We describe the MPE for the generic problem and then specialize to obtain the MPEs in the two trade regimes. Denote $\Upsilon(x_t)$ as an *arbitrary* policy function, mapping the period-t resource stock into the period-t tax. If the current stock and tax equal (x_t, T_t), and all future taxes equal $T_{t+i} = \Upsilon(x_{t+i})$, $i \geq 1$, then $\Upsilon(x_t)$ induces an asset price function (a functional in Υ). This function, $\sigma(x_t, T_t)$, is defined recursively:

$$\begin{gathered} p_t^{-\alpha}(x_t, T_t)\,\sigma(x_t, T_t) = \\ (1+\rho)^{-1}\left\{p_{t+1}^{-\alpha}(x_{t+1}, \Upsilon(x_{t+1}))\left[\pi(x_{t+1}, \Upsilon(x_{t+1})) + \sigma(x_{t+1}, \Upsilon(x_{t+1}))\right]\right\}. \end{gathered} \tag{8}$$

Equation (8) repeats the equilibrium savings condition, equation (1), showing the dependence of the endogenous function $\sigma(x_t, T_t)$ on x_t, T_t, and the function $\Upsilon(x_{t+1})$.

A MPE is a function $\Upsilon(x)$ for which: $\Upsilon(x) = \arg\max_T W_t(x, T)$, with $\sigma(x_t, T_t)$ the solution to equation (8), where χ_{t+1} satisfies equation (5), and next period tax is evaluated using $T_{t+1} = \Upsilon(x_{t+1})$. Finding $\Upsilon(x)$ is a standard fixed point problem, which can be solved using the collocation method and Chebyshev polynomials (Judd, 1998; Miranda and Fackler, 2002). (Appendix A.4).

One feature of the solution method requires comment. Instead of using equation (5) to determine χ_{t+1}, it is simpler to use this equation to determine the sign of δ underlying an equilibrium. If the sign of $\Upsilon(x)$ does not change with x (given the trade regime and parameter values), then the sign of the equilibrium R also does not change with x. In this case, equation (5) implies that for this particular equilibrium,

χ^* is constant, equal to either 0 or 1, in every period. In order to use this fact, we solve (for both trade regimes) the model with both $\chi^* = 0$ and $\chi^* = 1$, giving us four scenarios. In each of these scenarios we confirm that the (different) functions $\Upsilon(x)$ do not change signs with changes in x. For both values of χ, we also find that the equilibrium $\Upsilon(x)$ is positive in the closed economy and negative in the open economy, just as in any OLNE. Thus, R is positive in the closed economy and negative in the open economy. Given the assumed value of χ^*, we can then use equation (5) to infer the sign of δ, as Table 2 reports. For example, in the closed economy, $\chi^* = 1$ corresponds to $\delta > 0$, whereas in the open economy, $\chi^* = 1$ corresponds to $\delta < 0$. Thus, if the young generation has more weight than the old in the political preference function ($\delta > 0$), then in equilibrium in the closed economy the young receive all of the tax revenue ($\chi = 1$), and in equilibrium in the open economy the old pay the fiscal cost of the resource subsidy ($\chi = 0$).

	$\chi^* = 1$	$\chi^* = 0$
closed economy (where $R_t > 0$)	$\delta > 0$	$\delta < 0$
open economy (where $R_t < 0$)	$\delta < 0$	$\delta > 0$

Table 2: Relation between assumed value of χ^* and implied value of δ

Social planner To provide a benchmark, we also compute the equilibrium to a social planner with the same pure rate of time preference as individuals (Schneider, Traeger and Winkler, 2012) This planner has a standard (time consistent) control problem:

$$\max_{\{T_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} (1+\rho)^{-t} p(x_t, T_t)^{-\alpha} Y(x_t, T_t) \tag{9}$$

subject to equation (4) and an initial condition on the stock. The social planner has the same problem as agents with a perfect bequest motive.

4.1 Calibration

Our baseline calibration sets both α, the share of the resource-intensive commodity in the consumption basket, and β, the wage share in manufacturing, equal to 0.5. We use an annual pure rate of time preference of 1%/year, which gives $\rho = 0.41$ assuming that one period lasts 35 years. We normalize by setting the carrying capacity $C = 1$, so x equals the resource capacity rate. The productivity parameter γ equals the inverse of the amount of labor that would exhaust the resource in a single period, starting from the carrying capacity $x = 1$. We set $r = 0.68$, implying an uncongested growth rate of 1.5%/year, and we choose γ so that the closed economy BAU steady state is $x_\infty = 0.5$, implying that $\gamma = 0.513$. We choose the world price $P = 3.377$ so that the open and closed economy steady states are equal under BAU. System (10) collects these baseline parameter values:

$$\alpha = 0.5;\ \beta = 0.5;\ \rho = 0.41;\ r = 0.68;\ \gamma = 0.513;\ P = 3.377. \tag{10}$$

For this parameter set and $x > 0.5 = x_\infty$, production in the open economy remains diversified. For $1 > x > x_\infty$, the endogenous relative price p in the BAU closed economy ranges between $1.688 < p < 3.377$. Thus, for $x_0 > x_\infty$, opening the BAU closed economy to trade causes a jump in the relative commodity price. At the steady state, x_∞, opening the BAU closed economy to trade has no effect. The baseline results we report here are representative of those from a much larger set of parameter values. (Appendix A.5).

4.2 The MPE

The six panels in Figure 1 show the MPE corresponding to $\chi = 0$ (solid) and $\chi = 1$ (dashed) and to the social planner (dot-dash) and BAU (dotted). The left panels correspond to the open economy and the right panels correspond to the closed economy. We first discuss the MPE and BAU.

The major conclusion is that, for both $\chi = 0$ and $\chi = 1$, the MPE policy is a tax in the closed economy and a subsidy in the open economy, just as in the OLNE. The closed economy equilibrium ad valorem tax is approximately 10%, and the open economy resource subsidy is approximately 20%, depending on χ and x.

The equilibrium policy function (Figure 1, panels a and b) and asset prices (Figure 1, panels c and d) are insensitive to the value of χ in the closed economy, and

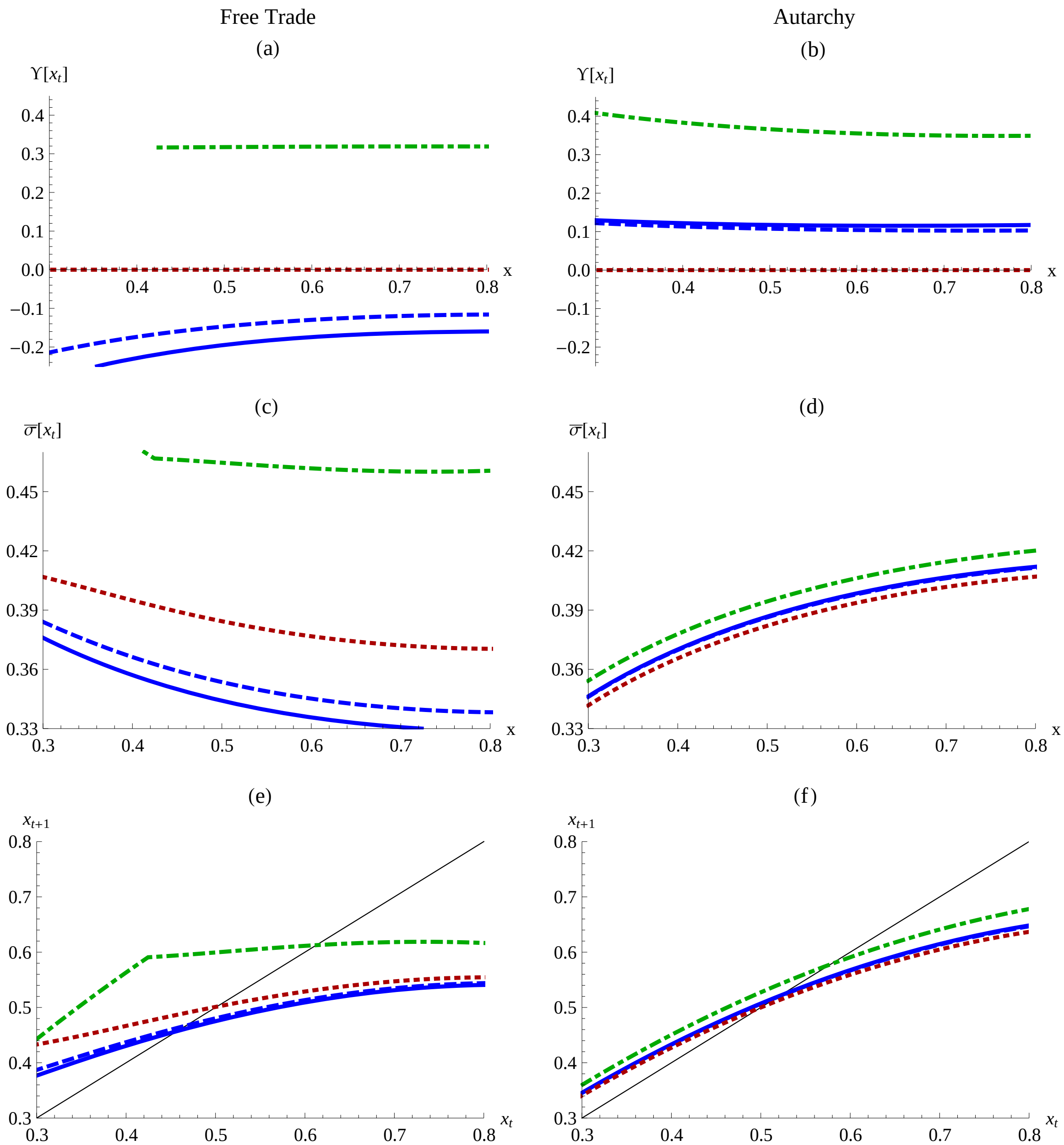

Figure 1: Left panels correspond to free trade and right panels to autarchy. Dashed graphs correspond to $\chi = 1$ and solid graphs to $\chi = 0$ in the MPE. Dotted graphs correspond to BAU and dot-dash to the social planner. The top panels show policy functions, the middle panels show the utility-denominated asset price, and the bottom panels show the equilibrium relation between current and next period stock.

moderately sensitive to χ in the open economy. In the open economy, where the equilibrium is a subsidy, Table 2 shows that $\chi = 0$ corresponds to $\delta > 0$: the old bear the fiscal cost of the policy. Changing from $\delta < 0$ to $\delta > 0$, i.e. giving the young generation greater weight in the preference function, slightly decreases the resource tax in the closed economy, and increases the resource subsidy in the open economy: greater political weight on the young harms the resource under both trade regimes.

Lemma 2 notes that a higher resource stock increases the asset price in the closed economy and decreases the asset price in an open economy. Panels c and d in Figure 1 confirm that this comparison also holds in the MPE. In the closed economy MPE, generations alive today want to tax resource use, because they know that the resulting higher stock increases future taxes, increasing the asset price. The closed economy MPE asset price is greater than the BAU asset price.

In the open economy, agents alive today want to use a subsidy, in order to lower the resource stock, and thereby increase the asset price. If these agents could commit to a tax, they could raise the asset price and their welfare (as occurs under the social planner discussed below). This type of commitment is not feasible in a MPE . Agents alive today understand that agents in the future have an incentive to subsidize production in the resource sector. Knowing this, agents alive today want to use a subsidy (just as in the OLNE). Agents face a version of the prisoner's dilemma in an intergenerational setting. The attempt to transfer welfare from the future toward the present backfires. The equilibrium subsidy reduces wealth (relative to BAU), reducing young agents' willingness to pay for the asset.

Panels e and f of Figure 1 show x_{t+1} as a function of x_t, together with the 45^0 line used to identify the steady state. These mappings are insensitive to χ. In the closed economy, the next period stock is higher in the MPE compared to BAU; free trade reverses this relation. The BAU steady state stock level equals 0.5 by calibration regardless of the trade regime. The steady state increases by 4% in the closed economy MPE and falls by 11% in the open economy MPE.

In the closed economy MPE, the domestic consumer price, $p\left(x_t, \Upsilon\left(x_t\right)\right)$, equals the world price at $x_t = 0.54$. If the economy were to suddenly open to trade at this value of the stock, the consumer commodity price remains constant, but the domestic

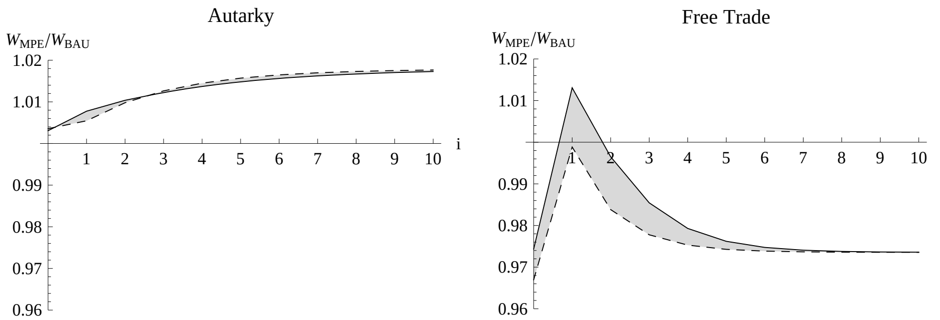

Figure 2: Welfare in MPE relative to BAU with initial resource stock $x_0 = 0.5$ (dashed) and $x_0 = 0.9$ (solid); autarchy left panel and free trade right panel, for $\chi = 1$. Period 0 shows combined lifetime welfare of old and young; subsequent periods show lifetime welfare of young.

resource tax switches to a subsidy, increasing harvest and causing the stock to fall more rapidly and toward a lower steady state. In previous papers, trade liberalization increases resource use because of differences in domestic and external prices. In our framework, trade reverses the direction of equilibrium policy and increases resource use even in absence of current commodity price changes.

Figure 2 shows present and future agents' lifetime welfare under the MPE with $\chi = 1$, relative to BAU levels. For future periods ($i \geq 1$) the figure shows the young agent's lifetime welfare change, and for the initial period ($i = 0$) it shows the aggregate lifetime welfare change for the current young and old generations. The dashed curve corresponds to the initial condition $x_0 = 0.5$ and the solid curve corresponds to $x_0 = 0.9$. For intermediate initial conditions, the welfare gain lies between these two curves. These conventions also apply to Figures 3 and 7.

In the closed economy, the MPE increases agents' welfare because the endogenous resource protection increases future and present wealth by protecting the resource. In the open economy, however, agents in period 0 and in every period after period 1 are worse off in the MPE compared to BAU, because the endogenous policy exacerbates the absence of property rights. If, in the open economy, the initial stock is sufficiently high, the young agent in period 1 has higher welfare in the MPE compared to BAU. This agent has no capital loss; see the comment below equation (3). Due to the high initial condition for x, the stock during this agent's lifetime is still relatively high, so she does not suffer (much) from the subsidy-induced fall in the stock; the subsidy-induced increase in her wage more than offsets the stock-related loss.

Trade liberalization creates the usual static utility gains, but has complicated dynamic welfare effects. Under BAU, at initial stocks above the steady state, the move from the closed to the open economy causes a fall in the asset price (Figure 1, panels c and d). Nevertheless, due to the usual static utility gain, trade increases BAU lifetime welfare of agents in the initial period. The lower future stock due to higher harvest reduces agents' welfare in all subsequent periods, except in the steady state (which under BAU is the same in both trade regimes, by calibration). The welfare effect of trade in the MPE is more pronounced than under BAU. The switch from a tax (in the closed economy) to a subsidy (in the open economy) causes a large fall in asset value; except for very high initial stocks, even the generations in the initial period have lower welfare in the open compared to the closed economy. All subsequent generations have lower welfare under trade, even at the steady state, because the economy under trade continues to use a resource subsidy.

4.3 The social planner

The dot-dash graphs in Figure 1 show the equilibrium policy functions, asset prices, and state transitions for the social planner. In both the open and closed economies, the equilibrium stock and tax trajectories are higher under the social planner compared with both BAU and MPE. Under free trade for $x < 0.42$, the social planner uses a prohibitive tax, allowing the resource to grow as fast as possible. Under diversified production, the tax remains close to its steady state level, $T_\infty = 0.32$, at $x_\infty = 0.61$. The closed economy steady state tax is higher, $T_\infty = 0.36$, but the steady state stock is lower, $x_\infty = 0.58$. The social planner achieves greater protection of the resource at a lower tax, in the open compared to closed economy.

In Ramsey models, optimal resource policy requires sacrifices from those cur-

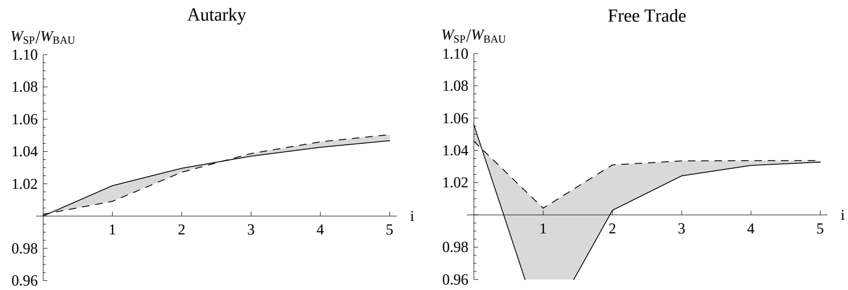

Figure 3: Welfare under the social planner relative to BAU with the initial resource stock $x_0 = 0.5$ (dashed) and $x_0 = 0.9$ (solid) for the closed economy (left) and the open economy (right).

rently alive, to benefit agents in the future. Our social planner solves the standard intertemporal problem (9), so it is no surprise that her program lowers aggregate period 0 *utility*. Using equation (8) in equation (3), together with a value of χ, the planner's policy function induces a trajectory of welfare for the old and young agents; here we use $\chi = 1$. The planner's program in the closed economy leads to a slight increase in period 0 aggregate lifetime *welfare* for initial conditions $x_0 < 0.91$ (Figure 3) and a small loss at larger stocks. The planner's program increases the asset price (Figure 1, panels c and d), and the old generation alive in period 0 obtains these capital gains. The young generation alive in period 0 also benefits from the higher stock in the second period of their life.

Those alive in the initial period have a more pronounced policy-induced welfare increase in the open economy, compared to the closed economy. This difference arises because the socially optimal policy creates a larger increase in the asset price in the open economy, compared to the closed economy. Without these capital gains, the initial generations suffer large losses in welfare under the social planner (compared to BAU), in the open economy. In both open and closed economies, the planner's intervention increases the steady state level of welfare, because intervention increases the steady state resource stock. In the closed economy, the planner raises welfare for all generations, if $x_0 < 0.91$. In the open economy, the planner reduces intermediate generations' welfare if the initial resource stock is high. Those generations would not have suffered much from a low stock under BAU, but they have a lower real wage when the planner taxes the resource.

The social planner corrects the open-access distortion. Opening the economy to trade eliminates the only remaining distortion, and necessarily increases the planner's maximand, the discounted stream of welfare. Some of this increased welfare appears as capital gains, which the first old generation appropriates. The planner's objective is to maximize the discounted stream of utility, not, for example, steady state utility. Thus, even in a standard Ramsey model, there is no presumption that trade, or any other movement from second to first best, increases utility in every period, e.g. in the long run steady state. In our calibration, except for initial conditions above 0.9, trade lowers all subsequent generations' welfare (under the social planner). Appendix A.6 reports the welfare effect of changing the trade regime, holding fixed the policy regime (BAU, MPE, social planner).

5 Conclusion

Many papers discuss the effect of trade when there are imperfect property rights to a natural resource. We depart from this literature by considering selfish overlapping generations rather than either a sequence of myopic agents or an infinitely lived agent. This modification allows us to study endogenous policy in a political economy equilibrium, and in particular to see how trade alters equilibrium policy. Our setting has two productive assets; property rights are perfect for capital and non-existent for the resource. These factors are used in different sectors, but the resource stock and tax influence labor allocation and thereby affect the return to capital.

In a closed economy, a resource tax is a best response for selfish agents, to a next-period resource tax. In an open economy, a resource subsidy is a best response for selfish agents, to a next-period resource subsidy. In both open loop and Markov perfect equilibria, the equilibrium policy is a tax in the closed economy and a subsidy in the open economy. Opening a closed economy converts a mutually beneficial policy to a mutually destructive one, and harms most agents.

Our results rely on three insights, which in isolation are unremarkable, but taken together have important implications. The first insight uses the fact that the relative commodity price is endogenous in a closed economy but exogenous in a small open economy. The qualitative effect of a change in policy or the stock, on the *nominal* return to a factor, does not depend on the trade regime. However, the qualitative effect of a stock or policy change on the *real* return to a factor does depend on the trade regime. For an open economy, a stock or policy change has the opposite qualitative effect on real returns to capital and to labor. For a closed economy, a stock or policy change has the same qualitative effect on the real returns to both factors, provided that the manufacturing sector elasticity of substitution between the two factors is sufficiently great – as it is in our Cobb-Douglas specification. Consequently, trade eliminates a commonality of interests between the two factor owners that exists in the closed economy.

The second insight is that factor market linkages cause policy and stock changes to alter the prices even of assets not used in the resource sector. Asset owners care about those price effects, regardless of their intrinsic concern for the resource stock or future generations' welfare. The asset market transfers, to agents currently alive, some of the future costs or benefits arising from changes in future resource stocks and policy. The old generation, who currently owns the capital, obtains the capital gains or losses resulting from policy-induced changes. Taken together, these two insights explain why a change in the stock or the policy have different asset price effects in the two trade regimes. They therefore provide the basis for understanding why selfish agents in the closed economy have an incentive to take actions that benefit their successors. In the open economy, these same agents have an incentive to take actions that harm their successors.

The third insight is that a higher next-period stock increases the next-period tax revenue or fiscal liability. If the next period uses a tax, and the current young generation obtains some of the tax revenue, current generations have an incentive to leave their successor a larger stock; the larger stock increases productivity in the resource sector, thereby increasing future tax revenue. Current generations therefore want to protect the resource stock using a tax. If the next period uses a subsidy, and the current young generation incurs some of the fiscal liability, current generations have an incentive to decrease the stock, using a subsidy; the lower stock decreases the next-period fiscal liability. This incentive remains even when the level of the policy responds to the stock, as in a Markov perfect equilibrium.

The asset price has significant implications even under a social planner who uses an efficient policy to protect the resource. The initial asset owner captures the capital gains resulting from trade. Subsequent generations may have lower welfare under trade, in the social planner setting.

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A Supplementary material (Not for publication)

The supplementary material, available on request, contains proofs not found in the text, and details of the numerical material. Appendix A.1 provides the proof of Lemma 1, which is standard.

Appendix A.2 provides the proof of Proposition 1. This proof is quite long, because it requires considering many special cases, but readers familiar with small general equilibrium models will find nothing surprising in it. The appendix also discusses the effect of relaxing assumptions made in the proposition.

The proof in the text of Lemma 2 contains the statement "Straightforward calculations (Appendix A.3) show that $\left[p_{t+1}^{1-\alpha}\gamma L_{t+1}x_{t+1}\right]$ is an increasing function of x_{t+1}, and thus an increasing function of T_t (using Assumption 1.iii)." Appendix A.3 contains those calculations.

Appendix A.4 contains details of the numerical algorithm. That algorithm relies on standard dynamic methods of function approximation. Appendix A.5 describes our sensitivity studies, mentioned in Section 4.1. Appendix A.6 compares the effect of moving from a closed to an open economy, holding fixed the policy environment (BAU, MPE, and the social planner). We think that this material is of general interest, but apart from brief comments, we have taken it out of the paper in order to focus on our principal research questions, and to keep the paper at a reasonable length.

A.1 Proof Lemma 1

(Sketch) Identical homothetic preferences make welfare aggregation unambiguous. Part (i) follows from the fact that a non-zero tax imposes the usual deadweight loss in current output; a 0-tax maximizes current real income. Part (ii) and (iii) follow from the facts that relative demand $(\frac{F}{M})$ depends on p but not T (by homotheticity), and relative supply depends on $p(1-T)$ (by concavity of the production possibility frontier) and on x (which affects labor productivity in F). An increase in T shifts in the relative supply function. In a closed economy, this shift leads to a higher equilibrium p and a reduction in equilibrium $\frac{F}{M}$. The higher tax therefore causes labor to flow to sector M, reducing w and increasing π. In the open economy, p is fixed, but the higher tax reduces $p(1-T)$, again causing L and w to fall, and π to increase. The policy-induced reduction in L lowers harvest, increasing next-period stock. A larger x increases productivity in the resource sector, attracting labor, increasing w and lowering π (in both the open and closed economy). A higher x also shifts out the production possibility frontier, increasing aggregate utility. (There is no possibility of immizerizing growth in this economy.)

A.2 Proof and discussion of Proposition 1

Remark 1 states that for the Cobb Douglas closed economy specification, higher T and lower x decrease both the real wage and the real rental rate. Using the fact that the equilibrium changes smoothly with all primitives, the following proposition confirms that this result is robust under a broad set of utility and manufacturing production technology. Roughly, the proposition (together with continuity) shows that under homothetic utility, if the sector-M elasticity of substitution between factors is sufficiently large, then changes in the tax or the resource stock have the same qualitative effects on the real return to the two factors, as in Remark 1. In contrast, if the elasticity is sufficiently small, changes in the tax or the stock have ambiguous effects; the changes may have the opposite qualitative effects on the real returns to the two factors. Thus, our results require sufficient substitutability between factors in sector M.

With identical homothetic preferences, the economy's aggregate consumption point lies on an Income Expansion Path, IEP(p), a ray from the origin in the (F, M) plane. A higher price causes the IEP to rotate counter-clockwise, towards the M (vertical) axis. The equilibrium production point depends on the producer relative price, $p - T$ and lies on the economy's Production Possibility Frontier (PPF). Because production equals consumption in the closed economy, the equilibrium consumption point must lie on both the IEP and the PPF. The equilibrium in this economy exists and is unique.

Proof. (Proposition 1) **Part i**. First consider infinite elasticity of substitution. In this case, the production function in sector M can be written $a_M K + a_L (1 - L)$, for constants $a_M > 0$ and $a_L > 0$. Sector M employs all capital, so output in the sector equals $a_M + a_L (1 - L)$. The PPF is kinked. For $L < 1$, the PPF is a negatively sloped line segment, $M = a_M + a_L \left(1 - \frac{F}{\gamma x}\right)$ and for $L = 1$ the PPF is a vertical line at $F = \gamma x$. The economy is never on the vertical part below the kink, because that region corresponds to unemployed capital. Because sector M employs capital, the nominal return to capital is fixed at $\pi = a_M$. Given the assumption that labor is employed in sector M under the initial tax (which for convenience we take to be 0), and the fact that in the closed economy there is always labor in sector F, the initial (zero tax) wage is $w^0 = a_L = p\gamma x$, and the initial closed economy price is $p^0 = \frac{a_L}{\gamma x}$.

An increase in the tax. Here we hold the resource stock constant. We show that a higher tax decreases the real returns to both factors. We use a proof by contradiction to establish that under the tax, labor remains employed in sector M. Suppose to the contrary that under the tax, all labor moves to sector F. In this case, the economy moves from a point on the negatively sloped portion of the PPF, above the kink, to the kink. The IEP must rotate down in order for markets to clear, so the new commodity price is $p^T < p^0$. Consequently, the wage falls: $w^T = p^T (1 - T) \gamma x < p^0 \gamma x = w^0 = a_L$. In this case, the wage is lower than its value of marginal productivity in sector M, which is not consistent with equilibrium. Therefore, if the economy was using some labor in sector M prior to the tax, it continues to do so after the tax. Because labor remains in sector F, equilibrium requires $w^T = a_L = p^T (1 - T) \gamma x$. Thus, $p^T = \frac{a_L}{(1-T)\gamma x} > p^0 = \frac{a_L}{\gamma x}$. The tax increases the equilibrium consumer price, without altering nominal factor prices. Thus, the tax causes exactly the same proportional fall in the real return to both factors.

An increase in the stock. Here we hold the tax constant, and consider the effect of a higher stock, $x' > x$. We show that a higher stock increases the real return to both factors. The higher stock shifts the kink to the right, without altering the kink's vertical coordinate. The M intercept of the PPF remains at $a_K + a_L$, so the negatively sloped portion of the PPF becomes flatter; its slope falls from $\frac{a_L}{\gamma x}$

to $\frac{a_L}{\gamma x'}$. There are two possibilities. If some labor remains in sector M, then the equilibrium commodity price falls. Nominal factor prices remain constant at $w = a_L$ and $\pi = a_K$, so both factors have the same percentage increase in real factor returns. If all labor goes to sector F, then again the commodity price must fall. The nominal rental rate remains constant, so the real return to capital increases. However, sector M can no longer afford to employ labor, so the nominal wage is now greater than or equal to a_L. Here the proportional increase in the real return to labor is greater than or equal to the proportional increase in the real return to capital.

Part ii. For zero elasticity of substitution, sector-M output equals $\min\left(a_K K, a_L\left(1-L\right)\right)$, where a_K and a_L are positive constants. (These coefficients are not the same as those used in Part i.) If capital is fully employed, there are $1 - L = \frac{a_K}{a_L}$ units of labor in sector M. Because production of F must be positive in the closed economy, $L > 0$, so it must be the case that $a_L > a_K$. Therefore, production of F equals $\left(1 - \frac{a_K}{a_L}\right)\gamma x$. The economy's PPF (with M on the vertical axis) is kinked; it has a horizontal portion at $M = a_K$, the maximum feasible level of M, and is a line with slope $-\frac{a_L}{\gamma x}$ for $M < a_K$. Given the opportunity to work in the resource sector, unemployment of labor is not consistent with equilibrium. Because labor is always fully employed, the economy is never on the horizontal portion strictly to the left of the kink. The length of the flat portion, $\left(1 - \frac{a_K}{a_L}\right)\gamma x$, increases with x. We first consider the effect of an increase in the tax, and then of an increase in the resource stock.

An increase in the tax. Here we take x as fixed. We show that a higher tax decreases the real return to labor and increases the real return to capital. Given the assumption that both factors are fully employed under an initial tax, the economy is at the kink, and the equilibrium closed economy price is the negative of the tangent to the indifference curve through this kink.

A proof by contradiction establishes that production remains at the kink under a higher tax. A resource tax cannot move the economy left, along the horizontal segment of the PPF, because in that case labor would be unemployed. If, contrary to the hypothesis, the tax results in unemployment of capital, then the tax causes the rental rate π to fall to 0, and the economy moves to a point south east of the kink, on the negatively sloped portion of the PPF. The IEP through this point is flatter than the original IEP through the kink, so the relative consumer price that supports the new production point (corresponding to the tax), p^T, is lower than the price under the initial tax, denoted, p^0: $p^T < p^0$. Consequently, the producer price under the tax, $p^T(1-T)$ is lower than the initial producer price. Because a larger T lowers both p^T and $1-T$, the higher tax lowers the wage: $w^T = p^T(1-T)\gamma x < p^0\gamma x$. In this case, the tax reduces both the rental rate and the wage. But the price of good M is normalized to 1, so there are now positive profits in sector M. Positive profits are not consistent with equilibrium in this economy. This contradiction establishes that if both factors are fully employed under a zero tax, then they remain fully employed under a positive tax. Production and consumption remains at the kink.

Because consumption under the tax remains at the kink, the consumer price remains constant at p^0. The higher tax therefore reduces the wage (equal to revenue-per-worker in the resource sector) $p^0(1-T)\gamma x$. To maintain zero profits in sector M, the rental rate, π, must increase. Because the higher tax leads to an increase in both π and $\frac{\pi}{p}$, the real return to capital increases. Because the tax reduces both w and $\frac{w}{p}$, the real return to labor falls.

<u>An increase in the resource stock.</u> Here we hold the tax fixed. (To simplify exposition we set the tax equal to 0.) We show that a higher stock has an ambiguous effect on the real returns to both factors. The larger x increases the length of the horizontal portion of the PPF, $\left(1-\frac{a_K}{a_L}\right)\gamma x$, causing the kink in the PPF to shift to the right; the larger x also causes the negatively sloped portion of the PPF to become flatter: $\frac{a_L}{\gamma x}$ becomes smaller. The production = consumption point must now involve higher F and the same or lower M, so the IEP must become flatter (in order for goods markets to clear). Thus, larger x lowers the commodity price, p: $\frac{dp}{dx} < 0$. Note also that the maximum feasible wage is $w = a_L$, which occurs when there is unemployed capital (so that the return to capital is 0).

First consider the effect of the higher resource stock on the real wage. There are only two possibilities: some capital is unemployed at the new (higher x) equilibrium, or all capital remains fully employed at the new equilibrium. In the first case, the wage equals its maximal level, a_L. Because the increase in x weakly increases w and strictly decreases p, the higher x increases the real wage.

In order to demonstrate that the relation between the real wage and the stock is ambiguous, we need only find a case where the higher stock lowers the real wage. From the previous remarks, this case requires that all capital remains fully employed; in this situation, the wage is related to the price and the stock by $w = \gamma px$. Totally differentiating this equality implies

$$\frac{dw}{dx} = \frac{dp}{dx}\gamma x + \gamma p. \tag{11}$$

We take the case where utility is constant returns to scale in consumption of F, M, so that indirect utility is linear in income. A worker with one unit of time earns income w from selling labor, and has indirect utility $v(w,p)$. The fact that utility is linear in income means that we can consider the real wage in isolation from any revenue obtained from tax receipts. We denote F^{worker} as the worker's equilibrium consumption of good F, and as before we use F to denote aggregate production = consumption. Thus, $\frac{F^{\text{worker}}}{F}$ is labor's share of national income. Differentiating the indirect utility function with respect to the stock gives

$$\begin{aligned}
\tfrac{dv}{dx} &= v_w\tfrac{dw}{dx} + v_p\tfrac{dp}{dx} \Rightarrow \\
\tfrac{\frac{dv}{dx}}{v_w} &= \tfrac{dw}{dx} + \tfrac{v_p}{v_w}\tfrac{dp}{dx} \Rightarrow \\
\tfrac{\frac{dv}{dx}}{v_w} &= \tfrac{dw}{dx} - F^{\text{worker}}\tfrac{dp}{dx} \Rightarrow \\
\tfrac{\frac{dv}{dx}}{v_w} &= \left(\tfrac{dp}{dx}\gamma x + p\gamma - F^{\text{worker}}\tfrac{dp}{dx}\right) \Rightarrow \\
\tfrac{\frac{dv}{dx}}{v_w} &= \left(\tfrac{dp}{dx}\tfrac{x}{p}\left(1 - \tfrac{F^{\text{worker}}}{\gamma x}\right) + 1\right)p\gamma \Rightarrow \\
\tfrac{\frac{dv}{dx}}{v_w} &= \left(\tfrac{dp}{dx}\tfrac{x}{p}\left(1 - \tfrac{F^{\text{worker}}}{F}\right) + 1\right)p\gamma.
\end{aligned}$$

The third line of this series of equations follows from Roy's identity, the fourth line uses equation (11),and the fifth uses the production function, $F = \gamma x$. If capital is fully employed (and has positive rent) then $\left(1 - \frac{F^{\text{worker}}}{F}\right) < 1$. In this case, a higher stock reduces the real wage if and only if

$$\frac{dp}{dx}\frac{x}{p}\left(1-\frac{F^{\text{worker}}}{F}\right)+1<0$$

or

$$\frac{F}{F-F^{w}}<\left|\frac{dp}{dx}\frac{x}{p}\right|.$$

The expression on the left is the reciprocal of the capital owner's share of national income, and is always greater than 1. Thus, the condition that real wage falls is that the absolute value of the elasticity of the equilibrium price, with respect to the stock, exceeds 1 by a sufficiently large amount. This condition is satisfied if the utility function is approximately (but not exactly) Leontieff. In this case, a large change in the relative commodity price is required to change the consumption bundle from the original kink, $(F, M) = \left(\left(1-\frac{a_K}{a_L}\right)\gamma x, a_k\right)$, to the new kink, $\left(\left(1-\frac{a_K}{a_L}\right)\gamma \tilde{x}, a_k\right)$, where x is the original stock and the higher stock is $\tilde{x} > x$.

We now need only establish that the effect of the higher stock on the real rental rate is ambiguous. Here we adopt the stronger assumption that agents have identical Cobb Douglas (not merely homothetic) preferences, with α equal to the budget share in sector M, as in the text. The location of the kink is $(F, M) = \left(\left(1-\frac{a_K}{a_L}\right)\gamma x, a_K\right)$: a larger stock causes the kink to shift to the right. The assumption that capital was fully employed prior to the increase in x, together with zero profits in sector M, implies $1 = \frac{\pi}{a_K}+\frac{w}{a_L} = \frac{\pi}{a_K}+\frac{p\gamma x}{a_L}$, or $\pi = a_K\left(1-\frac{p\gamma x}{a_L}\right)$. Equilibrium in the goods markets requires $\frac{(1-\alpha)}{\alpha} = \frac{M}{pF}$, or $p = \frac{\alpha M}{(1-\alpha)F}$ Given the assumption that consumption is initially at the kink, equilibrium requires

$$p = \frac{\alpha a_K}{(1-\alpha)\left(1-\frac{a_K}{a_L}\right)\gamma x}. \tag{12}$$

Therefore,

$$\pi = a_K\left(1-\frac{p\gamma x}{a_L}\right) = a_K\frac{a_L(1-\alpha)-a_K}{(1-\alpha)(a_L-a_K)} > 0 \tag{13}$$

where the strict inequality is assumed in the Proposition. Thus, the assumption that initially capital is fully employed, with positive rental rate, implies the parameter restriction

$$a_L(1-\alpha) - a_K > 0. \tag{14}$$

Following the increase in x, the production point might move to the new kink (higher F, constant M) or it might move onto the sloped portion of the PPF (higher F, lower M). If production moves to the new kink then (except in a knife-edge case), π remains constant, given by equation (13), but p falls, so the real return to capital increases. If production moves to the sloped portion of the PPF, then there is unemployed capital, and π falls to 0. In this case, the real return to capital falls. In these two cases, the nominal wage either stays the same or increases; because the price falls, the real wage increases in both cases. However, we showed above that the effect of a higher stock on the real wage is ambiguous under other circumstances.

■

Discussion of Proposition 1 Here we consider the effect of weakening assumptions in both parts of the proposition.

Part (i) assumes that initially capital is fully employed. If that assumption does not hold, the nominal return on capital is zero. A higher tax or resource stock has no effect on capital's nominal return, and thus no effect on its real return. The higher tax decreases, and the higher stock increases, the real return to labor.

Part (ii) assumes that initially sector M uses some labor. If that assumption does not hold, the economy produces at the kink, and the autarchy price is greater than or equal to $\frac{a_L}{\gamma x}$. The nominal factor returns are $\pi = a_K$ and $w = p\gamma x \geq a_L$. An increase in the resource tax reduces the nominal wage. If, despite the tax, all labor remains in sector F, the commodity price is unchanged, so the tax has no effect on the real return to capital, and strictly decreases the real return to labor. In this case, the tax merely extracts some rent that workers would otherwise receive. If the tax induces some labor to move to sector M, then production of F falls and the commodity price rises. In this case, the tax lowers the real return to both factors, but the percentage changes differ.

Now consider the effect of a larger stock, when initially all labor is in sector F. Because there is always some labor in sector F, $w = p\gamma x$, so $\frac{w}{p} = \gamma x$. The increase in x increases $\frac{w}{p}$. If all labor remains in sector F, then the new production point involves the same amount of M and increased F. Therefore, the IEP must rotate clockwise in order for the goods market to clear: $\frac{dp}{dx} < 0$. The effect of the increased stock on the nominal wage is $\frac{dw}{dx} = \gamma p \left(1 + \frac{dp}{dx}\frac{x}{p}\right)$. Depending on the magnitude of the elasticity $\frac{dp}{dx}\frac{x}{p}$, the higher stock can raise or lower the nominal wage. If it lowers the nominal wage, then the effect of the higher stock on the real wage is ambiguous. If it increases the nominal wage, then the higher stock also increases the real wage. The nominal return to capital is unchanged, $\pi = a_k$, but the commodity price falls, so the real return to capital increases.

If initially all labor is in sector F, and the higher stock causes some labor to move to sector M, the commodity price falls. (To confirm this claim, it helps to draw the original and the new PPF and the IEP through the original kink. The tangent to the indifference curve through the point of intersection of the new PPF and the original IEP is steeper than the new PPF. Thus, only prices lower than the original price can support an equilibrium on the sloped portion of the new PPF.) Because the price falls and π is unchanged, the higher stock increases the real return to capital. As before, $\frac{w}{p} = \gamma x$ increases, but w falls from $w = p\gamma x \geq a_L$ to a_L. Thus, the effect of the higher stock on the real wage is ambiguous.

In summary, when K and L are perfect substitutes in sector M, and initially all labor is in sector F: a higher tax lowers the real return to both factors, and a higher stock increases the real return to capital and has an ambiguous effect on the real return to labor.

A.3 Details of the proof of Lemma 2.ii

We suppress time subscripts. Tax revenue is $R = Tp\gamma Lx$. We use the following formula for equilibrium values in the closed economy:

$$L = \frac{1-T}{\frac{1-\alpha}{\alpha}\beta + 1 - T}; \quad w = \beta\left(1 + \frac{1-T}{\frac{1-\alpha}{\alpha}\beta}\right)^{1-\beta}; \quad p = \frac{w}{(1-T)\gamma x} = \frac{\beta\left(1 + \frac{1-T}{\frac{1-\alpha}{\alpha}\beta}\right)^{1-\beta}}{(1-T)\gamma x}.$$

Thus, in the closed economy,

$$p^{-\alpha}R = Tp^{1-\alpha}\gamma Lx = T\left(\frac{\beta\left(1 + \frac{1-T}{\frac{1-\alpha}{\alpha}\beta}\right)^{1-\beta}}{(1-T)\gamma}\right)^{1-\alpha} x^{\alpha-1}\gamma\left(\frac{1-T}{\frac{1-\alpha}{\alpha}\beta + 1 - T}\right)x = Tg(T)\,x^{\alpha},$$

using the definition

$$g(T) \equiv \gamma\left(\frac{\beta\left(1 + \frac{1-T}{\frac{1-\alpha}{\alpha}\beta}\right)^{1-\beta}}{(1-T)\gamma}\right)^{1-\alpha}\left(\frac{1-T}{\frac{1-\alpha}{\alpha}\beta + 1 - T}\right) > 0.$$

Consequently, the sign of $\frac{d\left(p^{-\alpha}R\right)}{dx}$ equals the sign of T in the closed economy.

In the open economy (where p is fixed) equilibrium in the labor market requires $w = \beta(1-L)^{\beta-1} = p(1-T)\gamma x$ or $L = 1 - \left(\frac{p(1-T)\gamma x}{\beta}\right)^{\frac{1}{\beta-1}}$, so

$$p^{-\alpha}R = p^{1-\alpha}\gamma TLx = p^{1-\alpha}\gamma Th(x)\,x,$$

using the definition

$$h(x) \equiv 1 - \left(\frac{p(1-T)\gamma x}{\beta}\right)^{\frac{1}{\beta-1}}.$$

A calculation establishes that

$$\frac{d(h(x)\,x)}{dx} = \frac{1}{1-\beta}\left(\beta\left(p\frac{x}{\beta}\gamma(1-T)\right)^{\frac{1}{\beta-1}} + 1 - \beta\right),$$

which is positive by inspection. Consequently, the sign of $\frac{d(p^{-\alpha}R)}{dx}$ equals the sign of T in the open economy.

A.4 Solution algorithm

Agents at time t take the functions $\Upsilon(x_{t+1})$ and $\sigma(x_{t+1})$ as given, but they are endogenous to the problem. We solve $\max_T W(x,T)$ using a standard dynamic programming algorithm. An arbitrary policy function, $\Upsilon^k(x_t)$, induces the real asset price, $\sigma^k(x_t,T_t)$, given by equation (8); the superscript k denotes the functional dependence of $\sigma^k(x_t,T_t)$ on the function $\Upsilon^k(x_t)$. Replacing $\sigma(x_t,T_t)$ with $\sigma^k(x_t,T_t)$ and W_t with W_t^k in the maximand, we denote

$$\Upsilon^{k+1}(x) = \arg\max_T W^k(x,T).$$

This relation is a mapping from Υ^k to Υ^{k+1}. An equilibrium Υ is a fixed point to this mapping, which we approximate using the collocation method and Chebyshev polynomials (Judd, 1998; Miranda and Fackler, 2002)

Infinite horizon models (but not finite horizon models) of this genus typically have multiple equilibria. Experiments suggest that our numerical approach always returns a unique equilibrium. An algorithm that iterates over the value function can be interpreted as the limit as the horizon goes to infinity of a finite horizon model. In view of the generic uniqueness of finite horizon models, the (apparent) uniqueness of the numerical results is not surprising.

To simplify notation, we introduce a new function, the value of the asset in units of utility (rather than in units of the numeraire good):

$$\bar{\sigma}(x_{t+1},\Upsilon(x_{t+1})) = p^{-\alpha}(x_{t+1},\Upsilon(x_{t+1}))\,\sigma(x_{t+1},\Upsilon(x_{t+1})).$$

We approximate $\Upsilon(x_{t+1})$ and $\bar{\sigma}(x_{t+1},\Upsilon(x_{t+1})) \equiv \Phi(x_{t+1})$ as polynomials in x_{t+1}, and find coefficients of those polynomials so that the solution to

$$\max_{T_t} P^{-\alpha}(x_t, T_t)\, Y(x_t, T_t) + \\ \tfrac{1}{1+\rho}\left\{P^{-\alpha}(x_{t+1}, \Upsilon(x_{t+1}))\pi(x_{t+1}, \Upsilon(x_{t+1})) + \Phi(x_{t+1}))\right\}$$

subject to equation (4) approximately equals $\Upsilon(x_t)$. We use Chebyshev polynomials and Chebyshev nodes. At each node, the recursion defining $\bar{\sigma}(x_{t+1}, \Upsilon(x_{t+1}))$,

$$\Phi(x_t) = \frac{1}{1+\rho}\left\{p^{-\alpha}(x_{t+1}, \Upsilon(x_{t+1}))\pi(x_{t+1}, \Upsilon(x_{t+1})) + \Phi(x_{t+1}))\right\} \tag{15}$$

and the optimality condition

$$\frac{d}{dT_t}\left[P^{-\alpha}(x_t, T_t)\, Y(T_t) + \frac{1}{1+\rho}\Omega\right] = 0, \tag{16}$$

with

$$\Omega \equiv \left\{P^{-\alpha}(x_{t+1}, \Upsilon(x_{t+1}))\pi(\Upsilon(x_{t+1})) + \Phi(x_{t+1}))\right\}$$

must be satisfied.

Starting with an initial guess for the coefficients of the approximations of $\Phi(\cdot)$ and $\Upsilon(\cdot)$, we evaluate the right side of equation (15) for at each node. Using these function values, we obtain new coefficient values for the approximation of $\Phi(\cdot)$. We then use the optimality condition (16) to find the values of $\Upsilon(\cdot)$ at each node; we use those values to update the coefficients for the approximation of $\Upsilon(\cdot)$. We repeat this iteration until the coefficients' difference between iterations, relative to the estimated value of the coefficient, falls below 10^{-6}. See chapter 6 of Miranda and Fackler (2002) for details.

The social planner's solution involves a prohibitive tax under free trade. We approximated the point of specialization through numerical experiments and at first limited the approximation space to the range of diversified production $x \in [0.4246, 1]$. Under a prohibitive tax, this set also contains all x_{t+1} for $x_t \in [0.3, 0.4146)$. Given the approximations of $\Phi(\cdot)$ and $\Upsilon(\cdot)$ for the set of diversified production, one can use recursion (15) to approximate $\Phi(\cdot)$ for the range of specialized production. As $\Phi(\cdot)$

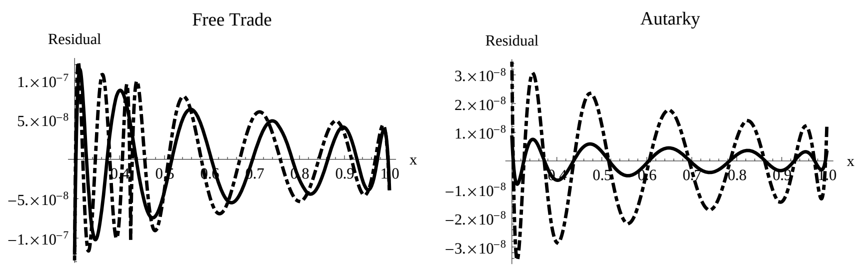

Figure 4: Approximation error for asset price function (LHS-RHS of (15)): the MPE (solid) and the social planner's (dot-dashed) problems, the open economy (left) and closed economy (right).

might not be smooth at $x = 0.4246$, we used separate polynomials for the ranges of diversified and specialized production.

Figures 4 and 5 graph the differences (the "residuals") between the right and left sides of equations (15) and (16), respectively. These residuals equal 0 at the nodes, because we set both the degree of the polynomial and the number of nodes equal to n. We choose $n = 12$, yielding residuals are at least 6 orders of magnitudes below the solution values on the $[0.3, 1]$ interval. In the case of a social planner under free trade, we chose $n = 10$ for $x \in [0.4246, 1]$ interval and $n = 6$ for $x \in [0.3, 0.4246)$.

A.5 Numerical sensitivity

Corollary 2 establishes that, except for the last period, the equilibrium policy is a sequence of subsidies under trade and of taxes under autarchy. The numerical results reported in the text show that these qualitative differences also hold in the MPE, where cannot establish the result analytically. To confirm that our numerical results (a sequence of subsidies in the open economy and of taxes in the closed economy) are not an artifact of one particular parameter set, we conduct extensive parameter sensitivity analysis. We define the following values for the model's parameters (with bold numbers indicating the baseline value used in the text), and determine the

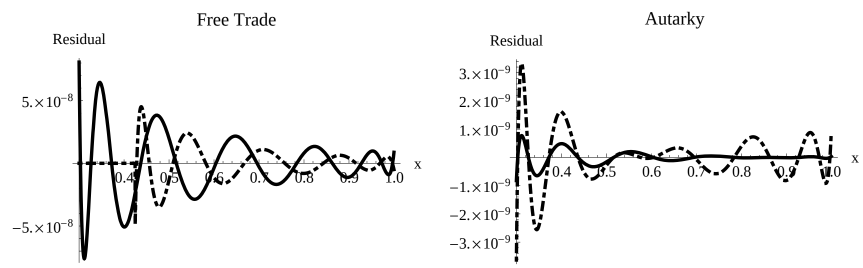

Figure 5: Approximation error for asset price function (LHS-RHS of (16)): the MPE (solid) and the social planner's (dot-dashed) problems, the open economy (left) and closed economy (right).

corresponding equilibrium policy for each combination of parameters that satisfy certain restrictions described below.

$$\alpha = \{0.1, 0.3, \mathbf{0.5}, 0.7, 0.9\}$$
$$\rho = \{0.1,\ \mathbf{0.41},\ 0.7\}$$
$$\beta = \{0.4,\ \mathbf{0.5},\ 0.6\}$$
$$r = \{0.1,\ 0.5,\ \mathbf{0.68},\ 0.9,\ 1.1\}$$
$$\gamma = \{0.1,\ 0.3,\ \mathbf{0.513},\ 0.7,\ 0.9\}$$
$$P = \{1,\ 2,\ 3,\ \mathbf{3.377},\ 4,\ 6,\ 9\}$$

In the sensitivity runs we set $\chi = 1$ and reduce the number of collocation points to 8.

For both the open and closed economy, we include only parameters that, under BAU, lead to monotonic adjustment (the BAU x_{t+1} is an increasing function of x_t, and crosses the 45^o line with slope less than 1). For the open economy, we additionally restrict parameter combinations so that at a particular value of x the BAU economy is diversified . Under specialization, equilibrium policy is indeterminate, so we do not consider that case. For lower values of x, this "diversification restriction" is more binding, so our open economy results use few combinations of parameters, the smaller is x. For the closed economy examples, we used the state space $[0.05, 1]$,

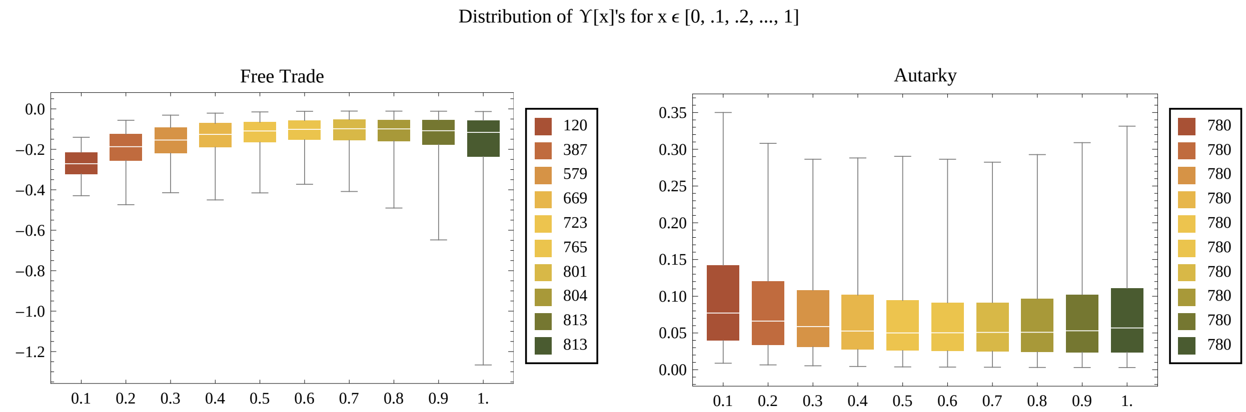

Figure 6:

rather than $[0, 1]$.

Preliminary experiments show that the value of α has negligible effect on the open economy equilibrium; therefore, to reduce the dimension of parameter space we hold α constant at the baseline value, $\alpha = 0.5$. Consequently, we begin with $3 \times 3 \times 5 \times 5 \times 7 = 1575$ combinations of parameter values. Of these, 915 combinations lead to monotonic BAU growth paths. At $x = 0.9$, there are 813 parameter combinations that imply both monotonic BAU paths and diversification; at $x = 0.1$, there are 120 such parameter combinations.

For the closed economy, we allow α to vary, but the world price P is irrelevant. We therefore begin with $5 \times 3 \times 3 \times 5 \times 5 = 1125$ parameter combinations. Of these, 1065 parameter combinations lead to monotonic BAU adjustment; 780 combinations lead to both monotonic BAU adjustment and BAU steady states in the interval $[0.05, 0.95]$.

Figure 6 shows box plots for the distribution of the equilibrium policy, at different values of x. For all parameter combinations included in these plots, the policy is a subsidy for the open economy and a tax for the closed economy. The numbers at the right of each figure show the number of parameter combinations used for each value of x; this number increases with x in the open economy and is constant in

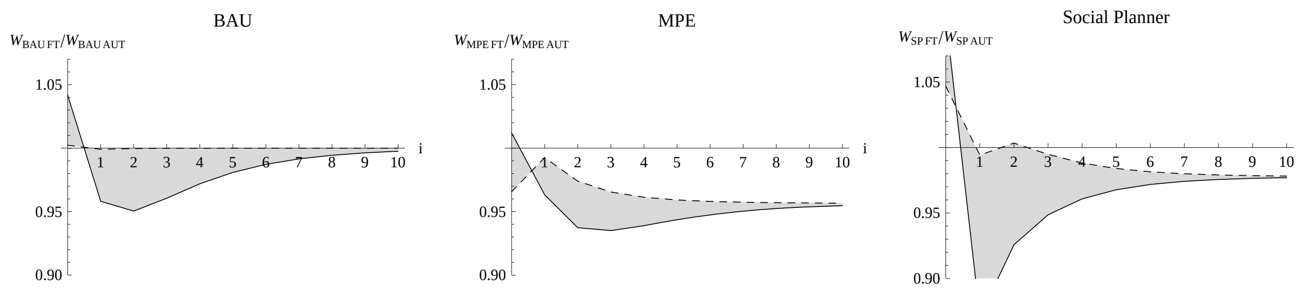

Figure 7: Welfare under free trade relative to autarchy under BAU (left), MPE (center) and social planner (right), with the initial resource stock $x_0 = 0.5$ (dashed) and $x_0 = 0.9$ (solid).

the closed economy. Each box contains the middle quartiles (Q2 and Q3) while the lower and upper whiskers give Q1 and Q4. The white line in the box shows the median subsidy/tax for a given value of x. The sensitivity results summarized in Figure 6 confirm that the equilibrium in the open economy involves a subsidy, and the equilibrium in the closed economy involves a tax for a large parameter space. Note, we do not find any parameter combinations that overturn these results; but we did not consider combinations that violate the monotonicity and diversification (under BAU) restrictions.

A.6 Further Discussion of Numerical Results

Figure 7 shows the welfare effect of changing the trade regime, holding fixed the policy regime: BAU, MPE, and social planner. Here we use the baseline parameters from the text, and set $\chi = 1$. The horizontal axis (labelled i) shows the number of periods from the time at which the closed economy opens to trade. The left panel shows the ratio of open-economy to closed economy welfare under BAU; the middle panel shows this ratio in the MPE, and the right panel shows this ratio under the social planner. As in the text, the dashed curve shows the ratio given the initial condition $x = 0.5$, and the solid curve gives the ratio at initial condition $x = 0.9$.

Under BAU, both open and closed economies have the steady states $x = 0.5$, so at this value, opening the closed economy has no effect (the welfare ratio equals 1). If the initial stock is high, current generations have higher welfare under trade. At stocks above the steady state, the domestic price is below the world market price. The higher price leads to high extraction in the current period and lower ones in subsequent ones, increasing aggregate welfare of currently living agents ($i = 0$) and lowering welfare of each future young generation.

In the MPE, all generations are worse off under free trade, except possibly the first generation if initial stocks are large (Figure 7 middle panel). The economy reaps the standard static gains from trade, but trade reverses the incentives to protect the resource stock. The lower resource stock lowers future generations' welfare. If the initial stock is high, then the initial closed economy price is low. In this case, the standard gains from trade may be large enough that trade improves welfare for those alive in the first period. However, for most initial stock levels, and for all future generations, the switch from resource protection to increased exploitation is more important than the standard gains from trade; here, trade lowers welfare.

Under the social planner, opening up to trade puts the economy in a first best world and necessarily increases the present discounted sum of welfare, but need not increase welfare for every generation. The right panel of Figure 7 shows that trade lowers welfare for most future generations. The trade-induced fall in future generations' welfare comes from the fact that single period utility is linear in income. With a constant commodity price, the planner has no incentive to smooth consumption. Comparison of panels c and d of Figure 1 shows that trade increases the asset price. The old generation in the first period captures all of these capital gains, which exceed 100% of the gains from trade.