ECON 140 - EXERCISES

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Abstract

This Reader is intended for the students of Econ-140 in my section. It is not supposed to be used for any other class. It contains exercises that will be discussed during the discussion sections. The Reader will be updated every week. If we are not covering all the exercises during the practice sections, please feel free to ask me for the solutions during my office hours. Also, there may be typos, and I would greatly appreciate if you pointed them out to me. The person who will point out to me most typos, will receive at the end of the semester a free Milka Chocolate bar...

1 Introduction:

The methodology of Econometrics:

- Presentation of the theory and main hypothesis
- Specification of the mathematical model
- Specification of the econometric model
- Getting the data
- Estimation of the parameters of the econometric model
- Verification/proof of the hypothesis
- Prognostic and Prediction
- Utilization of the model for empirical/political means

2 Thursday 01/18/06: Warm-up

Exercise 1 Suppose you have a questionnaire with 3 questions. For each question, you can answer yes (Y) or no (N). Let X be a random variable equal to the number of times you answer yes (Y).
1. What is a random variable?
2. What values does X take? What is the domain of X?

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3. What is a probability?
4. What is a discrete probability distribution? What are the two basic properties a probability distribution must satisfy? What is the probability that X takes the value 2, i.e. $P(X = 2)$?
5. What is a cumulative probability distribution? What are the two basic properties a cumulative probability distribution must satisfy? What is the probability that X is less than 2: $P(X \leq 2)$?

Exercise 2 We add one question to the upper-mentioned questionnaire. The answer can take 5 values, from 1 to 5 ($Z \in \{1, 2, 3, 4, 5\}$).
1. What is a probability distribution function (p.d.f.)? What is the p.d.f. of Z ?
2. What is a cumulative probability distribution function (c.d.f.)? What is $P(1 \leq Z \leq 3)$?
3. What is the mean of Z, i.e. $E(Z)$? How do you calculate it?
4. What is the standard deviation of Z? How do you calculate it?

Exercise 3 [from S & W - 2.3] The following table gives the joint probability distribution between employment status and college graduation among those either employed or looking for work in the working age population based on the 1990 U.S. Census.

Joint Distribution of Employment Status and College Graduation in the U.S. Population Aged 25-64, 1990

	Unemployment (Y=0)	Employed (Y=1)	Total
Non-College Grads (X=0)	0.045	0.709	0.754
College Grads (X=1)	0.005	0.241	0.246
Total	0.050	0.950	1.000

Table 1: Probability Distribution

Compute $E(Y)$. Calculate $E(Y|X = 1)$ and $E(Y|X = 0)$. What is the probability that a worker is a College Graduate? And a Non-Collage Graduate? Are educational status and employment status independent? Why?

Exercise 4 You are given Table 2:

Rate of return, X_i (%)	Probability, p_i
-20	0.10
-10	0.15
10	0.45
25	0.25
30	0.05

Table 2: Probability Distribution

Compute the expected rate of return, $E(X)$. Compute the variance $V(X)$ and the standard deviation σ_X. Compute the coefficient of variation defined as: $CV = \sigma_x/E(X)$ (NB: CV is usually multiplied by 100 to have a percentage interpretation). Compute the asymmetry of the distribution[1]. Is the asymmetry positive or negative?

Exercise 5 The following table (Table 3) shows the joint probability $p_{X,Y}$ of X and Y.

Y, X	1	2	3
1	0.03	0.06	0.06
2	0.02	0.04	0.04
3	0.09	0.18	0.18
4	0.06	0.12	0.12

Table 3: Joint Probability Distribution

Compute the marginal distributions of X and Y. Compute the marginal distributions of $X|Y$ and $Y|X$. Compute the conditional expectations $E(X|Y)$ and $E(Y|X)$.

3 Thursday 01/26/06

Exercise 1 Let X be a variable uniformly distributed on [1,2]. Write down the p.d.f. of X and draw the according graph. Let Y be such that $Y = ln(X)$. Calculate $E(Y)$.

Exercise 2 Let $X \sim U[0,1]$, and consider the function: $y = \frac{x}{1+x}$. Find the p.d.f. of Y, and show that $E(Y) = 1 + ln(\frac{1}{2})$.

Exercise 3 Show that the random variables X and Y defined by the joint density function:

$$f(x,y) = \frac{1}{2}, \text{ if } x \in [0,2],\ y \in [0,1] \tag{1}$$

are mutually independent.

Exercise 4 Let $f(x) = \frac{3}{8}(x+1)^2$, $\forall x \in [-1,1]$ be the p.d.f. of X. Let Y be such that $Y = \sqrt{X+1}$. Calculate the p.d.f. of Y, $E(Y)$ and $V(Y)$.

Exercise 5 The joint p.d.f. of X and Y is:

$$\begin{aligned} f(x,y) &= 4 - x - y && \text{if } 0 \leqslant x \leqslant 1 \text{ and } 0 \leqslant y \leqslant 1 \\ & \quad 0 && \text{otherwise} \end{aligned}$$

Find the marginal density functions f_X and f_Y. Find the conditional density functions $f_{X|Y}$ and $f_{Y|X}$. Compute $E(X)$ and $E(Y)$. Calculate $E(X|Y = 0.4)$.

Exercise 6 A random variable has a mean of 2 and a variance of 3. Find the expected value for the random variable $Y = 2X^2 + 5X + 4$.

[1] A measure of asymmetry is: $A = \frac{[E(X-\mu)^3]^2}{[E(X-\mu)^2]^3}$.

Exercise 7 Suppose the joint distribution of X and Y is given by:

$$f(x, y) = \frac{2}{5}(2x + 3y), \quad \text{for} \quad 0 \leqslant x \leqslant 1 \quad \text{and} \quad 0 \leqslant y \leqslant 1 \tag{2}$$

Show that $f(x, y)$ is a proper density function. Find $P(X \leqslant 0.5, Y \leqslant 0.5)$. Find the marginal distribution of the random variable X. Find the mean of X: $E(X)$. What is the probability that $Y \leqslant 0.5$ given that $X \leqslant 0.5$? Are X and Y independent, and why?

Exercise 8 In one law school class, the entering students averaged 700 on the LSAT test with a standard deviation of 40. Assuming the distribution of the test scores was normal, what fraction of the class scored above 750?

Exercise 9 Calculate $P(6 \leqslant X \leqslant 8)$ and $P(6 \leqslant \bar{X} \leqslant 8)$ if X is normally distributed with a mean of 7 and a variance of 25 and the sample size is 100.